Chapter 17 Inertial Dynamics

Plane Motion
3 equations:
\[ \Sigma \text{Forces}_x \]
\[ \Sigma \text{Forces}_y \]
\[ \Sigma \text{Moments about } G \]

\[ \vec{r} \times \vec{F} = \vec{r} \times m \vec{a} \]

\[ I = \int r^2 \, dm \]
\[ I_G = \frac{1}{12} mL^2 \]
\[ I_{Ed} = I + mL^2 \left( \frac{1}{2} \right) \]
\[ = \left( \frac{1}{12} + \frac{1}{3} \right) mL^2 \]
\[ = \frac{1}{3} mL^2 \]
Problem 17.27
When the lifting mechanism is operating, the 400-lb load is given an upward acceleration of \( a = 5 \text{ ft/s}^2 \). (a) Determine the compressive force the load creates in each of the columns, AB and CD. Assume the columns only support an axial load.

**Translation:**
\[
\sum F_x = m \cdot \ddot{x}
\]
\[
\sum F_y = m \cdot \ddot{y}
\]

**Rotation:**
\[
\sum M_G = I_G \cdot \alpha
\]
Problem 17.29

The lift truck has a mass of 70 kg and mass center at G. (a) If it lifts the 120 kg spool with an acceleration of \(3 \text{ m/s}^2\), determine the reactions on each of the four wheels. The loading is symmetric. Neglect the mass of the spool.

\[\Sigma F_y = N_A + N_B - m_s g - m_t g = m_s a\]

\[\Sigma M_B = m_t g \cdot 0.5 + m_s g \cdot 0.7 - N_A \cdot 1.25 = -m_s g \cdot 0.7\]
Problem 17.53

The 50-kg uniform crate rests on the platform for which the coefficient of static friction is μ_s = 0.5. (a) If at the instant theta = 30 degrees the supporting links have an angular velocity \( \omega = 1 \text{ rad/s} \) and angular acceleration \( \alpha = 0.5 \text{ rad/s}^2 \), determine the frictional force on the crate.

\[
\mathbf{a} = (-\omega^2 \mathbf{r}) + \alpha \mathbf{r}
\]

We polar coords

\[
\mathbf{a} = (-1^2 \cdot 4) \mathbf{r} + 0.5 \cdot 4 \mathbf{r}
\]
Newton: 

\[ \Sigma F_x = F_D = 50.4 \sin 30^\circ + 50.2 \cos 30^\circ \]

\[ \Sigma F_y = N_D - 50 \cdot 9.81 = 50.4 \cos 30^\circ - 50.2 \sin 30^\circ \]

\[ F_D = 188.6 \, N \quad \max F_D \text{ for } \mu_s = 0.5 = \mu_s N_D \]

\[ N_D = 613.7 \, N \quad 0.5 \cdot 613.7 = 306.7 \, N \]

\[ \Sigma \tau_G \]

\[ \Sigma M = N_D \cdot x - F_D \cdot 0.75 = 0 \]

\[ x = 0.23 \, m < 0.25 \]
Chapter 17.4 Rotation about a Fixed Point

\[ F = \frac{mrv^2}{r} \]

\[ \vec{r} \times \vec{F} = 0 \]

\[ \alpha = \tan^{-1} \frac{1}{r} \]

\[ \vec{F} = 0 \]
\[ k \rightarrow 1 \Rightarrow 0 \]
\[ \frac{1}{k} \cdot \frac{1}{2} mL^2 \cdot 3 \ll( I_c + mL^2 ) \text{sph.} + \left( \frac{T}{-G_{Bar}} + mL^2 \right) \text{Rod} \]

\[ y = -r \vartheta \]
\[ \sum F = r \cdot F = I_0 \cdot \alpha \]
\[ \alpha = \frac{r \cdot F}{I_0} \]
\[ \sum F = F - Mg = M \ddot{y} = 40 \text{N} \]
\[ F = (Mg + 40 \vartheta) > 40 \]
\[ -rM(g + r \vartheta) = I_0 \cdot \alpha \]
\[ T = mg(y + \theta) \]
\[ \sum F_y = T - mg = m\ddot{y} \quad \theta \neq 0 \]
\[ T < T_{\text{static}} \]
Problem 17.69

The 150 kg wheel has a radius of gyration about its center of mass O of $k_O = 250$ mm. (a) If it rotates counterclockwise with an angular velocity of $\omega = 1200 \text{ (rev/min)}$ and the tensile force applied to the brake band at A is $T_A = 2000 \text{ N}$, determine the tensile force $T_B$.

