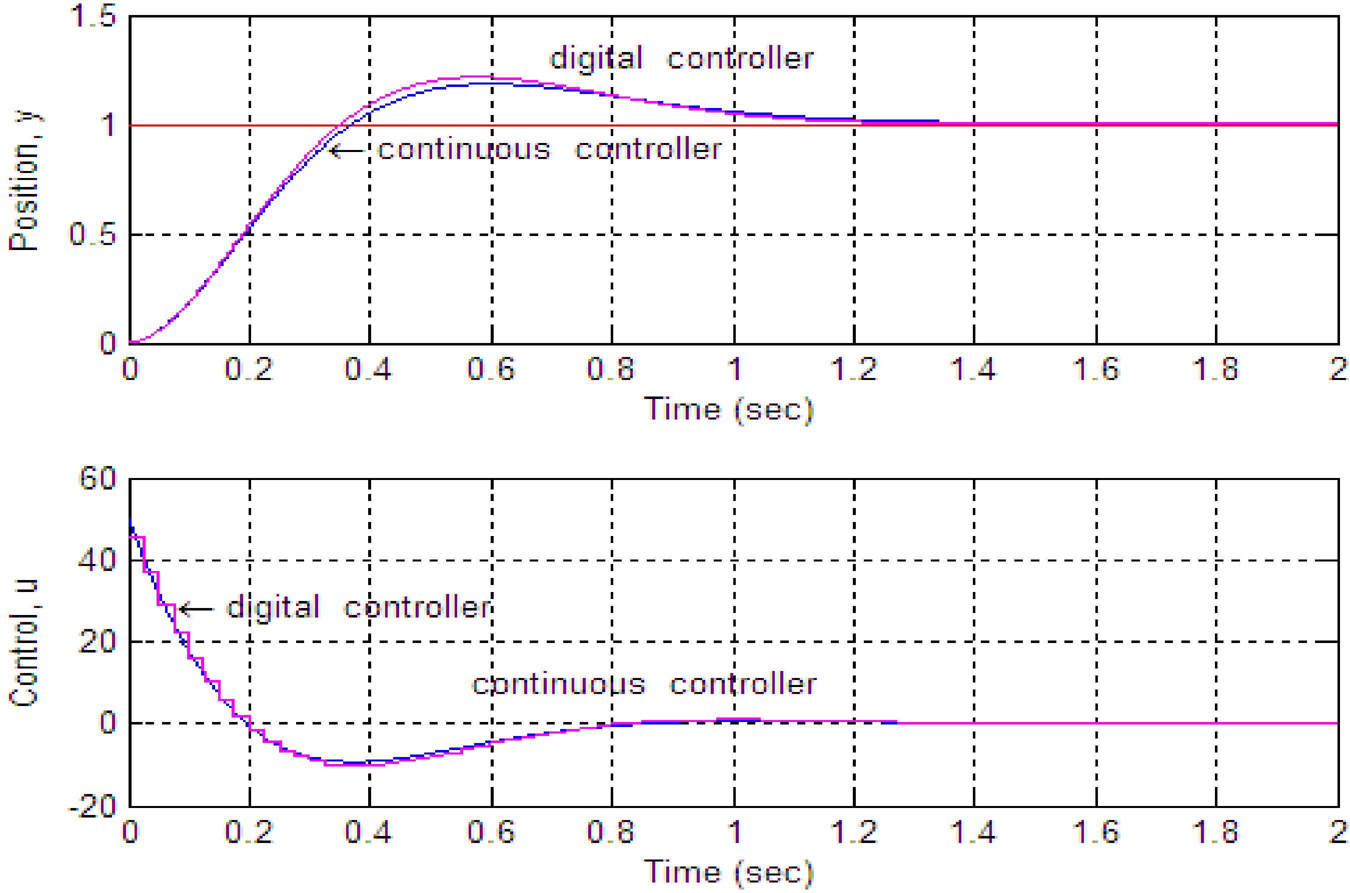
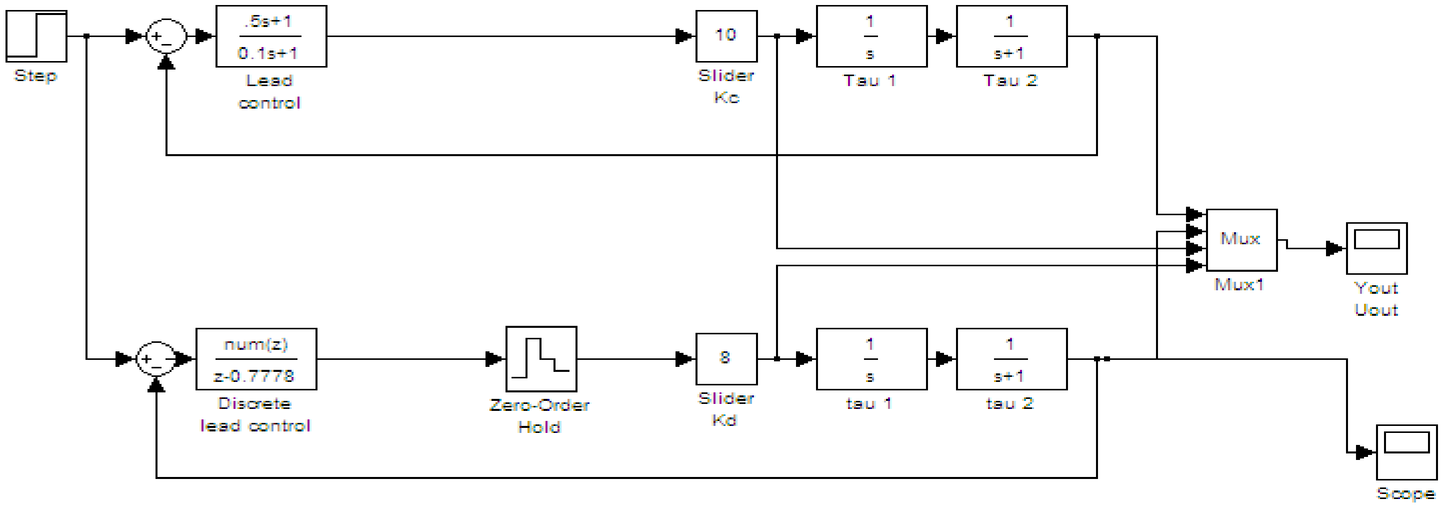
Discrete Systems Chapter 3

