Chapter 10

Dynamics Fundamentals

Objective: Create equivalent dynamics models for machines

Equations; $F = ma$ OR $T = I\alpha$

force

Resultant of all forces acting on the center of mass

mass

acceleration $= \frac{dv}{dt}$

linear system

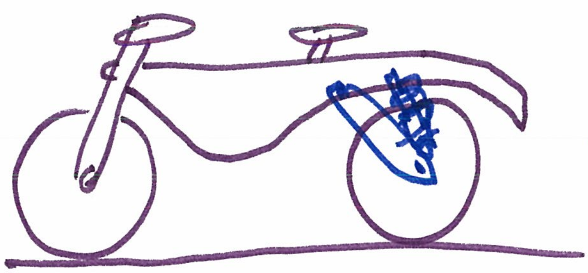
torque

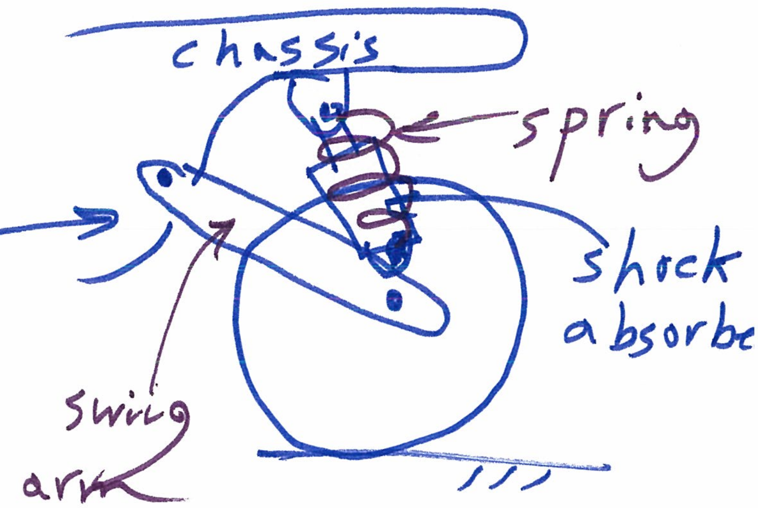
mass moment of inertia

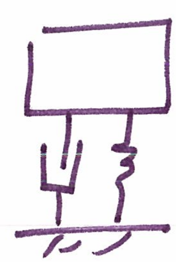
angular acceleration $= \frac{d\omega}{dt}$

rotational system

Dynamic Model







single degree of freedom equivalent system

* The objective is to create an equivalent dynamic model for the physical system.

* The model is not necessarily a 100% accurate representation of the physical system.

* It should be able to capture its overall performance.

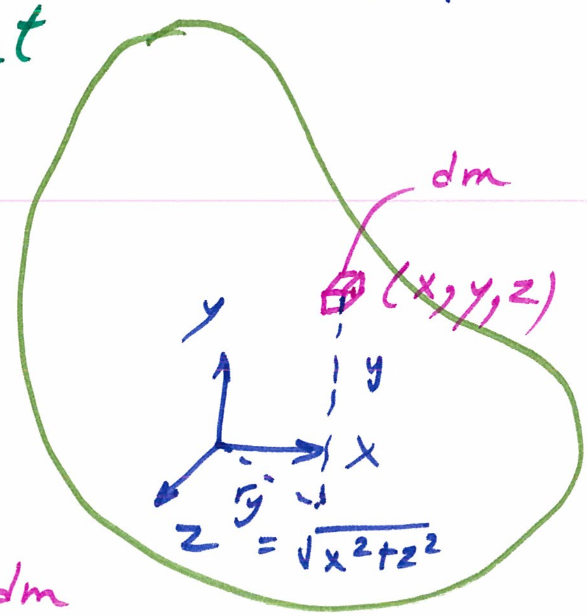
Conditions for creating an equivalent ~~SDS~~ Dynamic Model

1. Same mass
2. Same center of mass (center of gravity)
3. Same mass moment of inertia.

Center of mass

How to find the coordinates of the center of mass ??

* Use the moment of mass method



$dM_y = r_y \, dm = \sqrt{x^2+z^2} \, dm$

↓

moment of mass around the y-axis

Similarly,

$dM_x = r_x \, dm = \sqrt{y^2+z^2} \, dm$

$dM_z = r_z \, dm = \sqrt{x^2+y^2} \, dm$

by integration,

$$M_y = \int_{vol} \sqrt{z^2+x^2} \, dm$$

$$M_x = \int_{vol} \sqrt{y^2+z^2} \, dm$$

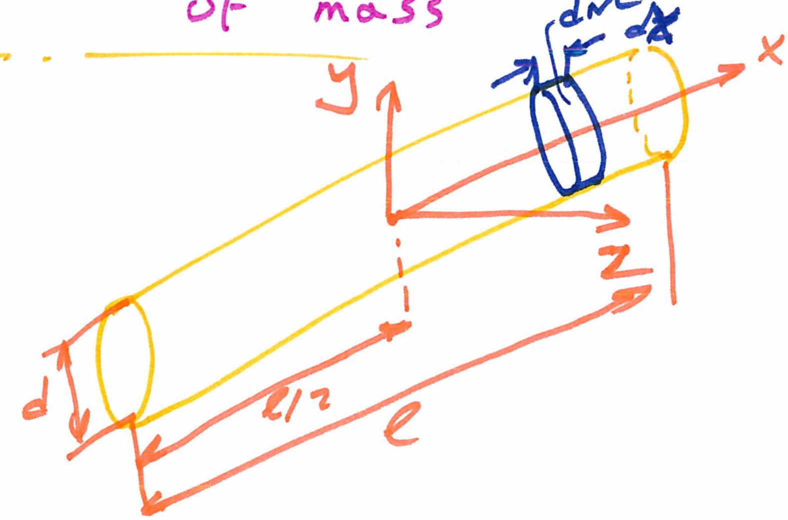
$$M_z = \int_{vol.} \sqrt{x^2+y^2} \, dm$$

