Chapter 3
Synthesis of Mechanisms (Design)

Objective:

To design a mechanism to achieve Kinematic (goals)

Syntesis of 4-bar mechanisms

Problem:

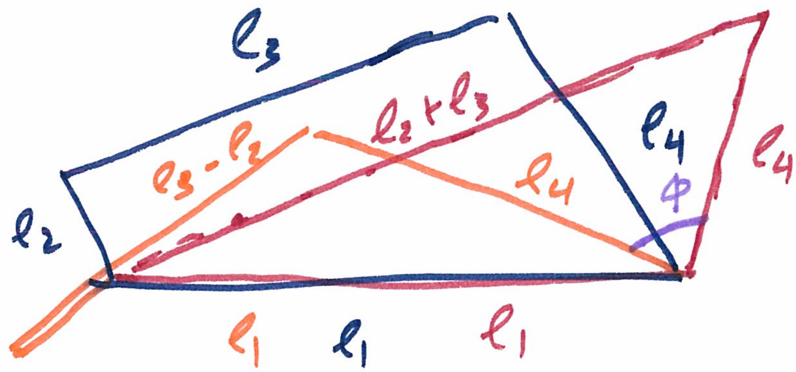
Design a 4-bar mechanism with a given oscillation angle (ϕ)

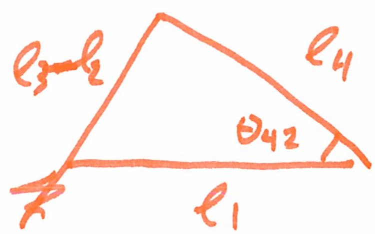
→ oscillation → crank-rocker mechanism

typically input is a motor rotating continuously

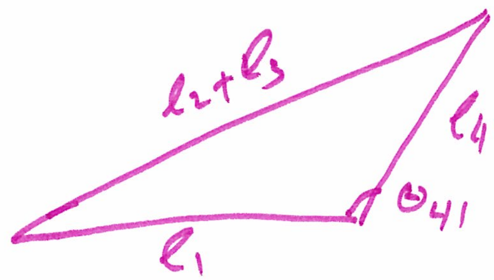
→ CR → satisfies Grashof criteria

shortest is the crank





ELP



ERP

$$\phi = \theta_{41} - \theta_{42}$$

ELP $\quad (l_3 - l_2)^2 = l_1^2 + l_4^2 - 2\, l_1 l_4 \cos \theta_{42}$

ERP $\quad (l_2 + l_3)^2 = l_1^2 + l_4^2 - 2\, l_1 l_4 \cos \theta_{41}$

3 equations

6 unknowns $l_1 \quad l_2 \quad l_3 \quad l_4 \quad \theta_{41} \quad \theta_{42}$

Solution

Provide θ_{42} ℓ_1 ℓ_4

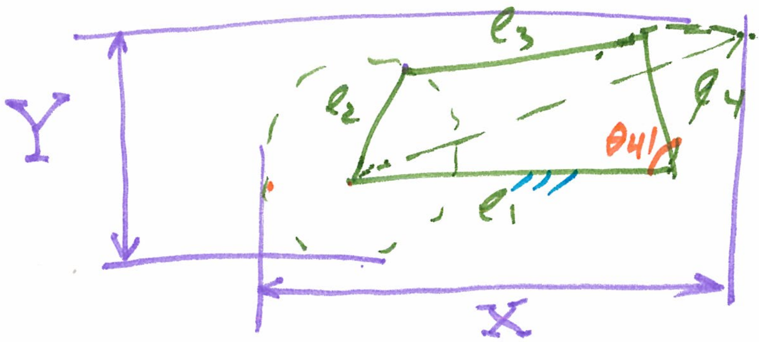
$$\theta_{41} = \varphi + \theta_{42}$$

$$\begin{cases} \ell_3 - \ell_2 = \left[\ell_1^2 + \ell_4^2 - 2\ell_1 \ell_4 \cos\theta_{42} \right]^{1/2} \\ \ell_2 + \ell_3 = \left[\ell_1^2 + \ell_4^2 - 2\ell_1 \ell_4 \cos(\varphi + \theta_{42}) \right]^{1/2} \end{cases}$$

solve ℓ_2 & ℓ_3

Additional Constraint

The mechanism should fit X by Y space



space is function of motion of ℓ_2 & ERP

$$X = \ell_2 + \ell_1 + \ell_4 \cos(180 - \theta_{41})$$

$$Y = \ell_2 + \ell_4$$

If the mechanism is bigger than the space, scale all links down to fit

Example:

Given an oscillation angle of 45°, design a 4-bar mechanism that produces this oscillation and can fit a 2"x 1" rectangle

Given $\varphi = 45°$

Let $\theta_{42} = 45°$ $\quad l_1 = 1"$ $\quad l_4 = \frac{3}{4}"$

$$\theta_{41} = \varphi + \theta_{42} = 90°$$

$$l_3 - l_2 = \left((1)^2 + (.75)^2 - 2(1)(.75)\cos(45°) \right)^{1/2}$$

$$l_3 + l_2 = \left((1)^2 + (.75)^2 - 2(1)(.75)\cos(90°) \right)^{1/2}$$

$$l_3 - l_2 = \cancel{1.23}\ .71$$

$$l_3 + l_2 = \cancel{1.23}\ 1.25$$

$$l_3 = \frac{\cancel{1.73}\,.71 + 1.25}{2} = \frac{\cancel{1}.98}{2} = \cancel{1.49}\ 0.99"$$

$$0.99 + l_2 = 1.25 \longrightarrow l_2 = 0.26"$$

check : Grashof

$$\begin{array}{ccc} l_1 + l_2 & & l_3 + l_4 \\ 1 + .26 & < & 0.99 + .75 \end{array} \longrightarrow \text{shortest is the crank}$$

