Genetic Tuning of Fuzzy Logic Controller for a Flexible-Link Manipulator

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Overview

- Fuzzy control presents a practical alternative to conventional controllers that is both computationally efficient and robust.
- While fuzzy logic control has been successfully implemented in many applications, several questions, concerning it, remain however unanswered.
- For example, design of fuzzy controllers usually depends on intuition or experience, which works well if the designer is familiar with the system. Fuzzy controller for flexible manipulators does not fit well within this category.
- While the rules for such controllers may be intuitive, selecting membership functions for input and output variables of the manipulator can be challenging. Wrong choices may lead to sluggish response, excessive vibration, or instability.
- This paper uses a distributed controller composed of two PD-like fuzzy logic controllers to control angle and link vibrations of this manipulator. It presents an approach for tuning the membership functions of the variables of these controllers using a genetic algorithm.
- The results of these controllers are analyzed to simplify the structure of controllers.
**Literature Review**

- A control law that consists of a fuzzy logic controller plus a nonlinear effects negotiator for a flexible manipulator arm was derived in [Lin, Y. and Lee, T., 1993]. The nonlinear effects negotiator was used to enhance the controller's ability in dealing with the model’s uncertainties. An error response plane method was proposed to obtain the fuzzy control rules. The optimal overlap of membership functions that results in the best performance of the manipulator was estimated.

- A fuzzy control strategy to control the rigid body and the first flexural mode of vibration in a single link robotic arm by designing a separate controller for each was described in [Kubica, E. and Wang, D., 1993].

- Controllers for manipulators with flexible links were implemented by using a class of fuzzy inference systems, [Arciniegas et al., 1993], where fuzzy inference systems were shown structurally similar to those of neural networks consisting of a collection of locally tuned units. This observation was used to generate the fuzzy rules function by a method similar to the problem of efficiently placing the centers of the receptive fields of units in neural networks.

- A fuzzy model reference learning controller for a manipulator with flexible links was developed in [Moudgal et al., 1995]. The proposed approach in that paper can synthesize a rule-base for a fuzzy controller and tune it to adapt to variations in the payload.

- Fuzzy logic was used to suppress vibration of a flexible arm, [Yoo et al., 1995] using a linguistic model of the system.

- A new structure for a distributed fuzzy logic controller and a method for tuning the membership functions of this controller while reducing its dimensionality were proposed in [Trabia, 1998].
**Objectives**
This paper is addressing several issues related to the design of a fuzzy logic controller for a flexible-link manipulator. The objective of this controller is to drive the manipulator through a desired trajectory without exciting vibration. The design of a fuzzy logic controller deals with these issues:

a. Identifying the variables of the controller.
b. Identifying the structure of the controller.
c. Choosing fuzzy inference rules that the controller uses.
d. Estimating the number of the membership functions that describe each of its variables.
e. Determining the shape and location of each of these membership functions.
f. Evaluating the performance of the controller to determine if any of the above elements, such as the number of membership functions that describe a variable, should be modified.
A Structure for a PD-Like Distributed Fuzzy Controller

- Fuzzy logic control has an intuitive nature, which may work well in controlling simple systems.
- It may be difficult to design a fuzzy logic controller for flexible manipulators, Figure 1, whose equations of motion are coupled.
- A controller that combines both joint angle and tip displacement, as well as their time derivatives, in a single controller will have large number of rules that may be difficult to determine. It will be also computationally intensive.

Figure 1 Schematic of a Flexible Manipulator
• To fully exploit the potential of fuzzy logic, the control action is distributed between two fuzzy controllers: joint angle controller and tip controller, Figure 2.
• Both controllers use the error as input. Error is defined as the difference between the desired value of a variable and its actual one. The output of both controllers is torque.
• Joint angle controller has two inputs: joint angular displacement and velocity errors, $e_\theta$ and $e_{d\theta}$ respectively, and one output: the torque needed to correct these errors: $T_\theta$.
• Similarly, the tip controller has two inputs: tip displacement and velocity errors, $e_{\text{tip}}$ and $e_{d\text{tip}}$ respectively, and one output: the torque needed to correct these errors: $T_{\text{tip}}$. The sum of the output of these two controllers is fed to the manipulator.

Figure 2 Structure of Distributed PD-Like Fuzzy Logic Controller for a Flexible Manipulator
Membership Functions

- The question of how many membership functions are sufficient to describe a fuzzy variable is difficult to readily answer. In this paper, an initial guess of seven membership functions is used to describe each of the six variables:
  - negative big (NB),
  - negative medium (NM),
  - negative small (NS),
  - zero (Z),
  - positive small (PS),
  - positive medium (PM), and
  - positive big (PB).
- Subsequent sections in this paper will help determine if this number is sufficient, excessive, or inadequate.
- Discussion at this paper is limited to Gaussian curve membership functions whose form is,
  \[ \mu(z, \sigma, c) = e^{-(z-c)^2 / 2\sigma^2} \] (1)
Gaussian curve membership function has the advantage of being described using only two parameters:
  - \( c \), the center of the function and,
  - \( \sigma \), curve shape parameter.
- Selecting the membership functions and ranges of variables is not as intuitive as determining the rules of the controllers.
- Details of the proposed algorithms for selecting membership functions and ranges of variables for optimal performance are shown in the next sections.
Fuzzy Rules

- The rules for both controllers are based on intuition and observations of inertial system.
- The goal of the joint angle fuzzy controller is to maintain the manipulator along a desired trajectory. Its inputs are selected to produce output similar to that of a typical PD controller. The rules of this controller, Table 1, are selected to avoid overshoot or lagging with respect to the desired joint trajectory.