Note Title 3/4/2008

Figure 8.8 Step Responses of Digital and Continuous Controllers





Step
.5s+1
0.1s+1
Lead control
10
Slider Kc
1
s
Tau 1
1
s+1
Tau 2
Mux
Mux1
Yout Uout
num(z)
z-0.7778
Discrete lead control
Zero-Order Hold
8
Slider Kd
1
s
tau 1
1
s+1
tau 2
Scope

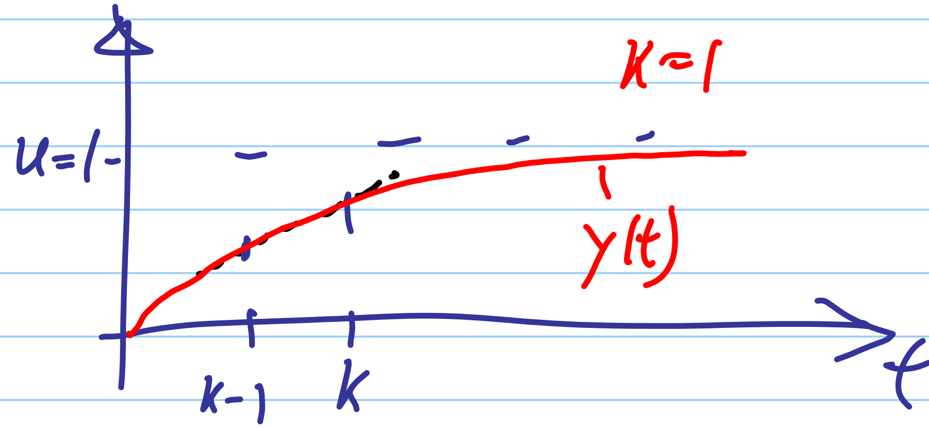
```
clear
clf
[tout,yout]=sim('fig8_08c');
r=[1 1];   %reference input
t=[0 2];
subplot(2,1,1)
plot(t,r,'r')
hold on
plot(ycd(:,1),ycd(:,2))
plot(ycd(:,1),ycd(:,3),'m')
title('Figure 8.8 Step Responses of Digital and Continuous Controllers')
ylabel('Position, y')
xlabel('Time (sec)')
text(.33,.9, '\leftarrow continuous  controller')
text(.7,1.3, 'digital  controller')
grid
hold off
subplot(2,1,2)
plot(ycd(:,1),ycd(:,4))
hold on
plot(ycd(:,1),ycd(:,5),'m')
ylabel('Control, u')
xlabel('Time (sec)')
text(.55,10, ' continuous  controller')
text(.08,29, '\leftarrow digital  controller')
grid
hold off
```