If $M_s = 0 \longrightarrow$ the axis passes through the center of mass

$\downarrow$

$s = x, y, \text{or } z$

Example



M_z ?

$$M_z = \int_{vol} \sqrt{x^2+y^2}\, dm$$

For a cylinder

$$dm = \rho \frac{\pi d^2}{4} d\cancel{x}$$

$$M_z = \int_{vol.} \sqrt{x^2+0} \left(\rho \frac{\pi d^2}{4} d\cancel{x} \right)$$

$$= \int_{-\frac{l}{2}}^{l/2} x \left(\rho \frac{\pi d^2}{4} dx \right)$$

$$= \frac{\rho \pi d^2}{4} \left(\frac{x^2}{2} \right)_{-l/2}^{l/2}$$

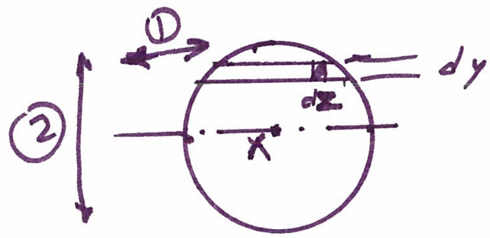
$$= \frac{\rho \pi d^2}{4} \left(\frac{l^2}{8} - \frac{l^2}{8} \right) = 0 \longrightarrow$$ z-axis passes through the center of mass

Continue with the same example.

M_x ?

$$M_x = \int_{vol} \sqrt{y^2+z^2}\, dm$$

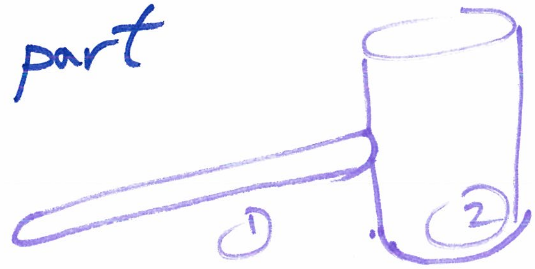
$$= \int_{-\frac{d}{2}}^{d/2} \int_{-\sqrt{\left(\frac{d}{2}\right)^2 - y^2}}^{\sqrt{\left(\frac{d}{2}\right)^2 - y^2}} \sqrt{y^2+z^2}\, (\rho\, dy\, dz\, \ell)$$



$$= \rho \ell \int_{-\frac{d}{2}}^{d/2} \left(\int_{-\sqrt{\frac{d^2}{4} - y^2}}^{\sqrt{\frac{d^2}{4} - y^2}} \sqrt{y^2+z^2}\, dz \right) dy$$

Center of mass of a compound object

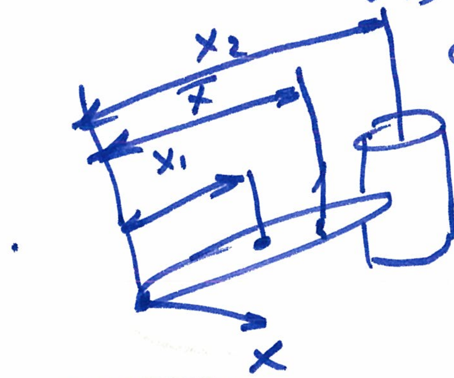
1. Find the center of mass of each part



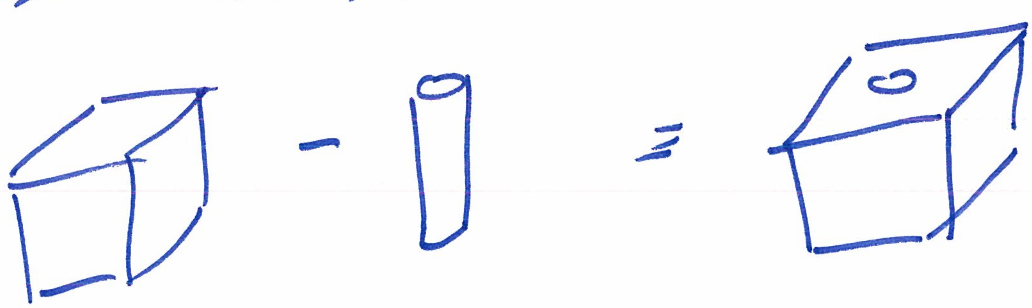
2. Take the moment of mass about a point and axis to find the location of the center of mass of the whole object