CR

Check space need for unobstructed motion of the mechanism

$$X = l_2 + l_1 + l_4 \cos(\cancel{\theta_4}\,180 - \theta_{41})$$

$$Y = l_2 + l_4$$

$$X = 0.26 + 1 + 0.75)\cos(180 - 90)$$

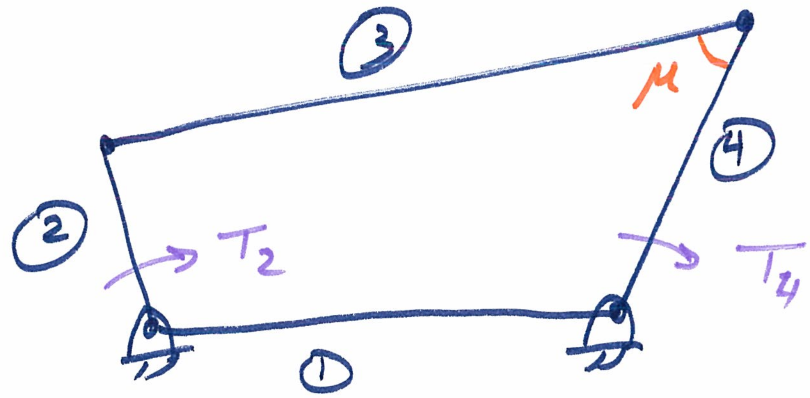
$$= 1.26''$$

$$Y = 0.26 + .75 = 1.01''$$

scale down by $\frac{1}{1.01}$ ratio

$\boxed{\text{All links}}$

Transmission Angle of 4-bar Mechanism



$\frac{T_4}{T_2}$ is function of μ

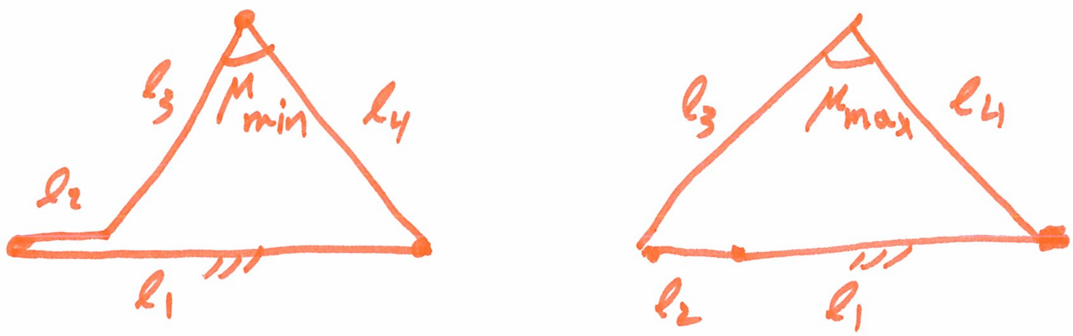
$\downarrow$

$\sin(\mu)$

- Typically, we would like to design machines with smooth output torque
- We would like μ to have reduce change range over the cycle of motion
- We would also like to ~~be~~ have μ around 90°

The general recommendation is to have $45^\circ \le \mu \le 135^\circ$

- Our task is to identify μ_{min} & μ_{max}



$$(l_1 - l_2)^2 = l_3^2 + l_4^2 - 2\, l_3\, l_4 \cos(\mu_{min})$$

$$(l_1 + l_2)^2 = l_3^2 + l_4^2 - 2\, l_3\, l_4 \cos(\mu_{max})$$

* Objective is to compare μ_{min} & μ_{max} to the 45° & 135°
* Redesign if either is outside this range

Example: check μ range for the previous example

$l_1 = 1''$ $\quad l_2 = 0.26''$ $\quad l_3 = 0.99''$ $\quad l_4 = 0.7$

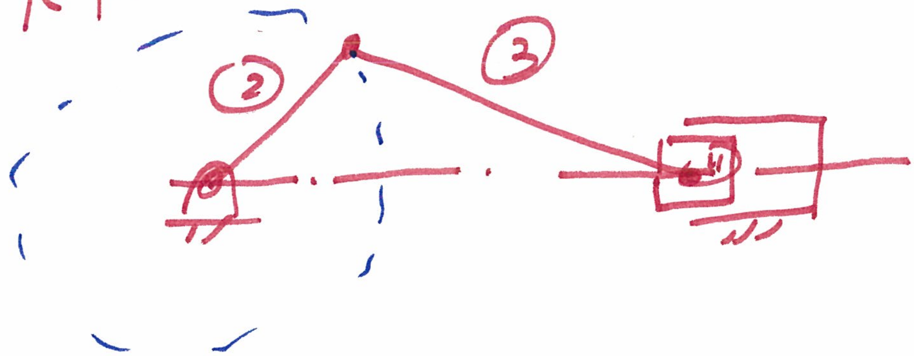
$$\mu_{min} = \cos^{-1}\left(\frac{l_3^2 + l_4^2 - (l_1 - l_2)^2}{2\, l_3\, l_4}\right)$$

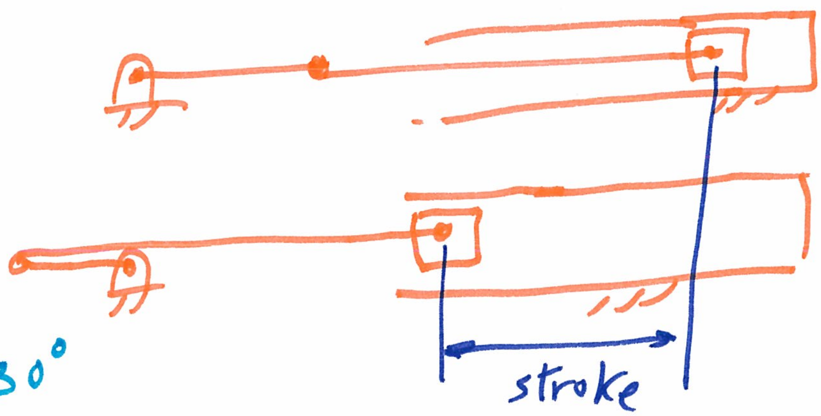
$$\mu_{max} = \cos^{-1}\left(\frac{l_3^2 + l_4^2 - (l_1 + l_2)^2}{2\, l_3\, l_4}\right)$$

$$\left.\begin{array}{l} \mu_{min} = 47.7^\circ \\ \mu_{max} = 93.5^\circ \end{array}\right\} \text{OK}$$