Discretization

Ex. continuous system

$$\tau \dot{y} + y = K \cdot u(t)$$

Numerical solution
use Difference equ.



$$\tau \cdot \frac{y_K - y_{K-1}}{T} + y_{K-1} = 1 \cdot u(t) \qquad T = \text{sampling rate}$$

solve for y_K: $y_K - y_{K-1} + \frac{T}{\tau}\left[y_{K-1} - u_K\right] = 0$

"Euler Method"

Difference eq. $\quad y_K = y_{K-1}\left(1 - \frac{T}{\tau}\right) + \frac{T}{\tau} \cdot u_K$

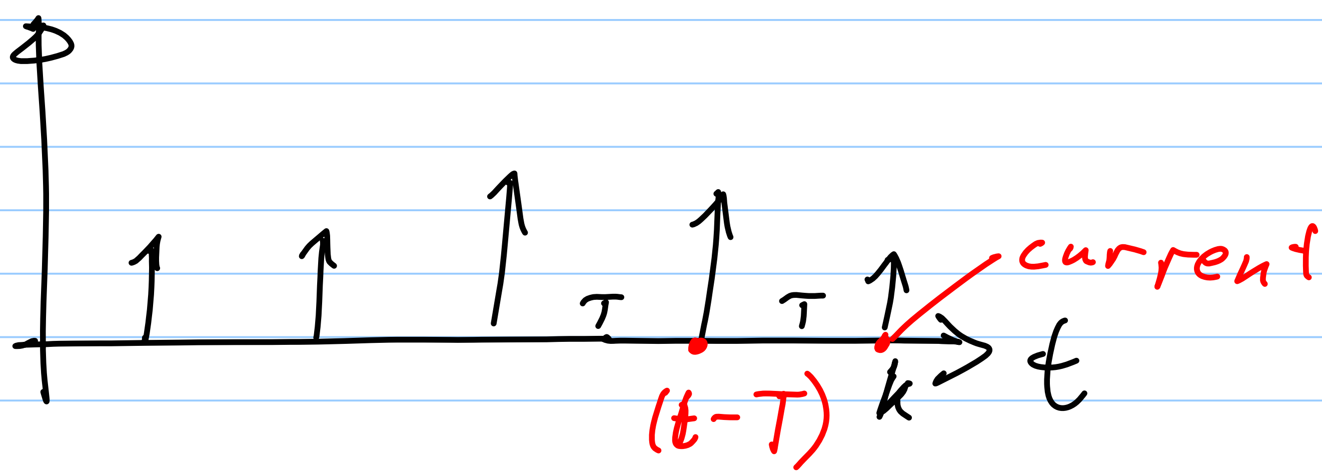
4.2 Discrete T.F.

Z – Transform

Continuous TF

$$\frac{Out(s)}{In(s)} = \frac{b_0 s^3 + b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$

Discrete : Z - Transform



$\mathcal{L}[f(t-T)] = e^{-sT}F(s)$ "Shifting Theorem"