$$\Sigma(m_i x_i) = \Sigma(m_i)\bar{X}$$

$\bar{X}$ → distance to center of mass



Use with subtraction as well



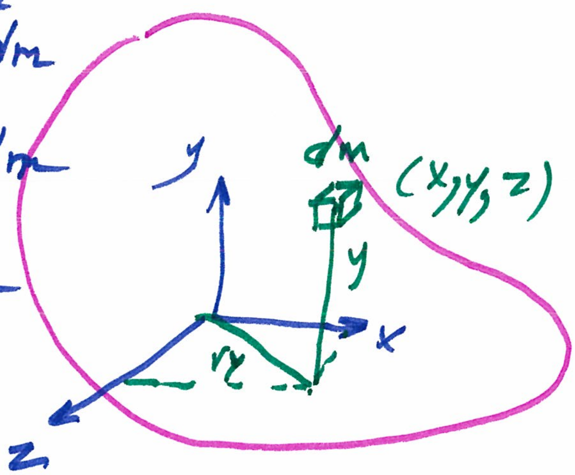
~~HW 10.3~~

Mass Moment of Inertia

$$dI_y = r_y^2\,dm = \left(\sqrt{z^2+x^2}\right)^2 dm$$

$$dI_x = r_x^2\,dm = \left(\sqrt{y^2+z^2}\right)^2 dm$$

$$dI_z = r_z^2\,dm = \left(\sqrt{x^2+y^2}\right)^2 dm$$



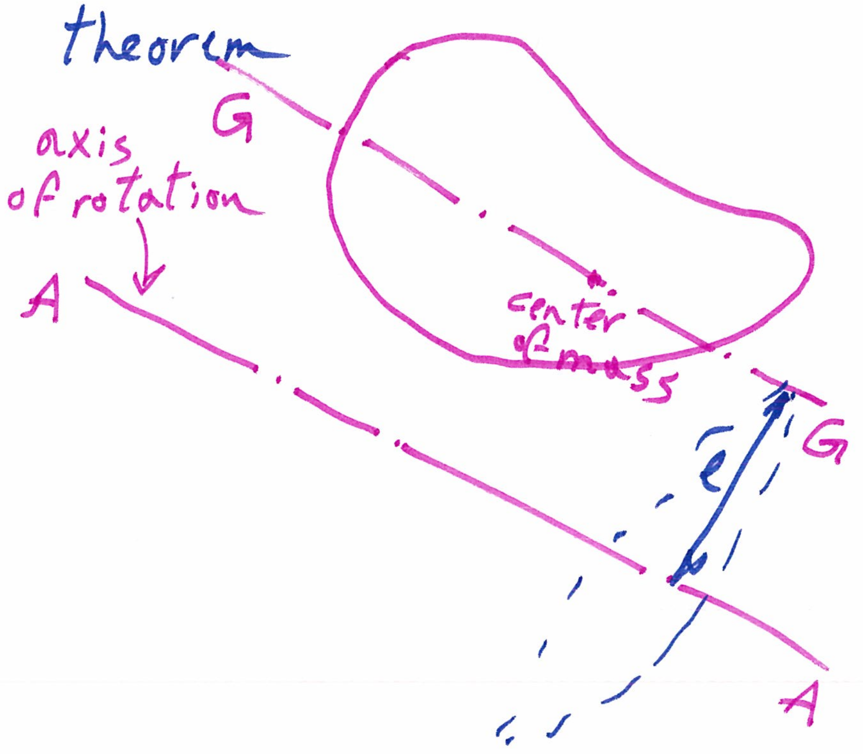
$$I_y = \int_{vol} (z^2+x^2)\,dm$$

$$I_x = \int_{vol} (y^2+z^2)\,dm$$

$$I_z = \int_{vol} (x^2+y^2)\,dm$$

Parallel axis theorem

$$I_{AA} = I_{GG} + m\ell^2$$



Kinetic Energy

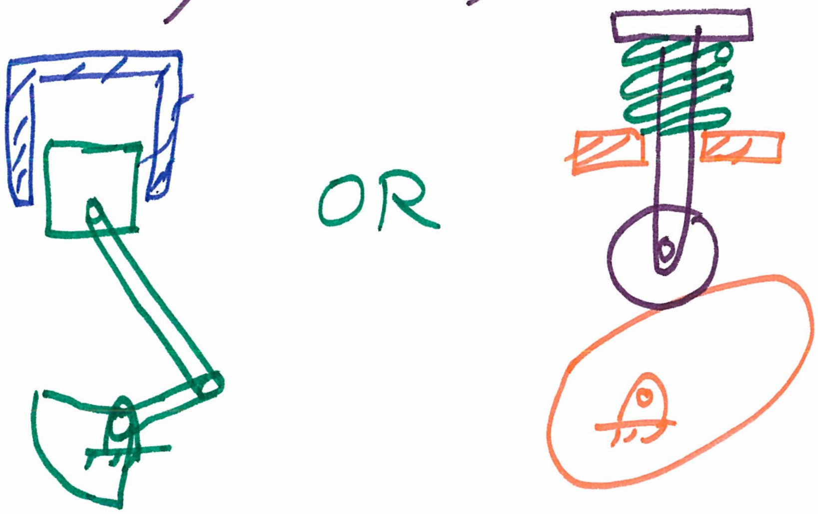
$KE = \frac{1}{2} m v^2 \longrightarrow$ linear systems

$= \frac{1}{2} I \omega^2 \longrightarrow$ rotating "

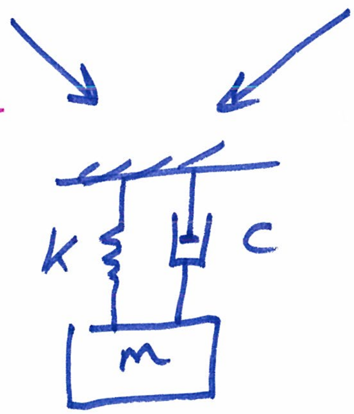
HW 10.1
10.3 (e)
10.5

Lumped Parameter Dynamic Models

Given: A dynamic system (machine)

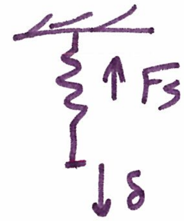


Simplification



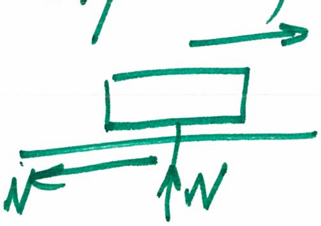
Spring $K\delta = F_s$

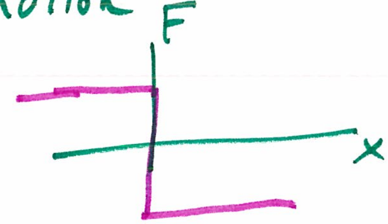
(linear)



Damper (energy dissipation)