Crank-slider Mechanism

RRRP





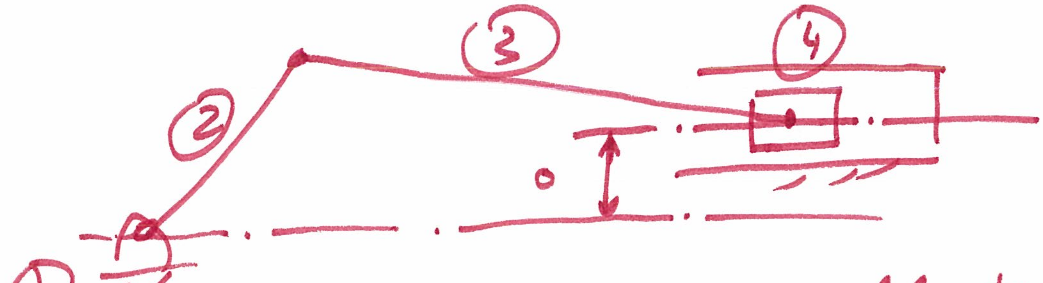
difference in crank ~~position~~ angles at extreme positions $= 180°$

$$\text{stroke} = 2 l_2$$

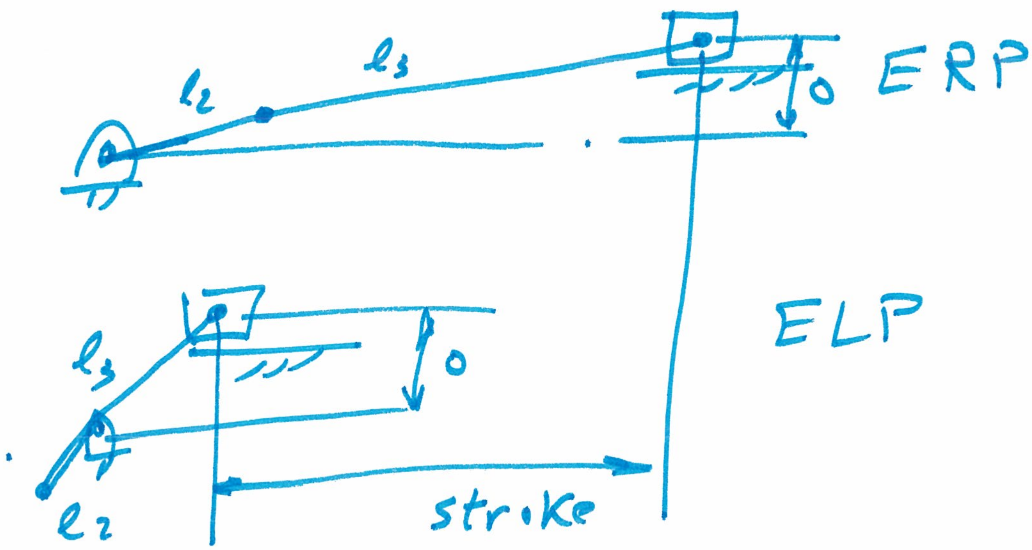
$$l_3 > l_2$$

$l_3 = l_2 \longrightarrow$ jamming

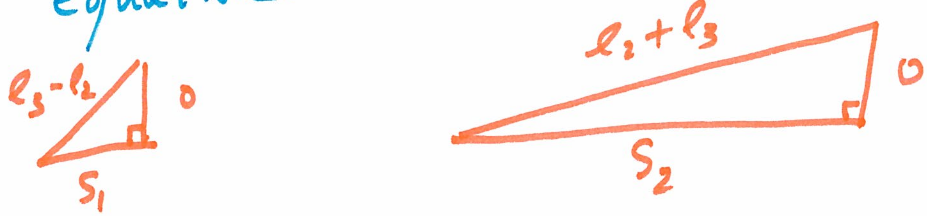
Offset Crank-Slider Mechanism



o: offset



stroke equation



$$\text{stroke} = s_2 - s_1$$

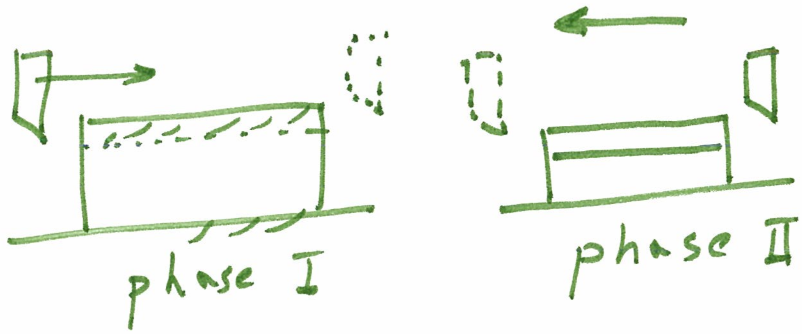
$$= \sqrt{(l_2 + l_3)^2 - o^2} - \sqrt{(l_3 - l_2)^2 - o^2}$$

Design of Quick-Return Mechanisms

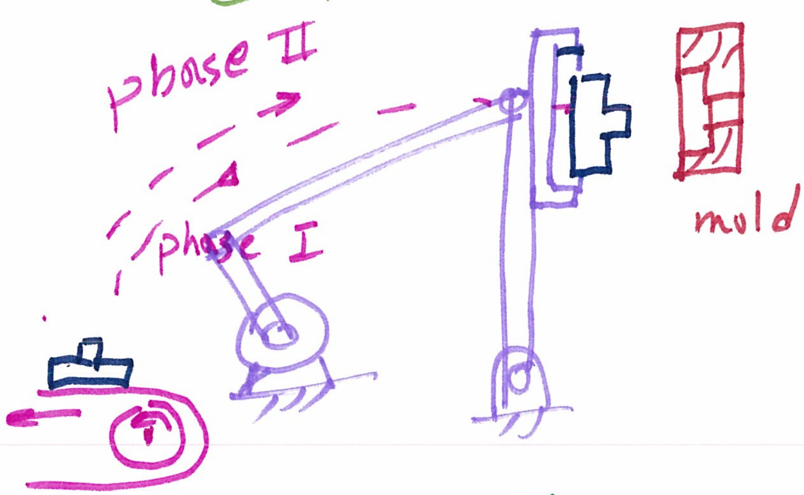
In many applications, the objective is to do a task then return to the original configuration