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<th>$e_\theta$</th>
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Table 1 Rules for the Joint Angle Fuzzy Controller
• On the other hand, the rules of the tip fuzzy controller, Table 2, are selected such that if the tip is approaching the correct position or if velocity error belongs to the zero function, the controller will produce no torque. These rules attempt to use the strain energy of the link to dampen the arm vibration. This controller produces torque only when the tip is moving away from the desired target position.

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<tr>
<th>$e_{tip}$</th>
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Tuning of a Distributed Fuzzy Controller Using Genetic Algorithm

- The performance of a fuzzy controller depends on the range of its input and output variables and shape of the membership functions that describe this function.
- An automated method to tune the membership functions of a fuzzy controller is presented.
- The algorithm chooses the ranges and the shapes of the membership functions of its variables using genetic programming.
- This algorithm reduces the need to having a required prior knowledge of the controlled system that is currently necessary for designing a fuzzy controller.
- The performance index chosen for the flexible manipulator is,

\[
PI = \sum_{i=1}^{nt} \left( e_{\theta_i}^2 + e_{d\theta_i}^2 + \left( \frac{e_{q_{\theta i}}}{L} \right)^2 + \left( \frac{e_{dq_{\theta i}}}{L} \right)^2 \right) + \\
\sum_{i=2}^{nt} \left( e_{d\theta_i} - e_{d\theta_{i-1}} \right)^2 + \left( \frac{e_{dq_{\theta i}}}{L} - \frac{e_{dq_{\theta_{i-1}}}}{L} \right)^2
\]  

(2)

where, \( nt \) is the total sampling time divided by sampling interval.
- The first term in performance index equation represents a measure for the manipulator errors while the second term represents a measure for the manipulator vibration.
In the absence of gravity, it is fair to assume that membership functions are symmetrical. The problem is then modeled as having forty-two variables (genes) that correspond to the shapes ($Z$, $PS$, $PM$, and $PB$) and centers of the membership functions ($PS$, $PM$, and $PB$) of: $e_{\theta}$, $e_{d\theta}$, $T_{\theta}$, $e_{tip}$, $e_{dip}$, and $T_{tip}$ respectively.

- Each variable is represented by real numbers.
- The problem is subject to spacing constraints in the form of:

$$g \frac{n_{mr} - 1}{2} + j \frac{n_{mr} + 1}{2} = c_{k,j} - c_{k,j-1} > 0$$

where

$$\frac{n_{mr} + 1}{2} + 1 \leq j \leq n_{mr}$$

$$1 \leq k \leq 7$$

These constraints ensure that the center of $PS$ ($PM$) membership function lies to the left of that of $PM$ ($PB$) membership function can be incorporated in the performance index using the bracket function. The modified function is,

$$\text{minimize, } FC = PI + \sum_{i=1}^{m} \Omega_{i}$$

$$\Omega_{i} = R \ast g_{i}(x)^{2} \quad \text{if } g_{i} \leq 0$$

$$\Omega_{i} = 0 \quad \text{if } g_{i} > 0$$

where, $R$ is a penalty term of a higher magnitude than the value of PI.
**Genetic Algorithm**

- The initial population of 200 chromosomes is randomly generated.
- The algorithm selects *fifty* percent of the population with the best fitness value as parents, as well as members, of next generation.
- The rest of the new population is generated by crossing over two randomly chosen parents using the *weighted average operator*:
  \[
  v'_{i} = a v_{i} + (1 - a) v_{j}
  \]
  \[
  v'_{j} = a v_{j} + (1 - a) v_{i}
  \]

  where, \( a \) is a randomly generated number from \([0, 1]\).
- A mutation rate of *0.01* is selected. At each generation, the number of mutated strings is equal to,
  \[
  Mutate\_Number = Mutation\_rate \times Population\_size \times Number\_of\_strings
  \]

  The positions of the mutated strings are included in an array of random integer numbers that are selected from the array:
  \([1, 2, \ldots, Population\_size \times Number\_of\_strings]\).
  The values of these strings are randomly generated.
- The process continues for a maximum number of 200 generations.
• Tuning the fuzzy logic controller can be represented by the block diagram of Figure 3. The system has three blocks:
  (1) **Plant**: The system that will be controlled. It receives controller inputs and produces sensors outputs.
  (2) **Fuzzy Controller Trainer**: This trainer uses a genetic algorithm to evaluate the system performance index. It suggests modifications of the membership functions to minimize this performance index.
  (3) **Fuzzy Controller**: Fuzzy controller produces the inputs for the plant.