$\boxed{\text{Def z-Transform} \quad z = e^{sT}}$

$z^{-1} = e^{-sT}$

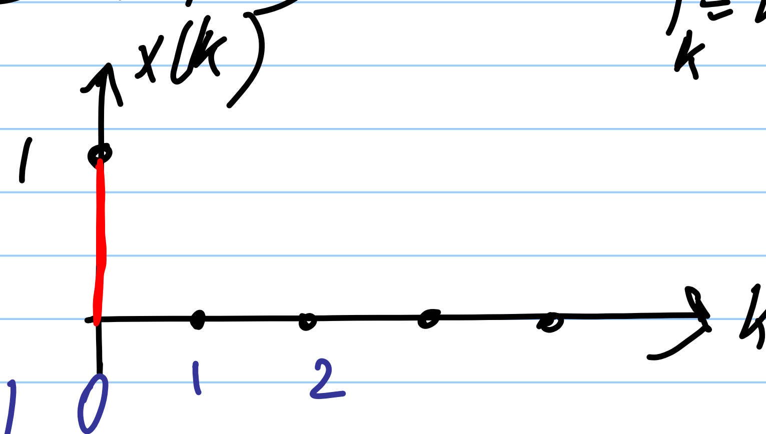
Examples of discrete TF's

$T_k = k \cdot T$

1. Unit pulse $\delta(k)$

$\delta(k) \quad x(k) = 1 \quad k = 0$

$\qquad\qquad = 0 \quad k \neq 0$

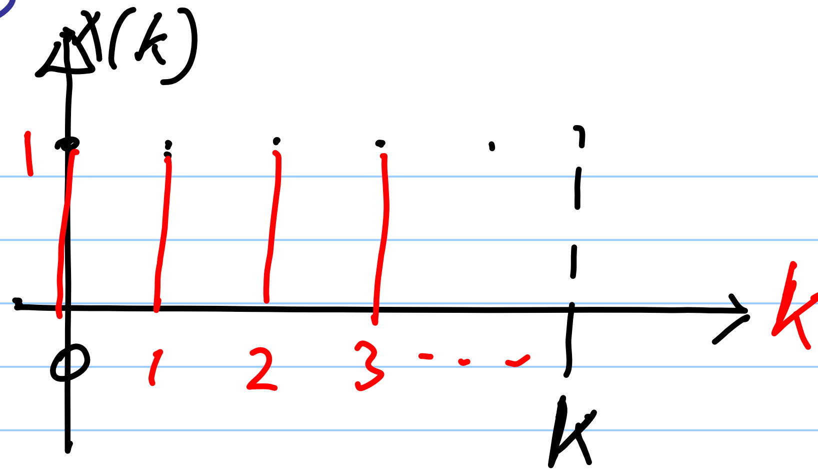


$X(z) = \sum \delta(k) \cdot z^{-k} = z^0 = 1$

Example 2 Unit Step

$x(k) = 1 \quad k \geq 0$

$$X(z) = \sum_{k=0}^{\infty} 1 \cdot z^{-k} = 1 + z^{-1} + z^{-2} + \cdots$$



$$X(z) = 1 + z^{-1} + \cdots z^{-n}$$

$$z \cdot X(z) = z + 1 + \cdots z^{-n+1}$$ subtract from upper

$$X(z)(1-z) = -z + \cancel{z^{-n}} \rightarrow 0$$

$$Z(u_{step}) = X(z) = \frac{-z}{1-z} = \frac{z}{z-1}$$

Example 3 Exponential $x(t) = e^{-at} \cdot \underbrace{1(t)}_{\text{unit step}}$

$$x(k) = e^{-akT} \cdot 1(kT)$$

$$X(z) = \sum e^{-akT} z^{-k} = \sum \left(e^{-aT} z^{-1}\right)^k$$

$$= \frac{1}{1 - e^{-akT} z^{-1}}$$

Time shift

$$\mathcal{Z}\{x(k-1)\} = z^{-1} X(z)$$

Discrete Transfer function

$\left(1-e^{-\frac{t}{\tau}}\right)$ revisited

$$\underbrace{y_K}_{\text{out}} = y_{K-1}\left(1-\frac{T}{\tau}\right) + \frac{T}{\tau}\cdot \underbrace{u_K}_{\text{in}}$$

$$TF(s) = \frac{Out(s)}{In(s)} \quad ; \quad TF(z) = \frac{Out(z)}{In(z)}$$

Sort for y_K and u_K

$$y_K + y_{K-1}\cdot\frac{T}{\tau} - y_{K-1} = \frac{T}{\tau}\cdot u_K$$

$$Y(z) + \frac{T}{\tau}\cdot Y(z)\cdot z^{-1} - z^{-1}\cdot Y(z) = \frac{T}{\tau}\cdot U(z)$$

$$\frac{Y(z)}{U(z)} = \frac{T/\tau}{1 + z^{-1}\left(\frac{T}{\tau} - 1\right)} = \frac{a}{1 + b \cdot z^{-1}}$$