Examples

* Milling



* Moving product on assembly line



In all these examples, there are two phases:

1. Power stroke (task is performed)
2. Return stroke (go back to original configurations)

* To increase productivity,

return stroke duration

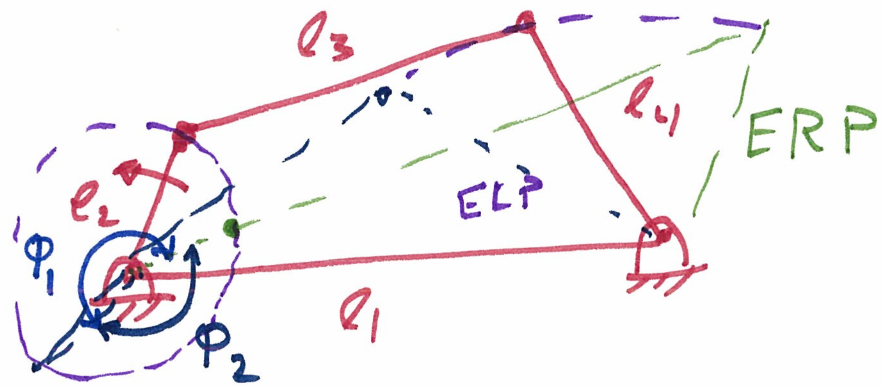
<

power stroke duration

If input is running at a uniform velocity, can we synthesize a mechanism to achieve this objective?

4-Bar Mechanism as a Quick-Return Mechanism (QRM)

↳ discussion is limited to CR



Introduce time ratio (TR)

ERP → ELP: φ_1

ELP → ERP: φ_2

$$TR = \frac{\varphi_1}{\varphi_2}$$

Special case: $\varphi_1 = \varphi_2 \rightarrow TR = 1$

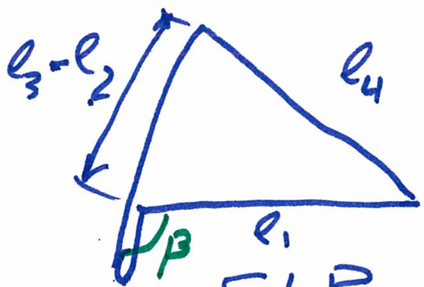
$\boxed{\text{NOT}}$ a QRM

$$\varphi_1 + \varphi_2 = 360^\circ$$

$$TR = \frac{360 - \varphi_2}{\varphi_2}$$

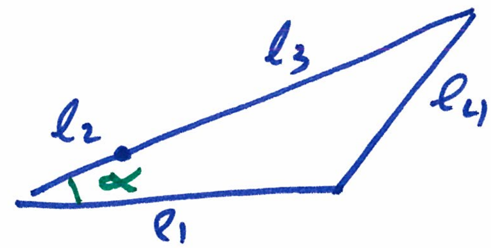
Design Problem:

~~Given~~ Design a 4-bar mechanism with a specific time ratio



ELP

$\varphi_2 = \alpha + \beta$



ERP

ERP $\longrightarrow$ $l_4^2 = l_1^2 + (l_2 + l_3)^2 - 2\, l_1 (l_2 + l_3) \cos\alpha$

ELP $\longrightarrow$ $l_4^2 = l_1^2 + (l_3 - l_2)^2 - 2\, l_1 (l_3 - l_2) \cos(180 - \beta)$

2 equations

5 unknowns l_1 l_2 l_3 l_4 α $\Big\}$ provide 3 variab.

if TR is known φ_2 ✓

$\hookrightarrow$ α and β are related

Example:

Design a 4-bar mechanism with $TR = 2$

$$TR = \frac{360 - \varphi_2}{\varphi_2}$$

$$2 = \frac{360 - \varphi_2}{\varphi_2} \longrightarrow 3\varphi_2 = 360$$

$$\varphi_2 = 120^\circ$$

$$\varphi_2 = \alpha + \beta$$

let $\beta = 90^\circ$ $\qquad \alpha = 30^\circ$

$$l_4^2 = l_1^2 + (l_2 + l_3)^2 - 2 l_1 (l_2 + l_3) \cos 30$$

$$l_4^2 = l_1^2 + (l_3 - l_2)^2 - 2 l_1 (l_3 - l_2) \cos(180 - 90)$$

provide l_2 & l_3 $\longrightarrow$ solve for: l_4 l_1

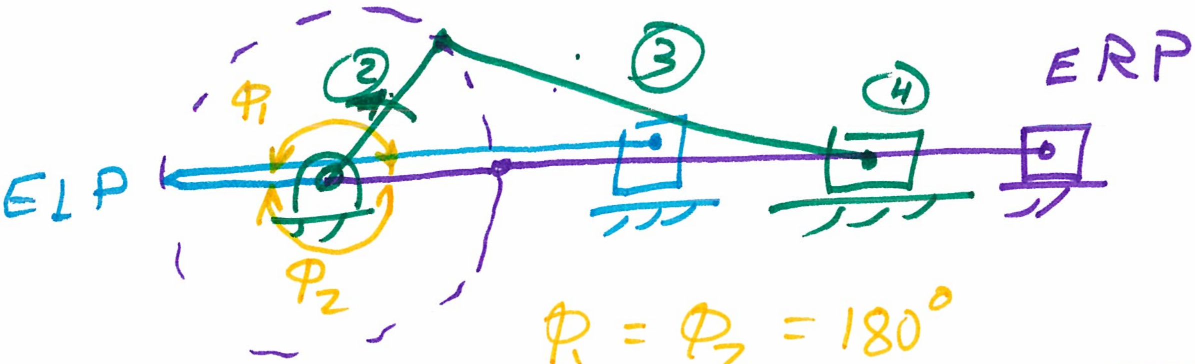
Special case if TR=1

$$TR = \frac{360-\varphi_2}{\varphi_2} = 1$$

$$2\varphi_2 = \cancel{180}\ 360$$

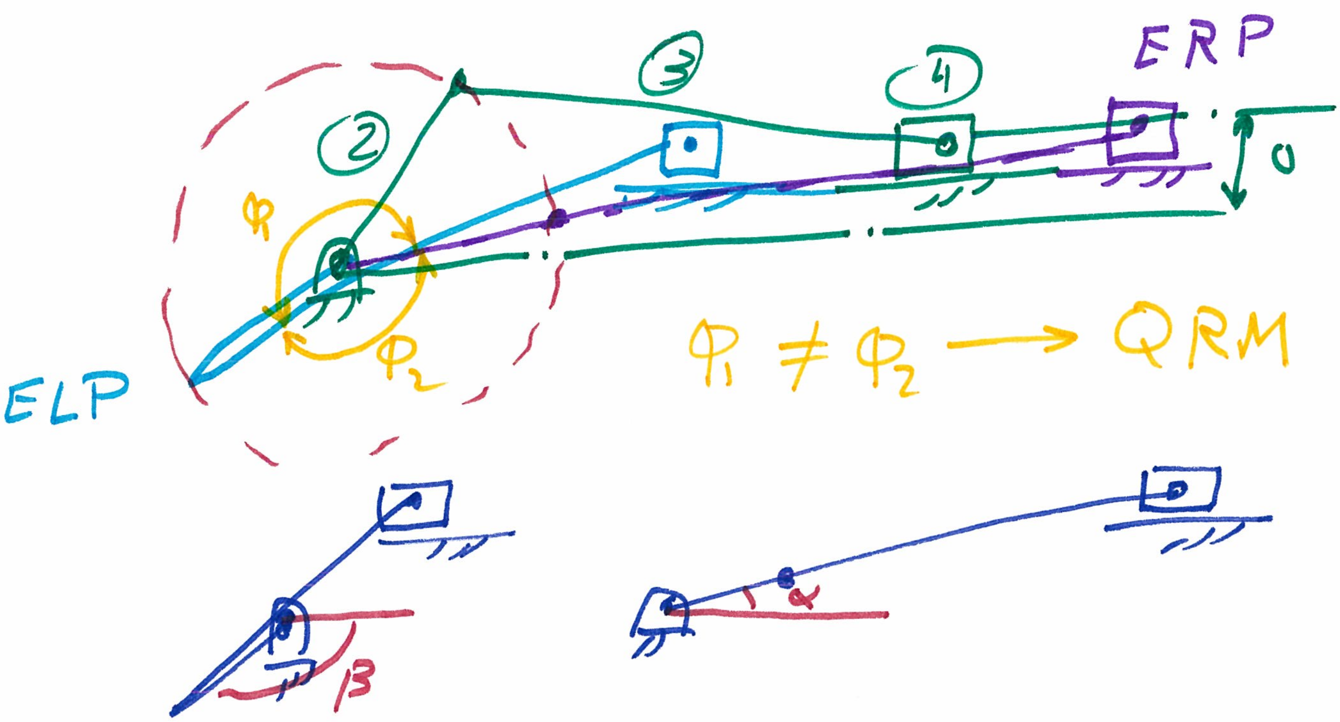
$$\varphi_2 = 180^\circ$$

Crank-Slider as a QRM



$\varphi_1 = \varphi_2 = 180°$

TR=1 → $\boxed{\text{NOT}}$ a QRM



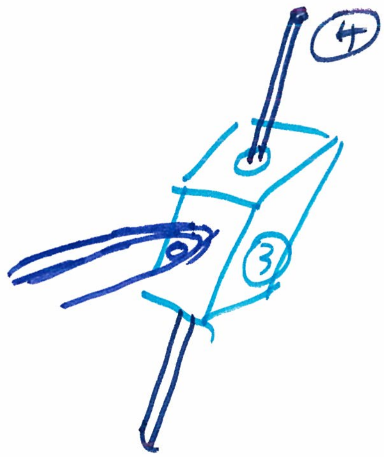
$\varphi_1 \neq \varphi_2$ → QRM

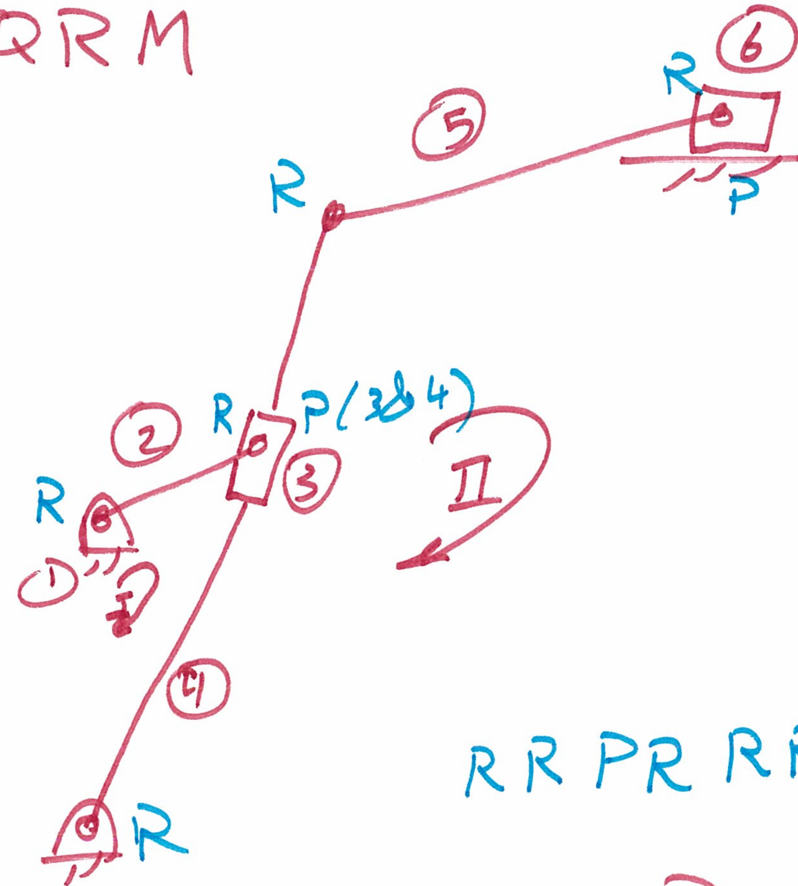
$\boxed{\text{HW}}$ Design a QRM using offset crank-slider with the following specifications:

TR=2

stroke=4"

Shaper QRM

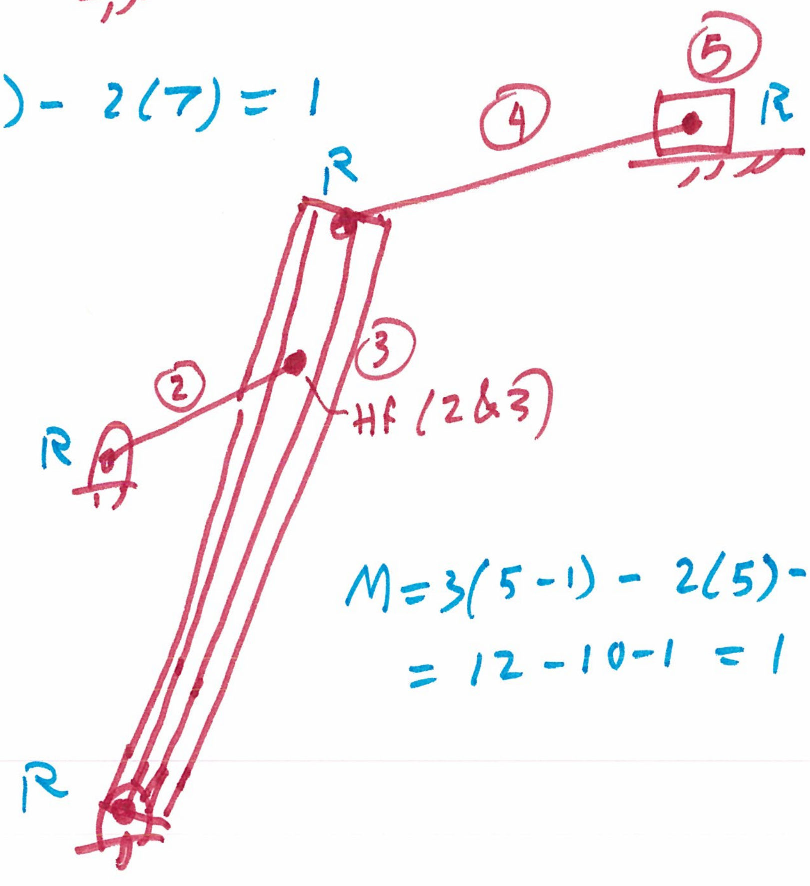




RR PR RR P

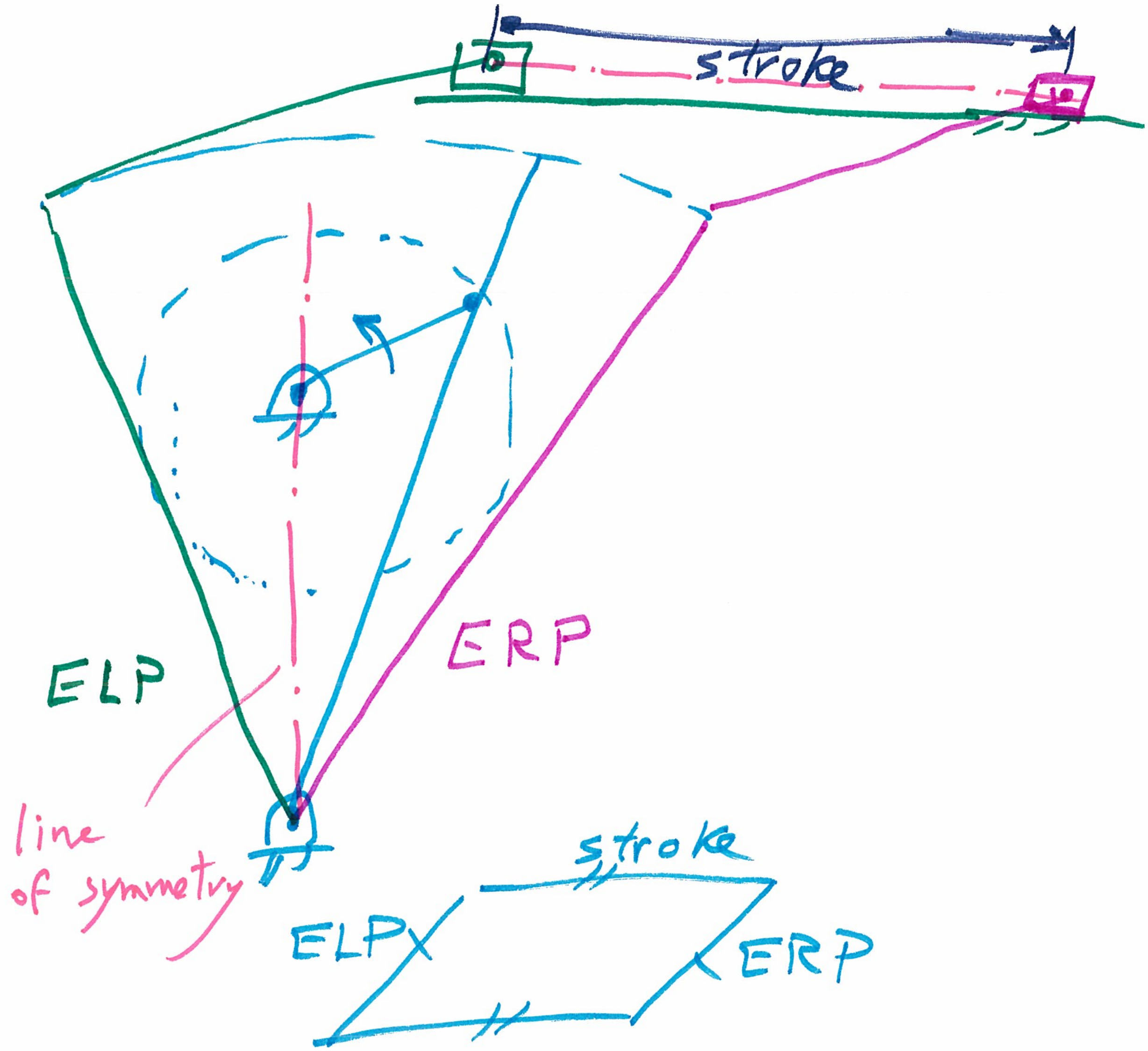
$M = 3(6-1) - 2(7) = 1$

$\equiv$



$M = 3(5-1) - 2(5) - 1$

$= 12 - 10 - 1 = 1$



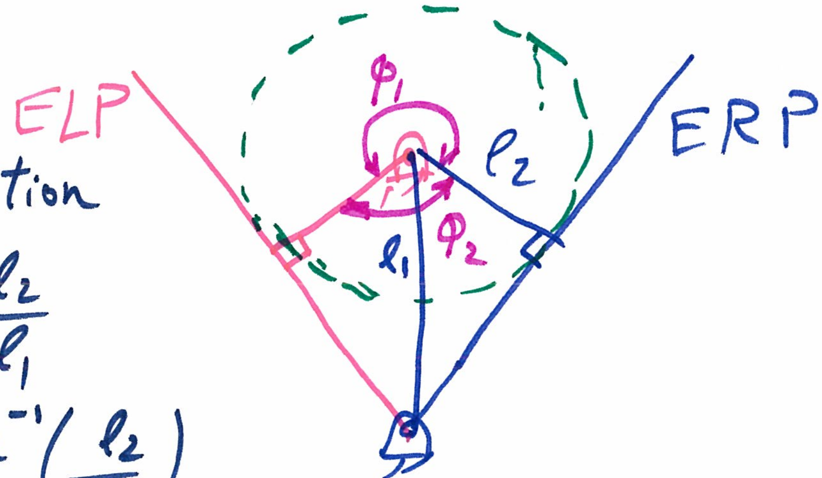
stroke
ELP
ERP
line of symmetry
stroke
ELP
ERP

Objective use this mechanism as a QRM

Design to produce specific time ratio (TR) and a stroke

start with the time ratio

Identify ERP & ELP



by observation

