Chapter 18 Energy Methods

\[ dW = F \cdot ds \]

Pure roll about \( A \), release from horizontal position

\( T_1 = 0 \)

\[ I_A = \frac{1}{3} mL^2 \]

\[ T_2 = \frac{1}{2} I_A \omega^2 \]

\[ U_{1/2} = 0 + mg \frac{L}{2} \]

\[ T_1 + U_{1/2} = T_2 \]

\[ 0 + mg \frac{L}{2} = \frac{1}{2} \cdot \frac{1}{3} mL^2 \omega^2 \]

\[ \omega^2 = \frac{3g}{L} \]

Updated mid-term estimates posted. (242)
Problem 18.4

The spool of cable, originally at rest, has a mass of 200 kg and a radius of gyration of \( k_G = 325 \) (mm). (a) If the spool rests on two small rollers A and B and a constant horizontal force of \( P = 400 \) N is applied to the end of the cable, determine the angular vel. at 8m. No friction.

\[
\begin{align*}
T_{\text{trans}} &= \frac{1}{2} m v_G^2 \\
T_{\text{rot}} &= \frac{1}{2} I_G \cdot \omega^2
\end{align*}
\]

\[
T_1 + U_{1\rightarrow 2} = T_2
\]

Work of \( P \): \[
\frac{1}{2} \cdot 400 \text{N} \cdot 8 \text{m} = \frac{1}{2} \cdot 200 \cdot \omega_2^2 \cdot 0.325^2 \]

\[
I_G = m k_G^2 = 200 \text{kg} \cdot 0.325^2 \text{m}^2
\]

solve for \( \omega_2 = 17.4 \text{ rad/s} \)
Problem 18.15
(a) If the system is released from rest, determine the speed of the 20kg cylinder A after A has moved downward a distance of 2m. The differential pulley has a mass of 15 kg with a radius of gyration about its center of mass of $k_O= 100$ mm.

\[ T = \frac{1}{2} I_O \omega^2 + \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 \]

\[ T_{disk} = \frac{1}{2} I_O \cdot \omega^2 = \frac{1}{2} \cdot 15 \cdot 0.1^2 \cdot \omega^2 = 0.075 \cdot \omega^2 \]

\[ T_A = \frac{1}{2} m_A v_A^2 = \frac{1}{2} \cdot 20 v_A^2 = 10 v_A^2 \]

\[ T_B = \frac{1}{2} \cdot 15 \cdot 1.0^2 \]

\[ T = T_D + T_A + T_B = 15.8 v_A^2 \]

for $S_A = 2m$:

\[ \omega = \frac{v_A}{r_A} = \frac{v_A}{0.15} = \frac{6.67 v_A}{0.15} = 6.7 \cdot v_A \]

\[ v_B = 0.5 v_A \]

Work:

\[ m_A g \cdot S_A = 20 \cdot 9.81 \cdot 2 = 392.4 \ J \]

\[ B: -m_B g \cdot S_B = -196.2 \ J \]
Problem 18.30

The 100-lb block is transported a short distance by using two cylindrical rollers, each having a weight of 35-lb. (a) If a horizontal force $P=25$ (lb) is applied to the block, determine the block's speed after it has been displaced 12 ft to the left. Start from rest.
Roller: $T_{\text{trans.}} + T_{\text{rot.}}$

$T_{\text{rot.}} = \frac{1}{2} I \cdot \omega^2 = \frac{1}{2} m_k r^2 \left( \frac{1}{r} \right)^2$

$= \frac{1}{2} m_k \cdot \frac{\omega^2}{r} - \frac{1}{2} m_k \left( \frac{v_b^2}{2} \right)$

$v_b = 5.05 \text{ ft/s}$

for Monday: Read Rest of Ch. 18.

$V = m g h + \frac{1}{2} k (x)^2$
18.5 Conservation of Energy

3rd Midterm

Topics:
- Ch. 17, 18
- Ch. 15.1–3 Impulse

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2 revs, \( r = 2 \cdot 2\pi \cdot r = 4 \cdot 0.3 \cdot 2\pi = 12\pi \)

Vertical drop \( h = 1.2\pi \text{ m} \cdot \sin 30^\circ \)

\[ T_1 = \frac{1}{2} m v_1^2 + \frac{1}{2} I_1 \omega_1^2 \]

\( k = 200 \text{ N/m} \)

\[ V_2 = \frac{1}{2} k \cdot (1.2\pi)^2 - 1.2\pi \cdot \sin 30^\circ - 3 \text{ m/s} \]

at Position 2

\[ \omega = \frac{v}{r} \Rightarrow \omega_0 = \frac{3}{0.3} = 10 \text{ rad/s} \]

Clockwise

Datum

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Problem 18.40
At the instant shown, the 50 lb bar rotates clockwise at \( 2 \text{ (rad)/s} \). The spring attached to its end always remains vertical due to the roller guide at C. (a) If the spring has an unstretched length of 2 ft and a stiffness of \( k = 6 \text{ (lb)/(ft)} \), determine the angular vel.

\[
T_1 + V_1 = T_2 + V_2
\]

\[
T_1 = \frac{1}{2} m v_1^2 + \frac{1}{2} I_6 \omega_1^2 + 0 = \frac{1}{2} m v_2^2 + \frac{1}{2} I_6 \cdot \omega_2^2
\]

substitute \( \omega = \frac{1}{2} \pi \)

\( + \frac{1}{2} k \cdot (1.2\pi)^2 - 1.2 \text{ ft} \cdot \text{min} \cdot 30^\circ \)

can solve for \( \omega_2 \)
Problem 18.45 •
The system consists of a 20-lb disk A, 4-lb slender rod BC, and a 1-lb smooth collar C. (a) If the disk rolls without slipping, determine the velocity of the collar at the instant the rod becomes horizontal, i.e., theta=0 degree(s). The system is released from rest at theta= 45 deg.

\[ V_1^0 + V_1 = T_2 + V_2 \]  \hspace{1cm} (1) 
\[ V_1 = 0 + 1\text{ lb} \cdot 3\text{ in} \cdot 45^\circ + 4\text{ lb} \cdot 1.5\text{ in} \cdot 45^\circ \]
\[ V_2 = 0 + 0 + 0 \]
\[ T_2 = \frac{1}{2} I_B \cdot \omega_B^2 + \frac{1}{2} m_C \cdot v_C^2 + \frac{1}{2} m_B \cdot v_B^2 \]  \hspace{1cm} (2) 2 constraint equations

Rekap: Define Datum, 0 and 2
Problem 18.51
The 30-kg pendulum has its mass center at G and a radius of gyration about point G of $k_G = 300$ mm. (a) If it is released from rest when theta=0 degree(s), determine its angular velocity at the instant theta = 90 degree(s). Spring AB has a stiffness of $k = 300$ N/m and is unstretched at theta = 0.