ME 746 Spring 2008

Note Title 1/22/2008

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B-130, Contact Hours
Contact M W 2-3 pm

Common Base

Weeks 1-3 Review of UG Control concepts
- Modeling
- Laplace Transforms
- Stability (Root Locus)

(Freq. response/Bode Methods)

— Time-Domain Modeling and Analysis (Franklin Ch. 7)

$$m\ddot{x} + B\dot{x} + Kx = u(t) = \text{input}$$

Choose state vars:

$$x_1 = x$$

$$x_2 = \dot{x} = v$$

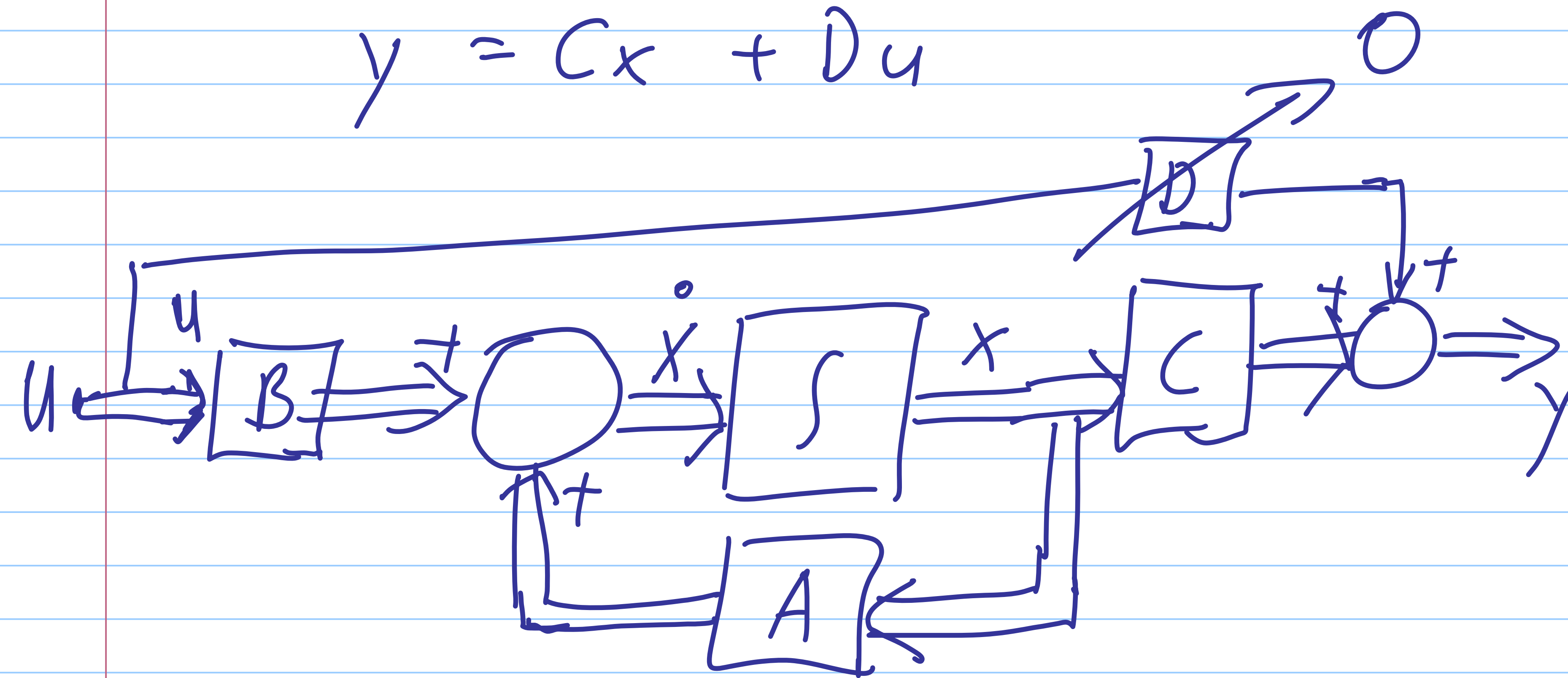
$$\dot{x}_1 = v$$

$$\dot{x}_2 = \frac{1}{m}\left(u(t) - Bx_2 - Kx_1\right)$$

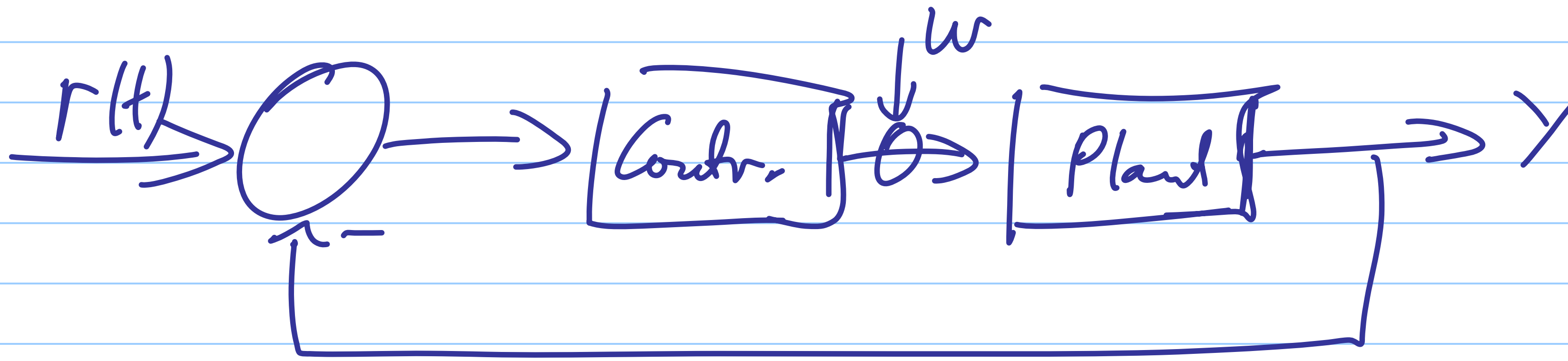
Common formulation

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