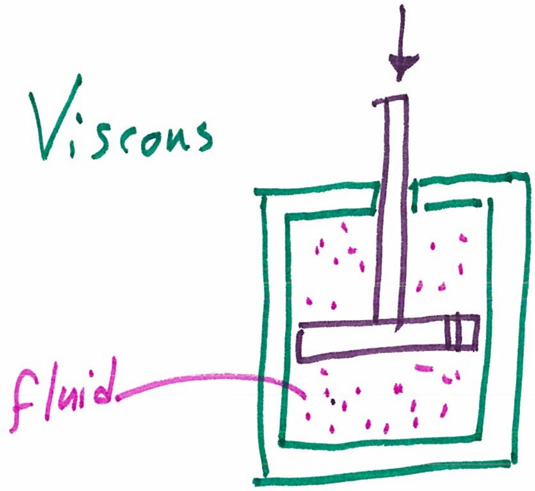
Frictional

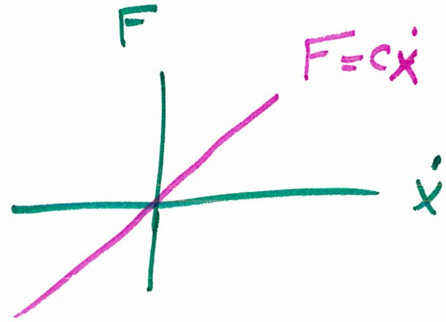




coefficient of friction

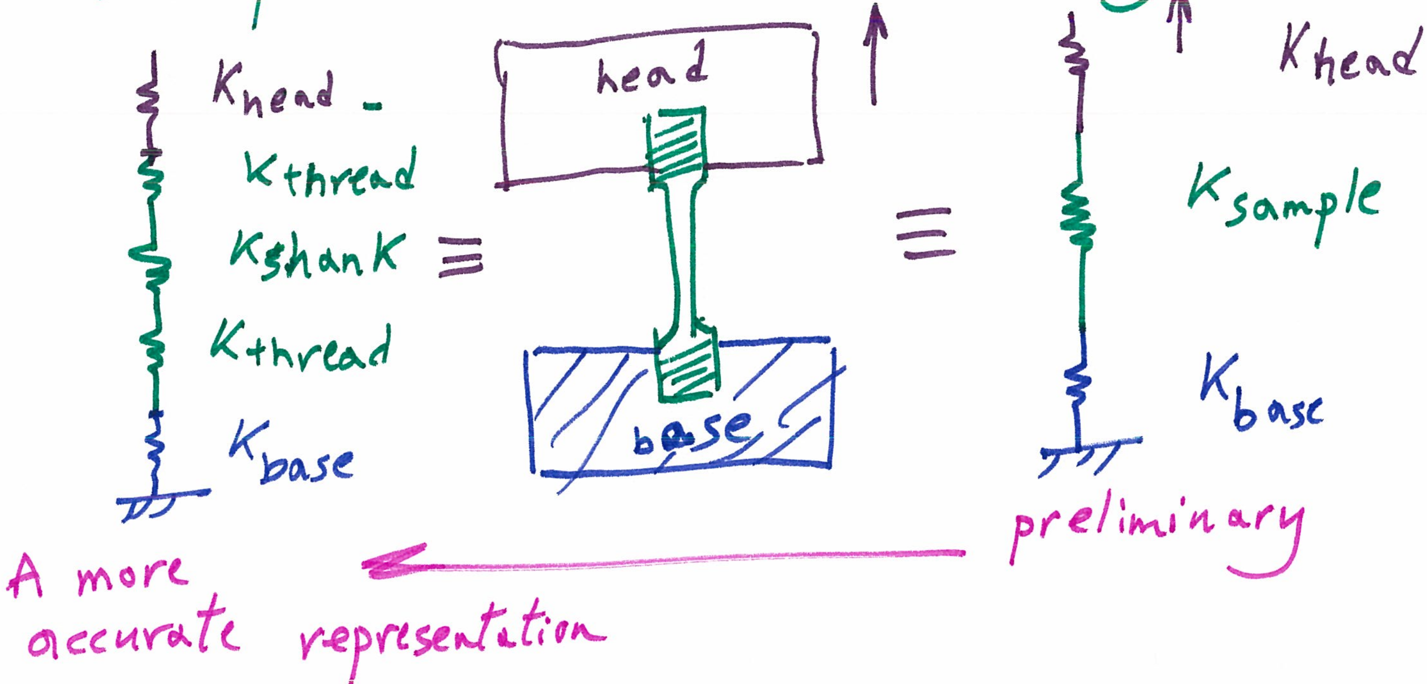
Viscous



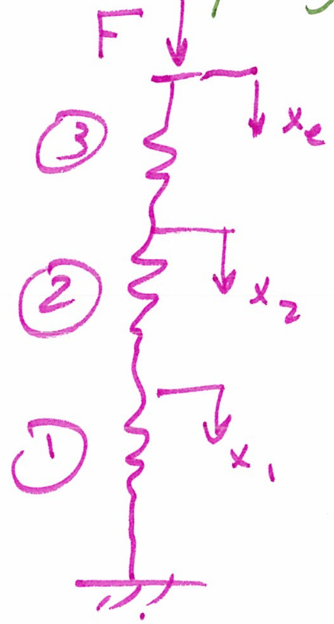


Dealing with multiple springs/dampers

Example: Axial Tension Testing



Equivalent System of springs/damper



Series

Common : Force

$F = K_e \, x_e$

end of
the spring chain

$F = K_i (x_i - x_{i-1})$

$x_e = (x_1 - x_2) + (x_2 - x_3) \ldots\ldots + x_n$ → for n-spring system

$(0 - x_1) +$

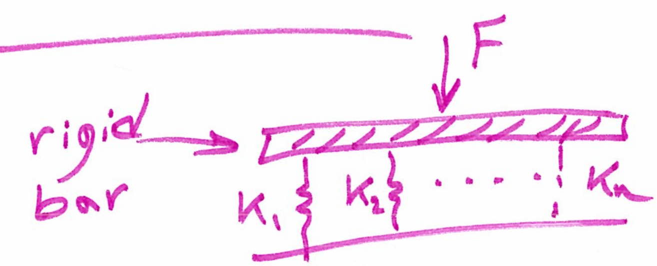
$= x_n$

$$x_e = \frac{F}{K_e} = \frac{F}{K_1} + \frac{F}{K_2} \ldots\ldots + \frac{F}{K_n}$$