$$\cos\left(\frac{\varphi_2}{2}\right) = \frac{\ell_2}{\ell_1}$$

$$\varphi_2 = 2\cos^{-1}\left(\frac{\ell_2}{\ell_1}\right)$$

$$TR = \frac{\varphi_1}{\varphi_2} = \frac{360 - \varphi_2}{\varphi_2}$$

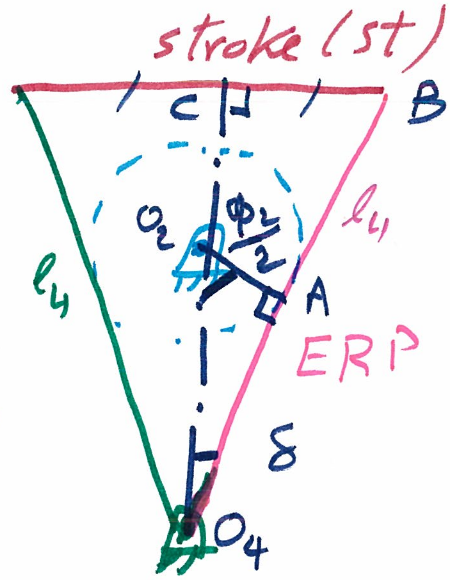
Example Desig a shaper QRM with time ratio = 2

$$2 = \frac{360 - \varphi_2}{\varphi_2} \longrightarrow \varphi_2 = 120^\circ$$

$$\cos\left(\frac{120}{2}\right) = \frac{\ell_2}{\ell_1} \longrightarrow 0.5 = \frac{\ell_2}{\ell_1} \rightarrow 1 \text{ eqn } 2 \text{ unk.}$$