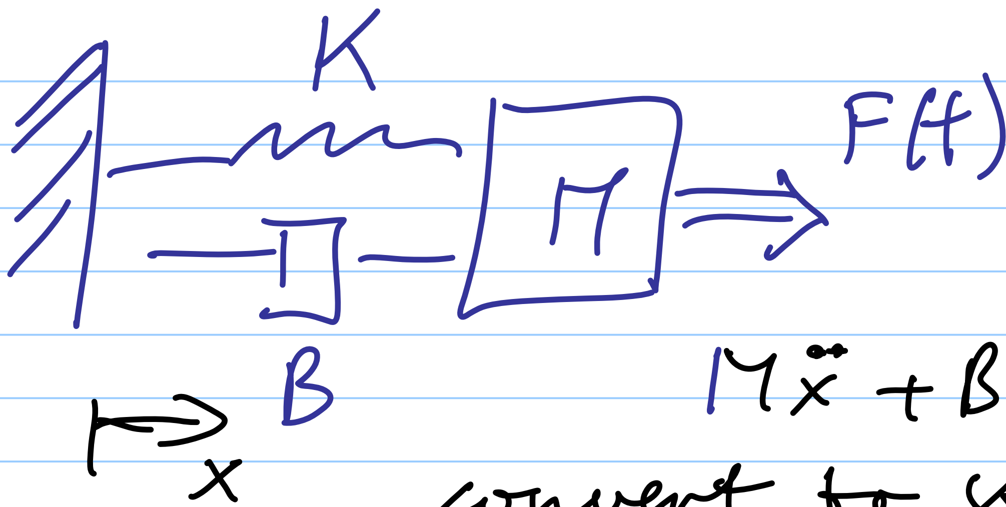


State Variables vs. Transfer Function



1 state var is measured

In class



$$M\ddot{x} + B\dot{x} + Kx = F(t)$$

convert to state variables.

choose state vector

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x \\ v \end{pmatrix}$$

$$\dot{x}_1 = v$$

$$\dot{x}_2 = \dot{v} = \frac{1}{M}\left(-Bv - Kx + \overset{u}{F(t)}\right)$$

$$\dot{x} = Ax + Bu$$

$$\begin{pmatrix} \dot{x} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{K}{M} & -\frac{B}{M} \end{pmatrix} \begin{pmatrix} x \\ v \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{M} \end{pmatrix} F(t)$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ v \end{pmatrix}$$ choose displacement