$$\frac{1}{K_e} = \sum_{i=1}^{n} \frac{1}{K_i}$$

Parallel

Common : displacement



$$x = \frac{F_1}{K_1} = \frac{F_2}{K_2} \ldots\ldots = \frac{F_n}{K_n}$$

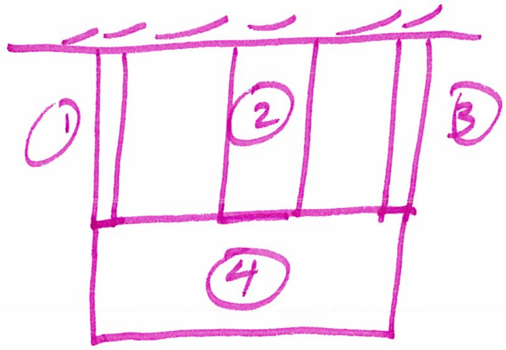
$\cancel{x = K_e}$ $F = K_e \, x_e$

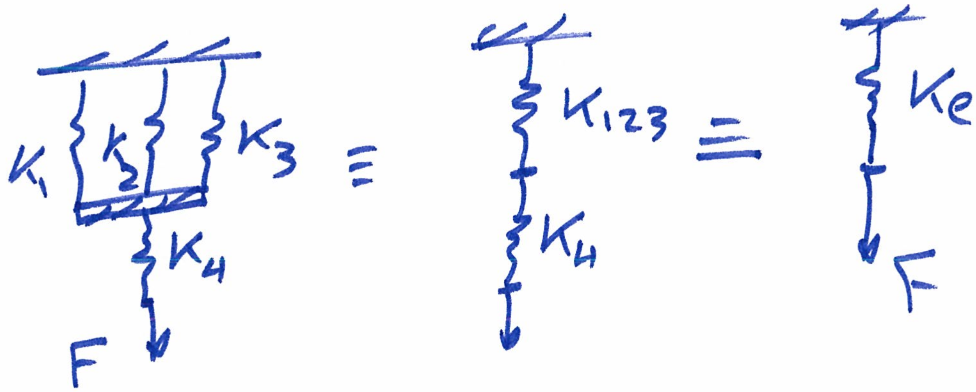
$F = \left(\sum_{i=1}^{n} K_i\right) x$

$K_e = \sum_{i=1}^{n} K_i$

Example

Find equivalent spring stiffness





$$K_{123} = K_1 + K_2 + K_3$$

$$\cancel{K_e} \quad \frac{1}{K_e} = \frac{1}{K_{123}} + \frac{1}{K_4}$$

$$K_e = \frac{K_{123}\,K_4}{K_{123} + K_4} = \frac{(K_1 + K_2 + K_3)K_4}{(K_1 + K_2 + K_3) + K_4}$$

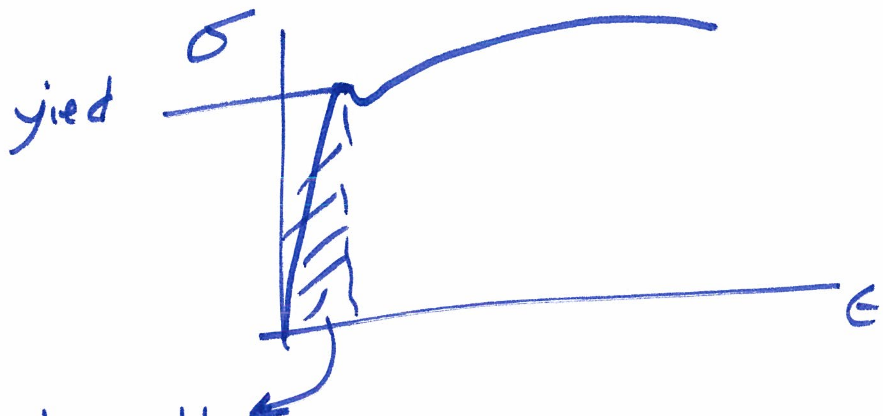
HW

10.6

Continuous Systems

- Discussion so far was limited to rigid bodies

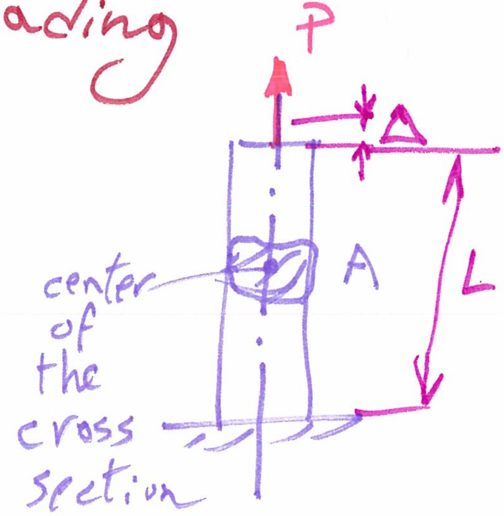
- Many materials behave in a linear fashion when loaded below the yield point.



deal with bodies as if they are linear springs

Examples:

Bar under Axial Loading



$$\sigma = \frac{P}{A}$$

Modulus of elasticity

Know $\sigma = E\epsilon$

$$\frac{P}{A} = E\left(\frac{\Delta}{L}\right)$$

$$P = \underbrace{\left(\frac{EA}{L}\right)}_{K}\Delta$$

K → equivalent spring stiffness

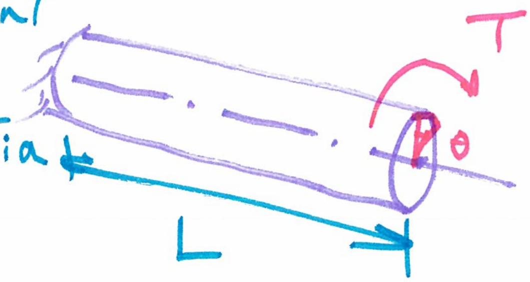
Equation are valid for compressive loads as long as they are less than buckling

Bar with Torsion

$$K = \frac{T}{\theta} = \frac{GJ}{L}$$

polar moment of inertia

modulus of rigidity



circle $\rightarrow J = \frac{\pi d^4}{32}$

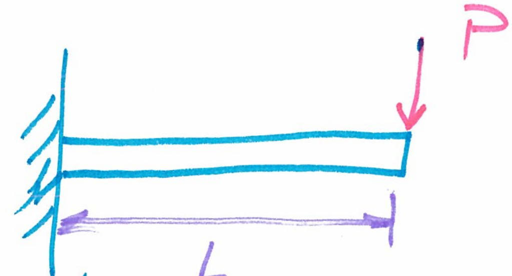
Beam under Bending

deflection equations depend on:

* Boundary conditions
* Loads

Example: Cantilever with load at the tip

$$\delta = \frac{PL^3}{3EI}$$



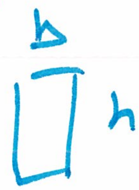
$I \rightarrow$ area moment of inertia

$$P = \frac{3EI}{L^3}\,\delta$$

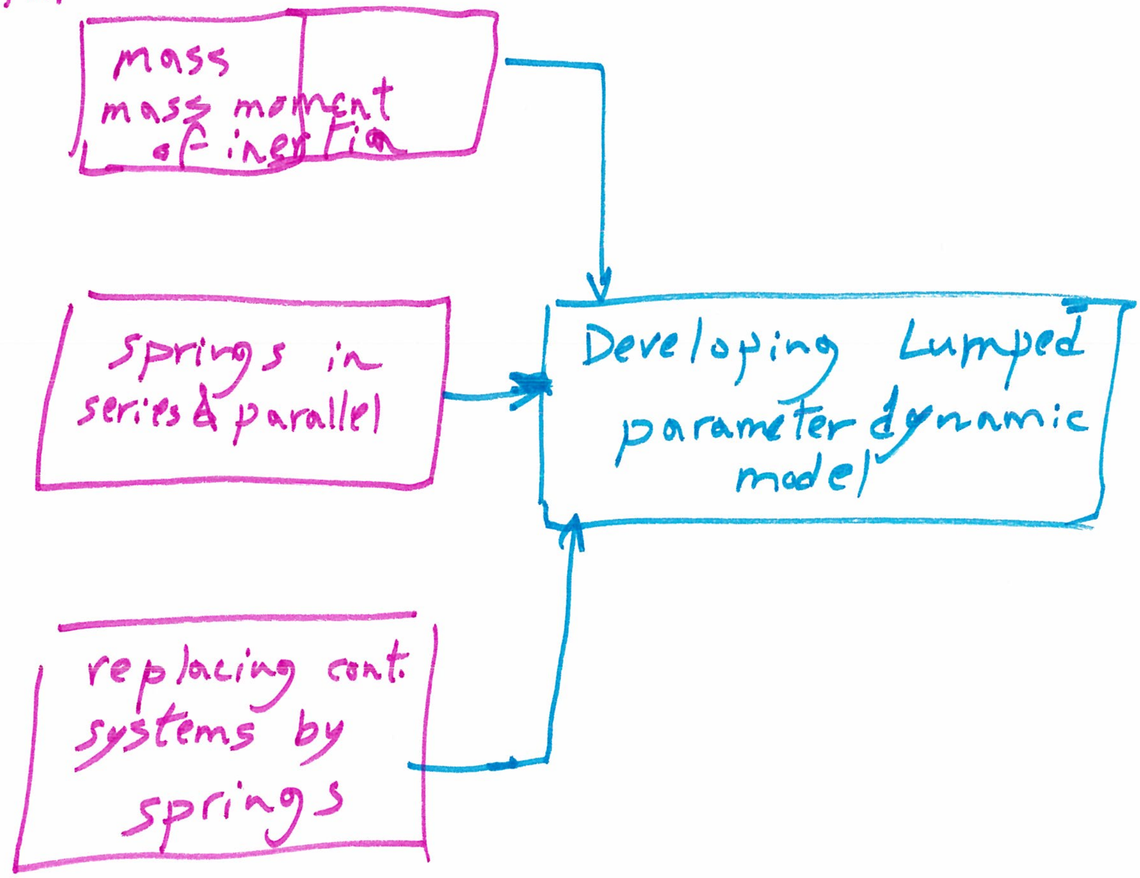
$\frac{3EI}{L^3} \rightarrow K$

circle $I = \frac{\pi d^4}{64}$

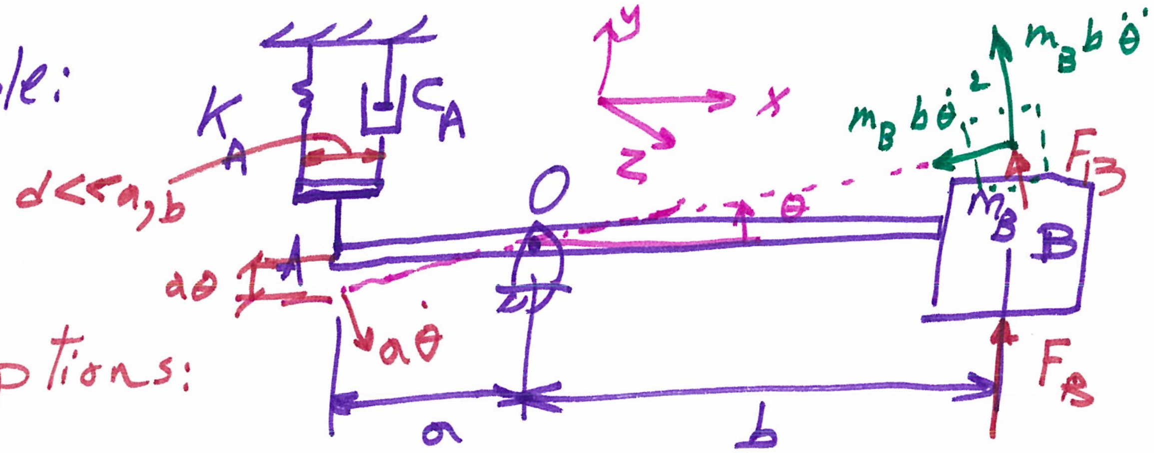
rectangle $I = \frac{bh^3}{12}$



So far


mass
mass moment of inertia
springs in series & parallel
replacing cont. systems by springs
Developing Lumped parameter dynamic model

Example:



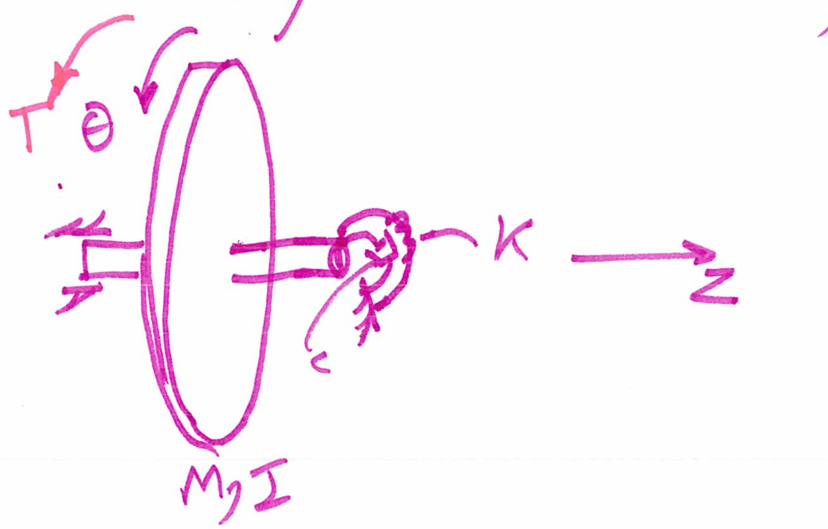
Assumptions:

* Only small oscillations occur
* Mass of the bar is negligible w.r.t. m_B
* The bar is assumed to be rigid

Step (1): Identify the degree of freedom

variable that will be used as the

Let θ (rotation about the hinge as the degree of freedom)



Apply Newton's $2^{\underline{nd}}$ Law to the moment aroun O

$\sum M_o$

$$\underbrace{(m_B\, b\, \ddot{\theta})\, b}_{\text{force (mass x accel.}} = F_B\, b - \underbrace{(K_A\, a\, \theta)}_{\text{spring force}}\, a - \underbrace{(c_A\, a\dot{\theta})}_{\text{damper force}}\, a$$

Rearranging

$$\text{equivalent}\ \underbrace{\boxed{m_B\, b^2}}_{\substack{I \\ \text{mass moment} \\ \text{of inertia}}}\ddot{\theta} + \underbrace{\boxed{c_A\, a^2}}_{c_{eff}}\dot{\theta} + \underbrace{\boxed{K_A\, a^2}}_{K_{eff}}\theta = \underbrace{\boxed{F_B\, b}}_{\substack{T \\ \text{(external torque}}}$$

$$\Downarrow$$

$$I\ddot{\theta} + c_{eff}\,\dot{\theta} + K_{eff}\,\theta = T$$

H.W. 10.16

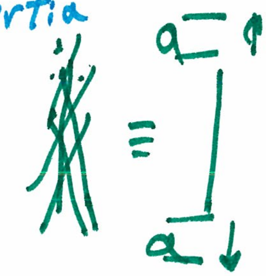
Redo this example if the flexibility of the bar is considered

Example

Develop a lumped-parameter model for a shaker

Assumptions:

* $L \gg l$
* $l \gg a$
* Flexibility of the crank (a) is negligible
* Rod (l) moves vertically (lateral motion is negligible)