The wheel stops after 50 revs.

\[ \omega = \frac{1200 \text{ revs}}{60 \text{ s}} = \frac{2\pi \text{ rad}}{1 \text{ rev}} = 40\pi \text{ rad/s} \]

\[ \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \]

\[ \theta = (40\pi)^2 + 2\alpha(100\pi - 0) \Rightarrow \alpha = -25.13 \text{ rad/s}^2 \]

\[ I_0 = mk^2 = 9.375 \text{ kg} \cdot \text{m}^2 \]

\[ \sum M(0) = 1 \cdot 0.3 - 2000 \cdot 0.3 = -9.375 \cdot 25.13 \]

Solve for $T_B = 1.21 \text{ kN}$.
Problem 17.73

The bar has a mass $m$ and length $l$. (a) If it is released from the position $\theta = 30^\circ$, determine its angular acceleration. (b) Determine the horizontal component of reaction at the pin $O$.

\[ MV(0) = I_0 \cdot \alpha \]

\[ \Rightarrow \alpha = \frac{1.3 \, g}{\ell} \]

\[ \Sigma F_x = Q_x = m \cdot \frac{l^2}{2} \cdot \sin \theta \cdot \frac{1.3 \, g}{\ell} \Rightarrow Q_x = 0.325 \, mg \]

\[ \Sigma F_y = Q_y - mg = -m \frac{l}{2} \cos 30^\circ \cdot \frac{1.3 \, g}{\ell} = 0.438 \, mg \]
Problem 17.83
At the instant shown, two forces act on the 30-lbf slender rod which is pinned at O. (a) Determine the magnitude of force \( F \) so that the horizontal reaction which the pin exerts on the rod is 5 lbf directed to the right. (b) Determine the initial angular of the rod so that the pin reaction = 5 lbf to the right.

\[
\sum \tau_{(0)} = I_0 \alpha
\]

\[
-20 \cdot 3 - F \cdot 6 = -19.88 \quad (1)
\]

\[
\sum F_x = 20 + F - 5 = \frac{30}{32.2} \cdot 4 \cdot \alpha \quad (2)
\]

\[
F = 30 \text{ lbf} \quad \alpha = 12.1 \text{ rad/s}^2
\]

Chapter 17.5 General Plane Motion

**Translation:** \( \sum F_x = m \cdot \ddot{x} \)

\[
\sum F_y = m \cdot \ddot{y}
\]

**Rotation:** \( \sum M_p = I_p \cdot \alpha \)

\[
\sum F_y = F - \mu_s \cdot N = m \cdot a_y
\]

\[
\alpha = \alpha_y \quad \gamma = \alpha \cdot r
\]
Plate

reaction in plate

any
Problem 17.103

The spool has a mass of \( m = 100 \text{ kg} \) and a radius of gyration of \( k_G = 0.3 \text{ m} \). (a) If the coefficients of static and kinetic friction at \( A \) are \( \mu_s = 0.2 \) and \( \mu_k = 0.15 \), respectively, determine the angular acceleration of the spool if \( P = 50 \text{ N} \).

\[
\text{Translation: } \sum F_x = m \dot{x} = P + 100 \text{ kg} \cdot g
\]
\[
\sum F_y = m \dot{y} = N_A - m \cdot g = 0 \Rightarrow N_A = 100 \cdot 9.81 \text{ N}
\]
\[
\sum M_G = I_G \alpha = -P \cdot 0.25 + F_A \cdot 0.4 = 100 \cdot 0.3^2
\]

\[
\alpha = 0.4 \cdot \alpha
\]
\[
\alpha = 1.3 \text{ rad/s}^2 \cdot k
\]

\[
N_A = 981 \text{ N}
\]
\[
\dot{\alpha} = 0.52 \frac{m}{s} \Rightarrow
\]
\[
F_A = 2 \text{ N} \ll F_{A,\text{max}} = 0.2 \cdot 981 \approx 196 \text{ N}
\]
Problem 17.112
The assembly consists of an 8-kg disk and a 10-kg bar which is pin connected to the disk. (a) If the system is released from rest, determine the angular acceleration of the disk. The coefficients of static and kinetic friction between the disk and...

No friction at B

Steps: 1. Frame

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Must be familiar with
FEA
CAD
specialized tools
Problem 17.120
(a) If the truck accelerates at a constant rate of 6 m/s\(^2\), starting from rest, determine the initial angular acceleration of the 20 kg ladder. The ladder can be considered as a uniform slender rod. The support at B is smooth.

\[ A_x \leq M = mg \cos 60^\circ = I \cdot \alpha \]

\[ A_y = I_A \cdot \frac{1}{2} \cdot \sin 60^\circ \cdot a \]

\[ 20 \cdot 9.81 \cdot 2 \cdot \cos 60^\circ = 20 \cdot a_{\min} \cdot 2 \cdot \sin 60^\circ \]

\[ a_{\min} = 5.66 \text{ m/s}^2 < 6 \text{ m/s}^2 \]