$+ 0 \cdot F(t)$

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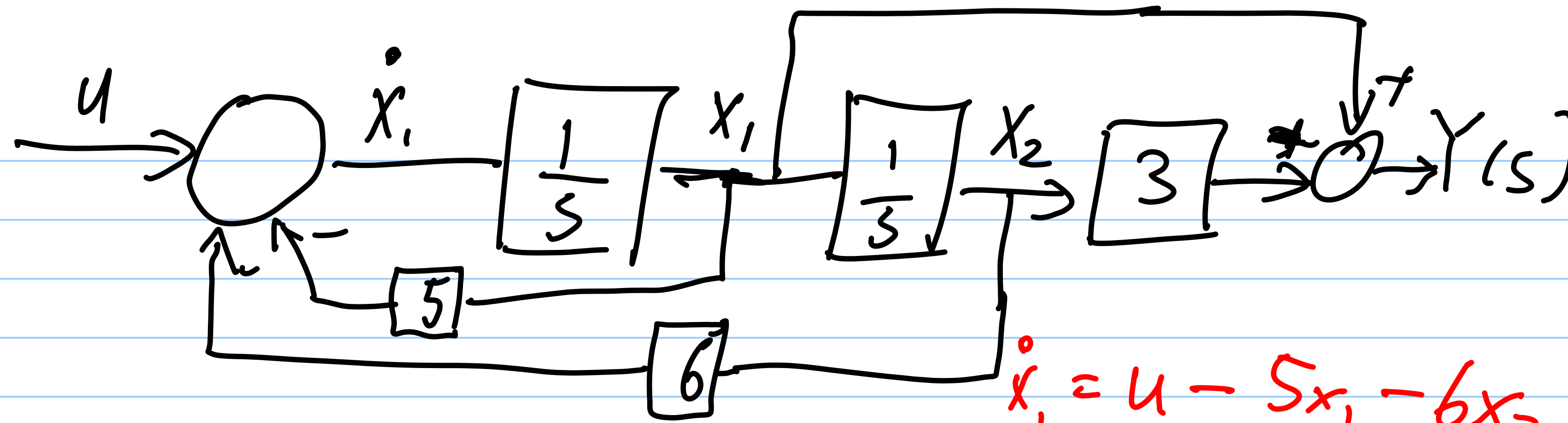
in SISO (single-input single output)

Ex:

$$U \longrightarrow \boxed{\frac{s+3}{s^2+5s+6}} \longrightarrow Y \qquad \frac{Y}{U} = \frac{s+3}{\cdots\cdots}$$

create blockdiagram



$$\dot{x}_1 = u - 5x_1 - 6x_2$$

$$Y(s)\left[s^2 + 5s + 6\right] = (s+3)\,U(s)$$

$$\dot{\vec{x}} = \begin{bmatrix} -5 & -6 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y(t) = \begin{bmatrix} 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Example $$\frac{Y}{U} = \frac{5}{s^3 + 8s^2 + 9s + 2}$$

