UNIVERSITY OF NEVADA, LAS VEGAS
DEPARTMENT OF MECHANICAL ENGINEERING

EGG 207, Spring 2000, **Final Examination**
Closed Book, three pages of handwritten notes allowed.

1. (10 points) The speed of a car is increased at a constant rate from 66 km/h to 90 km/h over a distance of 180 m along a curve with 240-m radius. Determine the magnitude of the total acceleration after the car has traveled 100 m along the curve.

\[ V_1 = 66 \cdot 10^2 \text{ m/s} \]
\[ \frac{3.6 \cdot 10^2}{2} = 18.3 \text{ m/s} \]
\[ V_2 = 25 \text{ m/s} \]

\[ a_0 = 0.2 \frac{100}{\text{m}} \cdot \frac{1}{2} (V_2^2 - V_1^2) = a_t \cdot 100 \text{ m} \]
\[ V_2^2 = V_1^2 + 2 \cdot 80 = 49.6 \cdot \frac{m^2}{s^2} \]
\[ V_1 = 22.3 \text{ m/s} \]

\[ a = \frac{a_t}{2} \cdot \frac{100}{180 \text{ m/s}^2} = 0.8 \frac{m}{s^2} \]

\[ \vec{a} = (a_t^2 + a_n^2) = 0.8 \frac{m}{s^2} + \left( \frac{V_2^2}{r^2} \right) = \left( \frac{25^2}{240} \right) = 2.2 \frac{m}{s^2} \]

**Answer:** \( a_{\text{total}} = 2.2 \) (\( \frac{m}{s^2} \) units)

2. (15 points) The two blocks with masses \( m_A = 8 \text{ kg} \) and \( m_B = 5 \text{ kg} \) are initially at rest. Neglecting the masses of the pulleys and all friction, determine \( a) \) the acceleration of block A

\( b) \) the velocity of block A immediately before hitting the floor at \( y = 0 \).

\[ B: T - m_B \cdot g = m_B \cdot \ddot{y}_B \quad (1) \]
\[ Y_B = Y_A = Y \quad (3) \]

\[ A: m_A g - T = m_A \cdot \ddot{y}_A \quad (2) \]

\( \text{Solve (1) for } T \text{ and insert into (2):} \)

\[ m_A g - m_B (\ddot{y} + g) = m_A \cdot \ddot{y} \]

\[ q (m_A - m_B) = (m_A + m_B) \ddot{y} \]

\[ \dot{v}_A = a \Delta x \]
\[ \frac{1}{2} (V_1^2 - V_0^2) = 2.26 \frac{m}{s^2} \cdot 1 \text{ m} \]

**Answer:**

\( a = 2.26 \) (\( \frac{m}{s^2} \) units)

\( v_A (y=0) = 2.13 \) (\( \frac{m}{s} \) units)
3. (20 points) A package (point mass) with mass $m$ is projected into a vertical return loop with radius $r$ at velocity $v_0$. Determine the smallest allowable velocity so that the package can reach the horizontal surface at C. No friction.

At Q: $\frac{v^2}{r} \geq g$ for staying on track.
Thus: $v_x^2 = rg$; only gravity does work.

$T_1 + U_{1\rightarrow 2} = T_2$

$U_{1\rightarrow 2} = -2rgm$

$\frac{1}{2} m v^2 - 2rgm = \frac{1}{2} m v_x^2$

$m v^2 - 2rg \cdot 2 = m v_x^2 = m v_0^2$

$v_0^2 = 5rg$

$v_0 = \sqrt{5rg}$

4. (20 points) The flywheel ($r = 0.1$ m) shown is rotating about C at constant $\omega_b = 50$ rad/s. Pin P slides in the slot of arm AB. Point A is fixed. AC = 0.3 m.

Determine the angular velocity of arm AB as $\Theta = 90$ degrees.

Unit vectors

Geometry:

$l = \sqrt{0.1^2 + 0.3^2} = 0.316$m

$\phi = \tan^{-1} \frac{0.1}{0.3} = 18.43^\circ$

Pin P: $\vec{v}_P = \vec{v}_A + \omega_{AB} \times r_{PA} + \vec{v}_{rel}$

Using $e_r - e_t$ frame attached to AP:

$\vec{v}_{rel} = \omega_{AB} \cdot l$

$\omega_{AB} \cdot l = 5 \sin 90^\circ \omega_{AB} = \frac{5}{0.316} \text{ m/s}$

Answer $\omega_{Ab} = \frac{5}{0.316}$ rad/s
5. (15 points) Roller C moves up the channel at constant velocity \( v_C = 2 \text{ m/s} \). For the instant shown at left, determine 
(a) the instantaneous center of motion of bar BC and its distance from point B. (construct the instantaneous center in the graph, and label it clearly)
(b) the angular velocity of bar AB

\[
\begin{align*}
\text{Using inst. center:} \\
U_C = 2 \frac{\text{m}}{\text{s}} = 0.12 \text{m} \cdot \omega_{BC} \\
\Rightarrow \omega_{BC} = \frac{2}{0.12} = 16.67 \text{ m/s} \\
V_B = r \cdot \omega_{BC} = 2 \text{ m/s}
\end{align*}
\]

\[
\begin{align*}
\omega_{AB} &= \frac{V_B}{r_{BA}} = \frac{2}{0.08} = 25 \text{ rad/s}
\end{align*}
\]

(a) Inst. Center : construct in graph, Distance Ctr - B = 0.12 ( m units)

(b) \( \omega_{AB} = -25 \text{ k rad/s} \)

6. (20 points) Gear D is stationary. Gear C has mass = 5 kg and radius of gyration = 75 mm. Bar AB has no mass. As the system is released from rest, determine the angular acceleration of gear C.

\[
\begin{align*}
\sum \tau &= -mg r = I_c \cdot \alpha_c \\
I_c &= m (k^2 + r^2) \\
\alpha_c &= -\frac{mg r}{k^2 + r^2} = \frac{-9.8 \cdot 0.1 \text{ m} \cdot \text{m}}{0.0156 \text{ m}^2} \\
\alpha_c &= -62.78 \text{ rad/s}^2 \text{ (clockwise)}
\end{align*}
\]

\( \alpha_c = 62.78 \text{ rad/s}^2 \)