Let $\ell_2 = 1''$, $\ell_1 = 2''$

Stroke Calculations



From $\Delta\, O_2 O_4 A$

$$\delta = 90 - \frac{\phi_2}{2}$$

From $\Delta\, O_4 C B$

$$\frac{st}{2} = l_4 \sin \delta$$

$$st = 2\, l_4 \sin\left(90 - \frac{\phi_2}{2}\right)$$

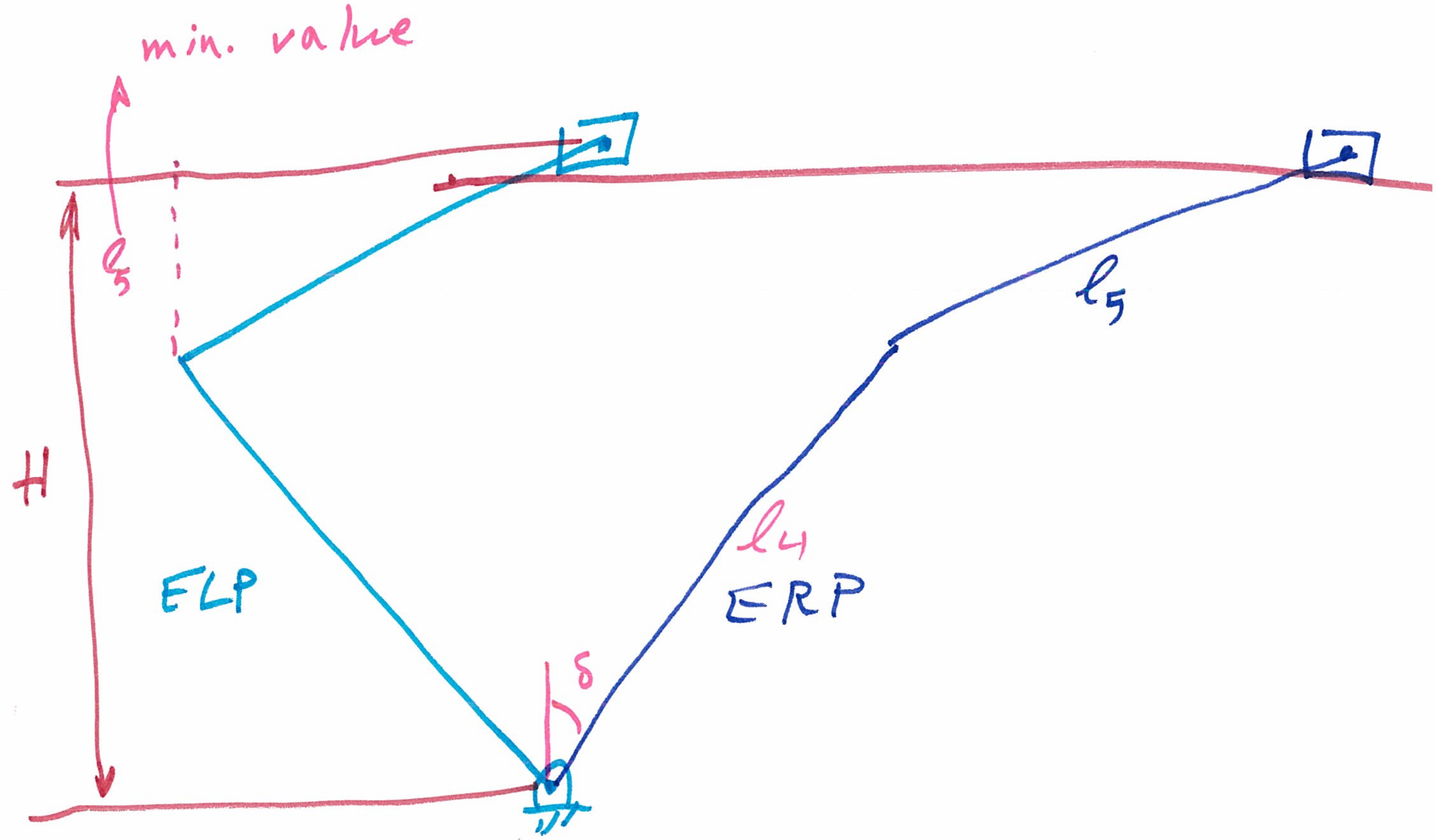
Continue the previous example!
Design for a stroke = 4"

$$st = 2\, l_4 \sin\left(90 - \frac{\phi_2}{2}\right)$$

$$4 = 2\, l_4 \sin\left(90 - \frac{120}{2}\right)$$

$$4 = 2\, l_4 \sin 30$$

$$l_4 = \frac{2}{\sin 30} = 4''$$



Assembly condition

$$l_4 \cos\delta + l_5 > H$$

$$l_4 \cos\left(90 - \frac{\phi_2}{2}\right) + l_5 > H$$

Finish the example

Let $H = 6''$

$$l_4 \cos\left(90 - \frac{\phi_2}{2}\right) + l_5 > H$$

$$4 \cos\left(90 - \frac{120}{2}\right) + l_5 > 6$$

$$l_5 > 2.8$$

Let $l_5 = 4''$

QRM Design Procedures

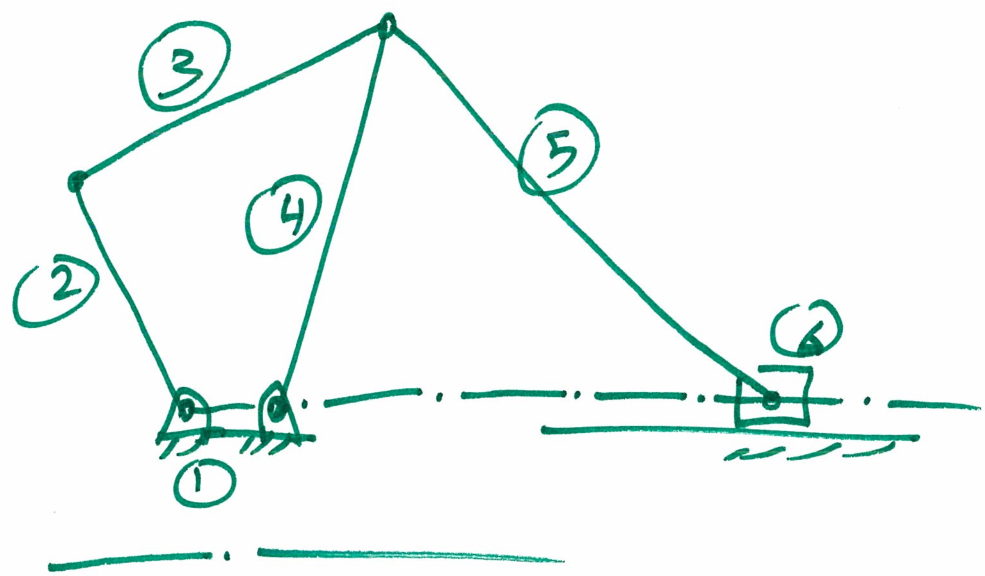
1. Start by identifying extreme right & left positions
2. Use these positions to define the angles of the crank (input link)
3. Find expressions relating ϕ_1 & ϕ_2 to ERP & ELP configurations
4. Consider the output motion which can be oscillation or translati
   and relate the output to the extreme positions

HW

Design a crank-crank crank-slider QRM with the following specifications.

Time ratio = 2

Stroke = 4"



HW

Simulate the above machine using Working Model 2D