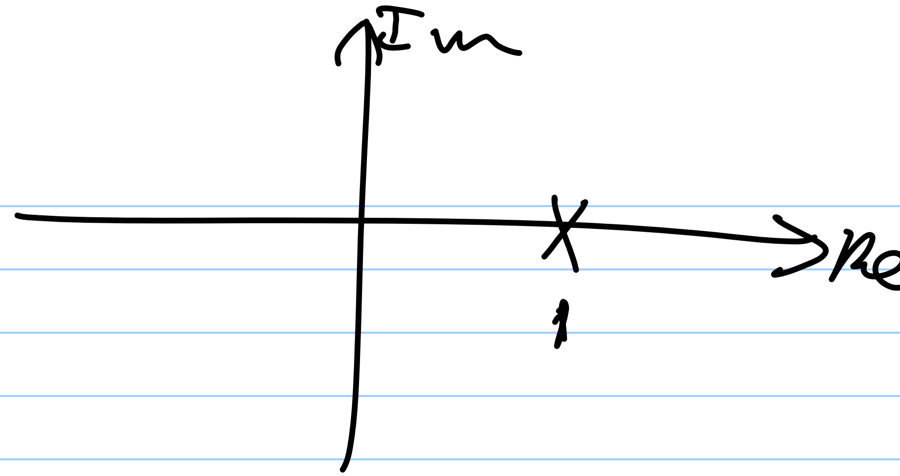
HW due Tue March 11 3.4

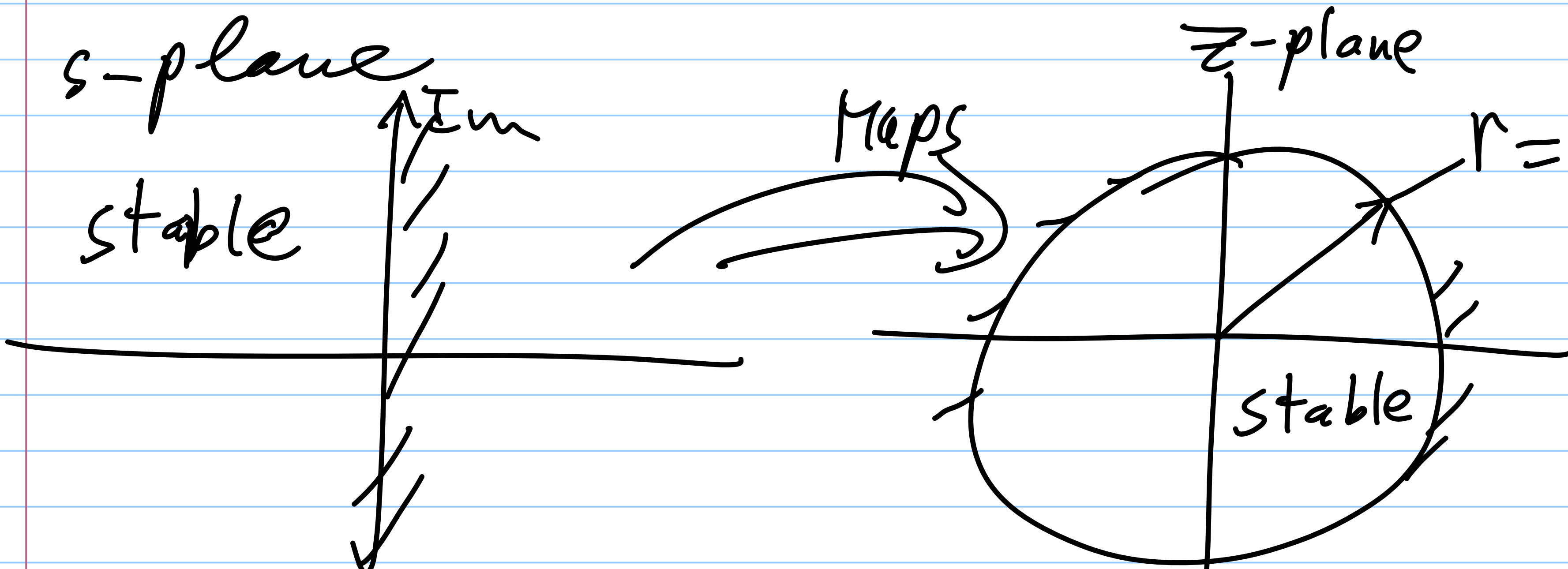
3.10

Discrete TF $z = e^{sT}$

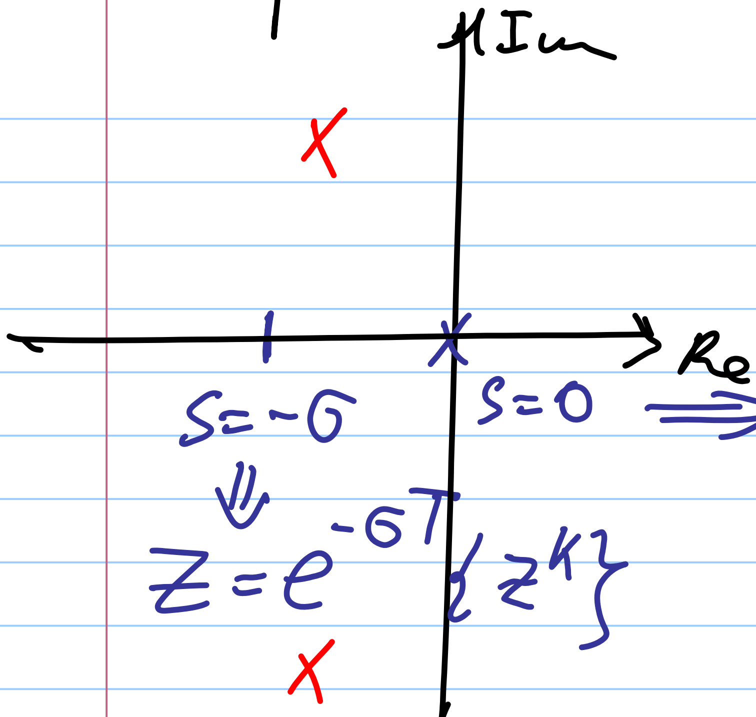
Example $T(z) = \frac{Num}{z + p}$

$$Z(u_{step}) = \frac{z}{z-1}$$