$$\dddot{y} + 8\ddot{y} + 9\dot{y} + 2y = 5u(t)$$

choose $x_1 = y$

$x_2 = \dot{y} = \dot{x}_1$

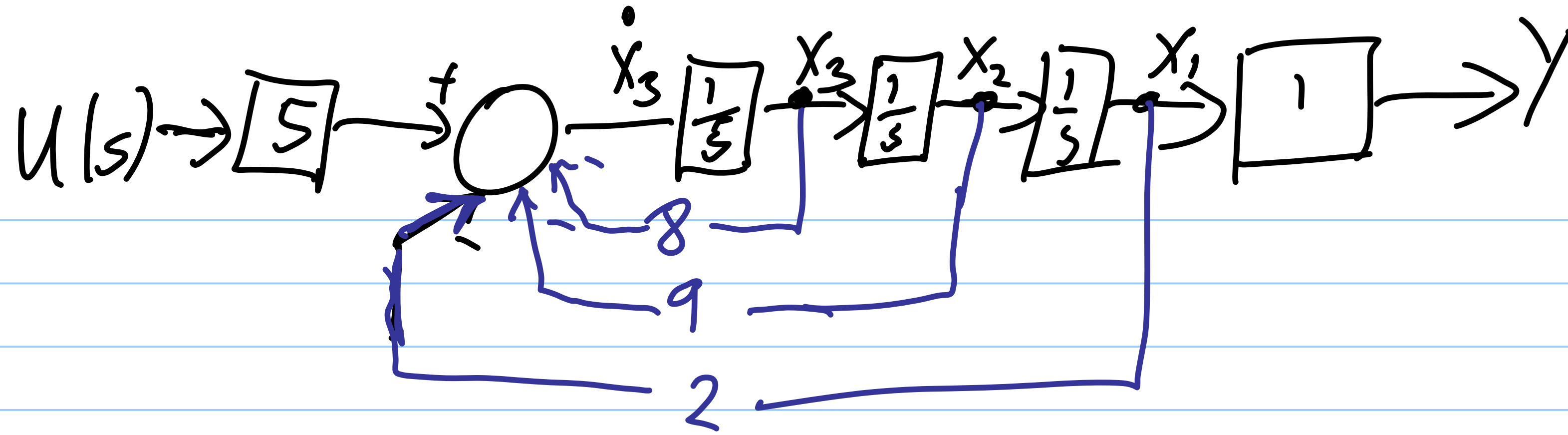
$x_3 = \ddot{y} = \dot{x}_2$

state equations

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$$\dot{x}_3 = -8x_3 - 9x_2 - 2x_1 + 5u(t)$$



$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -9 & -8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} u$$

$$y = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + 0 \cdot u$$

Some Matrix Algebra

Given $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ $\quad \mathrm{adj}A = \begin{pmatrix} a_{22} & -a_{21} \\ -a_{12} & a_{11} \end{pmatrix}$

invert A $\qquad A^{-1}$

$$A^{-1} = \frac{1}{\det A}\,\mathrm{adj}\,A$$

adj = Matrix of Cofactors $\cdot (-1)^{i+k}$

how to use:

$$M\vec{x} = \vec{b}$$

$$\vec{x} = M^{-1}\cdot b$$

Matrix inversion for dynamic systems

$\dot{\vec{x}} = A\vec{x}$ (homogeneous)

$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \end{pmatrix}$

Laplace:

initial cond

$$sX(s) - X(0) - AX(s) = 0$$

regroup: $X(s)[sI - A] = X(0)$

solution: $X(s) = X(0)\,[sI - A]^{-1}$

Def: "Transition Matrix $\Phi(s) = [sI - A]^{-1}$

Example of homogeneous solution:

$$A = \begin{pmatrix} -2 & 2 \\ 1/2 & -2 \end{pmatrix} \quad b = \begin{pmatrix} 0 \\ 3/2 \end{pmatrix} \quad C = \begin{pmatrix} 0 & 1 \end{pmatrix}$$

$$[sI - A] = \begin{bmatrix} s+2 & -2 \\ -\frac{1}{2} & s+2 \end{bmatrix}$$

$$\det A = (s+2)^2 - 1$$
$$= s^2 + 4s + 3$$
$$= (s+1)(s+3)$$

$$[sI - A]^{-1} = \frac{\begin{bmatrix} s+2 & +1/2 \\ 2 & s+2 \end{bmatrix}}{(s+1)(s+3)} = \frac{C_1}{s+1} + \frac{C_2}{s+3}$$

$$\Phi(s) = \frac{1}{2}\begin{pmatrix} 1 & 2 \\ 1/2 & 1 \end{pmatrix}\frac{1}{s+1} - \frac{1}{2}\begin{pmatrix} -1 & 2 \\ 1/2 & -1 \end{pmatrix}\frac{1}{s+3}$$

$$\frac{1}{(s+1)(s+5)} = \frac{A}{s+1} + \frac{B}{s+5}$$

$$1 = A(s+5) + B(s+1)$$

$$= s(A+B) + 5A + B$$