ROOT LOCUS CONSTRUCTION RULES

Rule #1: The number of root locus branches is equal to the order of the characteristic equation. Each branch of the root locus begins at an open-loop pole (K = 0) and ends at an open-loop zero or at a zero at infinity (K → ∞). The root locus is always symmetric with respect to the real axis.

Rule #2 (Real axis RL): For K > 0, the root locus lies on a section of the real axis if the number of finite poles and zeros to the right of the section is odd.

Rule #3 (Imaginary axis crossings): If branches of the root locus cross the imaginary axis, the locations of the crossings, jω = jω₁, and the values of the gain K at the crossing points can be found by using the Routh array. The value of K at each crossing will be the value that makes an entire row of the Routh array equal to zero. The crossing points jω₁ will be the roots of the auxiliary equation using that value of K.

An alternate method of finding the values of K and ω₁ is to form the closed-loop characteristic equation Char_eq(s) = Den(s) + K*Num(s) = 0. The variable s is replaced by jω, and the resulting expression is separated into its real and imaginary parts. At the imaginary axis crossing of the closed-loop pole, the real and imaginary parts of ∆CL(jω) must each be zero:
Re(Char_eq(jω₁)) = 0 and Im(Char_eq(jω₁)) = 0. The two equations can be solved for K and ω₁.

Rule #4 (Asymptotes): There will be n - m branches of the Root Locus as K → ∞. For large K, they the root locus branches going to infinity will follow asymptotes that meet at a common point on the real axis, and form specified angles with respect to the positive real axis. The angles of asymptotes, φₐ, and the center of asymptotes, σₐ, are given by

\[
φₐ = \frac{(2r+1) \pi}{(# p_i) - (# z_i)} \quad \text{and} \quad σₐ = \frac{\sum (p_i) - \sum (z_i)}{( # p_i) - (# z_i)}
\]

where pᵢ and zᵢ are the open-loop pole and zero locations, respectively. Complex poles and zeros are included in the calculation of σₐ.

Rule #5 (Breakaway Points): For K > 0, the root locus breaks away from the real axis at points of relative maximum K and re-enters the real axis at points of relative minimum K. i.e., breakaway and re-entry occur at points sₜ where

\[
\frac{dK}{ds} \bigg|_{s=sₜ} = 0
\]

See your textbook book for additional rules.