1. (20 points) Point A of linkage ABC moves downward vertically with constant velocity $v_A = 2 \text{ m/s}$. AB is coupled to rod BC, which is fixed at C. Determine, for AB at angle $\Theta = 30^\circ$ as shown:

(a) the angular velocity of rod AB.

(b) the velocity vector of point B of the linkage (i- and j-components)

(c) the coordinates of the instantaneous center of the rod AB at the angle $\Theta = 30^\circ$. Mark the instantaneous center location in the figure at left. All coordinate units are in meters. Use unit vectors i and j, and frame origin at C.

\[
\omega = \vec{\omega}_{AB} \times \vec{r}_{A/C} \\
0 \text{ } = -2 \text{ m/s} = 1 \text{ m} \cdot \omega_{AB} \Rightarrow \omega_{AB} = -2 \text{ rad/s}
\]

\[
\vec{v}_B = \vec{\omega}_{AB} \times \vec{r}_{B/C} \\
\vec{v}_B = -2 \frac{\text{rad}}{\text{s}} \cdot 1.732 \text{ m} = -3.464 \text{ i m/s}
\]
2. (20 Points) Arm AB rotates with a constant angular velocity of 10 rad/s clockwise. Gear A does not rotate. Determine
(a) the angular velocity of gear B
(b) the acceleration of gear B at its contact point, C, with fixed gear A in terms of the i-and j-coordinates shown at left.

(a) \( \omega_B = \frac{45}{3} = 15 \text{ rad/s} \) (clockwise)

(b) \( \vec{a}_C = \vec{a}_B + \vec{a}_{CB} \)

Only centripetal accelerations exist:

\[
\vec{a}_B = -4.5 \text{ m/s}^2 \begin{pmatrix} \cos 60^\circ \\ \sin 60^\circ \end{pmatrix}, \quad \vec{a}_{CB} = +3 \text{ m/s}^2 \begin{pmatrix} \cos 120^\circ \\ \sin 120^\circ \end{pmatrix}
\]

\[
\vec{a}_C = \vec{a}_B + \vec{a}_{CB} = \begin{pmatrix} -0.75 \\ 2.5 \end{pmatrix} \text{ m/s}^2
\]

Answer

\[
\omega_B = -15 \text{ rad/s} \quad \text{(Units)}
\]

\[
a_C = +225 \text{ m/s}^2 \quad \text{(Units)}
\]

3. (30 points) Masses A (3 kg) and B (2 kg) are released from rest as shown. Friction coeff.: \( \mu_k = 0.1 \) The rope length is constant. Using x- and y-coordinates at left, determine
(a) the acceleration of mass B.
(b) the velocity of block B after B has moved 2 meters in y-direction.

Mass A:

\[
\sum F_x = -T + \mu_k mg = m_A \ddot{x} = -m_A \ddot{y} \quad (1)
\]

\[
\ddot{y} = -\ddot{x}
\]

Mass B:

\[
\sum F_y = -T + m_B g \sin 60^\circ - \mu_k N_B = m_B \ddot{y} \quad (2)
\]

\[
\sum F_z = N_B = m_B g \cos 60^\circ = \frac{1}{2} m_B g \quad (3)
\]

Solving (1) for \( T \) and substituting \( T \) in (2) gives:

\[
\ddot{y} = \frac{mg \sin 60^\circ - \mu_k \frac{1}{2} m_B g + m_B g \sin 60^\circ}{m_A + m_B} \quad (4)
\]

Thus:

\[
\ddot{y} = \frac{9.81 (1.732 - 0.4)}{5}
\]

\[
\frac{0.2}{2} = \frac{a_B (x - x_0)}{x_0} \quad \Rightarrow \quad \dot{v}_B = 2.61 \text{ m/s}
\]

Answer

\[
a_B = \dot{v}_B = 2.61 \text{ m/s}^2
\]

\[
V_B(2m) = 3.23 \text{ m/s}
\]
4. (30 points) At the instant shown the end A of plunger AB has a velocity of 21.7 m/s to the left, and the angular velocity of plunger AB is $\omega_{AB} = -0.694$ rad/s (counterclockwise). The plunger speed at A is DECREASING at a rate of 0.25 m/s$^2$.

(a) Write the vector equation for the acceleration of contact point B, and graph schematically the acceleration at B as seen from points A and C, respectively. Clearly label each vector in the drawing with its corresponding expression in the vector equation (e.g. $r\times\omega^2$ etc).

(b) Determine the angular accelerations $\alpha_{AB}$ of the rod AB and $\alpha_{Gear}$ at this instant.

\[
\vec{a}_B = \vec{a}_{AB} + \vec{r}_{BA} \times \vec{\omega}_{AB} \times \vec{r}_{BA}
\]

Using $\sin \phi = \frac{5}{13}$ and $\cos \phi = \frac{12}{13}$ we get:

\[
5 \cdot \vec{\omega}_c \cdot \left(-\frac{12}{13} \hat{i} + \frac{5}{13} \hat{j}\right) + 5 \cdot \frac{4^2}{\left(\frac{5}{13}\right)^2} \left(-\frac{5}{13} \hat{i} - \frac{12}{13} \hat{j}\right) = -0.25 \hat{i} + \\
+ 12 \cdot \alpha_{AB} \left(-\frac{5}{13} \hat{i} - \frac{12}{13} \hat{j}\right) + 12 \cdot 0.694^2 \left(\frac{12}{13} \hat{i} - \frac{5}{13} \hat{j}\right)
\]

We have 2 equations for the unknown variables $\alpha$ and $\alpha_{AB}$:

\[
\begin{align*}
-60 \alpha_{gear} - 400 &= -3.25 - 60 \alpha_{AB} + 69.3 \\
25 \alpha_{gear} - 960 &= -144 \alpha_{AB} - 28.9
\end{align*}
\]

Simplifying:

\[
\alpha_{AB} - \alpha_{gear} = 7.76 \\
5.76 \alpha_{AB} + \alpha_{gear} = 37.24
\]

(If you assumed $a_A$ as 0.25 m/s$^2$ to the right, alpha values are -1.7 and -6.17.)

<table>
<thead>
<tr>
<th>Answer (b) $\alpha_{AB}$</th>
<th>+6.19 (rad/s$^2$)</th>
<th>C.W.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{Gear}$</td>
<td>-1.567 (rad/s$^2$)</td>
<td>C.C.W.</td>
</tr>
</tbody>
</table>