* Deflections of beam (L) and rod (l) are small compared to their lengths

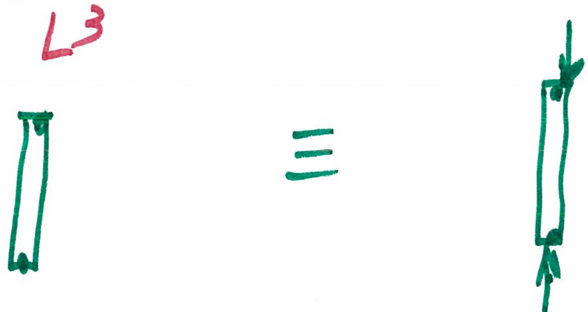
Results

→ $\theta = \sin\theta = \tan\theta$

$\cos\theta = 1$

→ use standard deflection equations

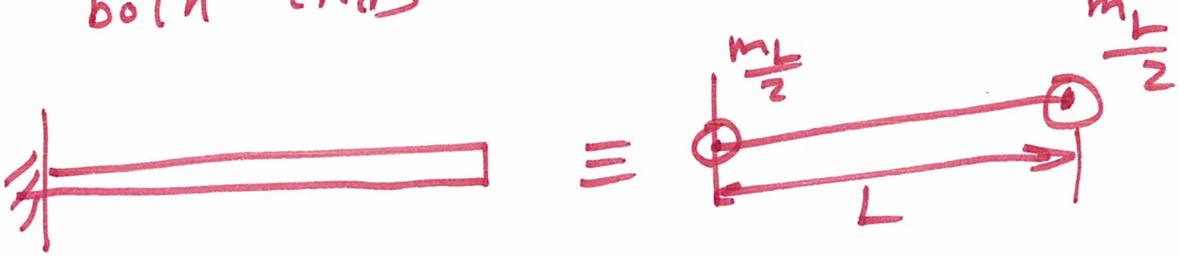
$$K_L = \frac{3 E_L I_L}{L^3}$$



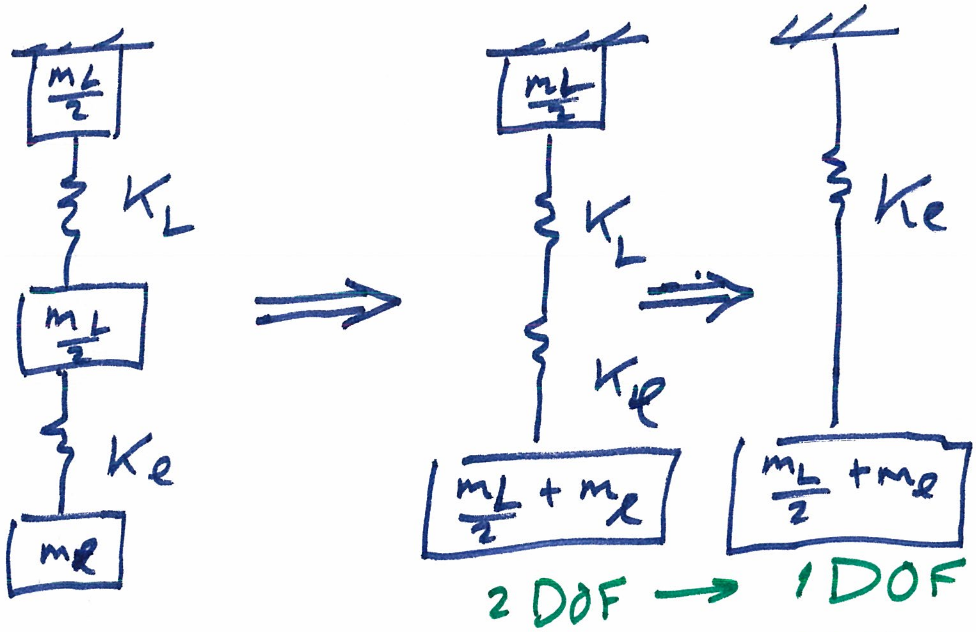
$$K_\ell = \frac{E_\ell A_\ell}{\ell}$$

Another Assumption:

* Observe that the mass of L is distribut uniformly
* Each point ~~with~~ along the beam will have different acceleration
* split the mass of L evenly at both ends



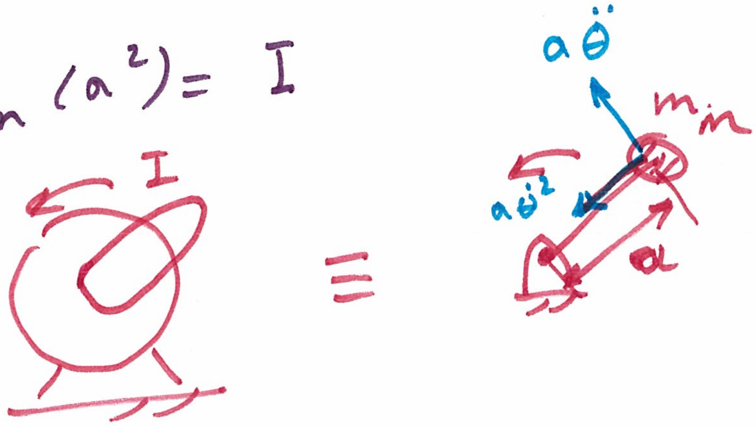
The system can be described as:

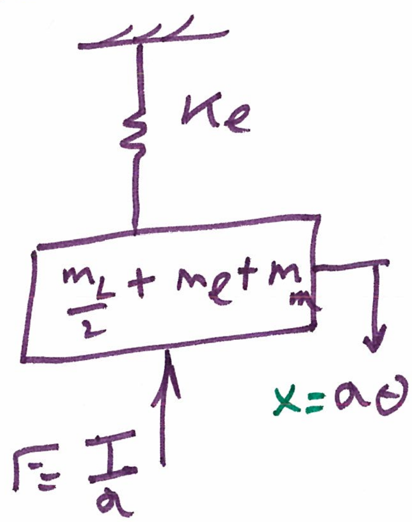


$$K_e = \frac{K_L K_\ell}{K_L + K_\ell} \longrightarrow \text{2 springs in series}$$

We need to include excitation

$m_m (a^2) = I$





Newton's 2nd Law

$$T = I\ddot{\theta} = (m_m a \ddot{\theta})\, a$$

$$I = m_m a^2 \longrightarrow m_m = \frac{I}{a^2}$$

Equation of Motion of the Lumped Parameter Model

$$\left(\frac{m_L}{2} + m_e + m_m\right)\ddot{x} + K_e x = F$$

$$\left(\frac{m_L}{2} + m_e + \frac{I}{a^2}\right) a\ddot{\theta} + \frac{K_L K_e}{K_L + K_e}\, a\theta = \frac{T}{a}$$

HW

~~10.16~~ 10.27

10.30