Figure 3 Block Diagram of the Algorithm used for Tuning Membership Functions of Fuzzy Controllers
Simulation Example

• A computer program has been developed to simulate the dynamics of a single-link flexible manipulator.
• The physical parameters and mechanical properties of the manipulator used in this simulation are listed in Table 3.
• Four elements of equal length are used to model the beam.
• The initial angle of the manipulator is equal to zero while desired final angle is equal to one radian.
• The number of samples, $n_t$, is equal to one thousand samples over the simulation period of ten seconds.

Table 3 Physical Parameters and Mechanical Properties of the Simulated Flexible Manipulator

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Link length, $L$</td>
<td>1.0 m</td>
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<tr>
<td>Density, $\rho$</td>
<td>0.1 kg/m</td>
</tr>
<tr>
<td>Bending stiffness, $EI$</td>
<td>2.0 Nm$^2$</td>
</tr>
<tr>
<td>Moment of inertia of the hub, $J_m$</td>
<td>0.05 Kgm$^2$</td>
</tr>
<tr>
<td>Radius of the hub, $L_0$</td>
<td>0.01 m</td>
</tr>
<tr>
<td>Tip mass, $m_t$</td>
<td>1.0 kg</td>
</tr>
<tr>
<td>Tip mass moment of inertia, $J_t$</td>
<td>$10^{-5}$ Kgm$^2$</td>
</tr>
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Results of Tuning the Flexible Manipulator Fuzzy Controller Using GA

- The figures show the membership functions for of the member with the lowest value of the performance index at the final generation.
- The memberships functions have significant amount of overlap.
- The results show an overshoot and steady state error of 0.0116 radians for $e_\theta$. Tip vibration disappears after six seconds.
- The torque curve is smooth.
Figure 4 Joint Angle Response for the GA Controllers

Figure 5 Tip Point Displacement Response for the GA Controllers

Figure 6 Applied Torque for the GA Controllers
Figure 7 Membership Functions of $e_\theta$ of the GA Joint Angle Fuzzy Controller

Figure 8 Membership Functions of $e_d\theta$ of the GA Joint Angle Fuzzy Controller

Figure 9 Membership Functions of $T_\theta$ of the GA Joint Angle Fuzzy Controller
Figure 10 Membership Functions of $e_{\text{tip}}$ of the GA Tip Vibration Fuzzy Controller

Figure 11 Membership Functions of $e_{\text{dip}}$ of the GA Tip Vibration Fuzzy Controller

Figure 12 Membership Functions of $T_{\text{tip}}$ of the GA Tip Vibration Fuzzy Controller
Reducing the GA Fuzzy Controller Observations:

- Membership function $Z$ of $e_{tip}$ overlaps the other functions completely. The value of this function is very close to one for the whole range of $e_{tip}$. This membership function effectively neutralizes the effect of $e_{tip}$ on the tip fuzzy controller.

- This controller was therefore reduced by changing it to a single input, $e_{dtip}$, single output, $T_{tip}$, controller.

- The rules of the reduced controller are listed in Table 4. Implementing these changes,

- The reduced controller effectively eliminates the steady state error while maintaining the tip vibration and the joint torque as before.

Table 4 Rules for the Reduced Tip Fuzzy Controller

<table>
<thead>
<tr>
<th>$e_{dtip}$</th>
<th>PB</th>
<th>PM</th>
<th>PS</th>
<th>Z</th>
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<tr>
<td>$T_{tip}$</td>
<td>NB</td>
<td>NM</td>
<td>NS</td>
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Conclusions

- Successful implementation of fuzzy logic control depends on the availability of knowledge of the controlled system. For complex systems, such as flexible manipulators, this knowledge may be lacking, which limits the scope of using fuzzy controllers.
- This paper uses a distributed controller composed of two fuzzy logic controllers to control joint angle and tip deflection independently.
- Separate controllers are used since simple rules can be proposed for each controller when its action is considered independently, based on observation of the dynamics of the sub-system.
- The outputs of these two controllers are combined and used as the commanded torque to the manipulator.
- Genetic algorithm is used to tune the performance of the controller by varying the ranges and shapes of the membership functions since it is difficult to estimate the appropriate values of these variables for every flexible manipulator.
- The method is satisfactorily applied to a single-link flexible manipulator.
- The results show that the initial number of membership functions used to describe some of the variables can be reduced from what was originally proposed.
- This reduction can produce a simpler and faster controller than the original without significant difference in the performance.
- The procedures presented in this paper can be applied to other systems that are difficult to characterize.
- Using hybrid genetic algorithms to this type of problems is currently under consideration.