s-plane $\qquad z = e^{sT} \qquad$ z-plane

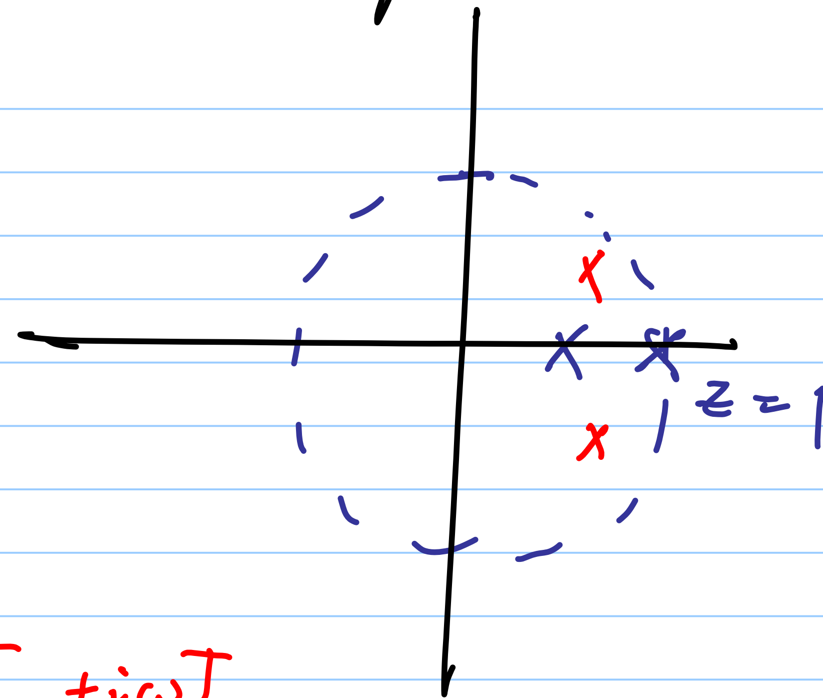


$s = 0 \Longrightarrow z = 1$

$s = -\sigma$

$\Downarrow$

$z = e^{-\sigma T} \quad \{z^k\}$



$s = -\sigma \pm j\omega \Longrightarrow z = e^{-\sigma T} \cdot e^{\pm j\omega T}$

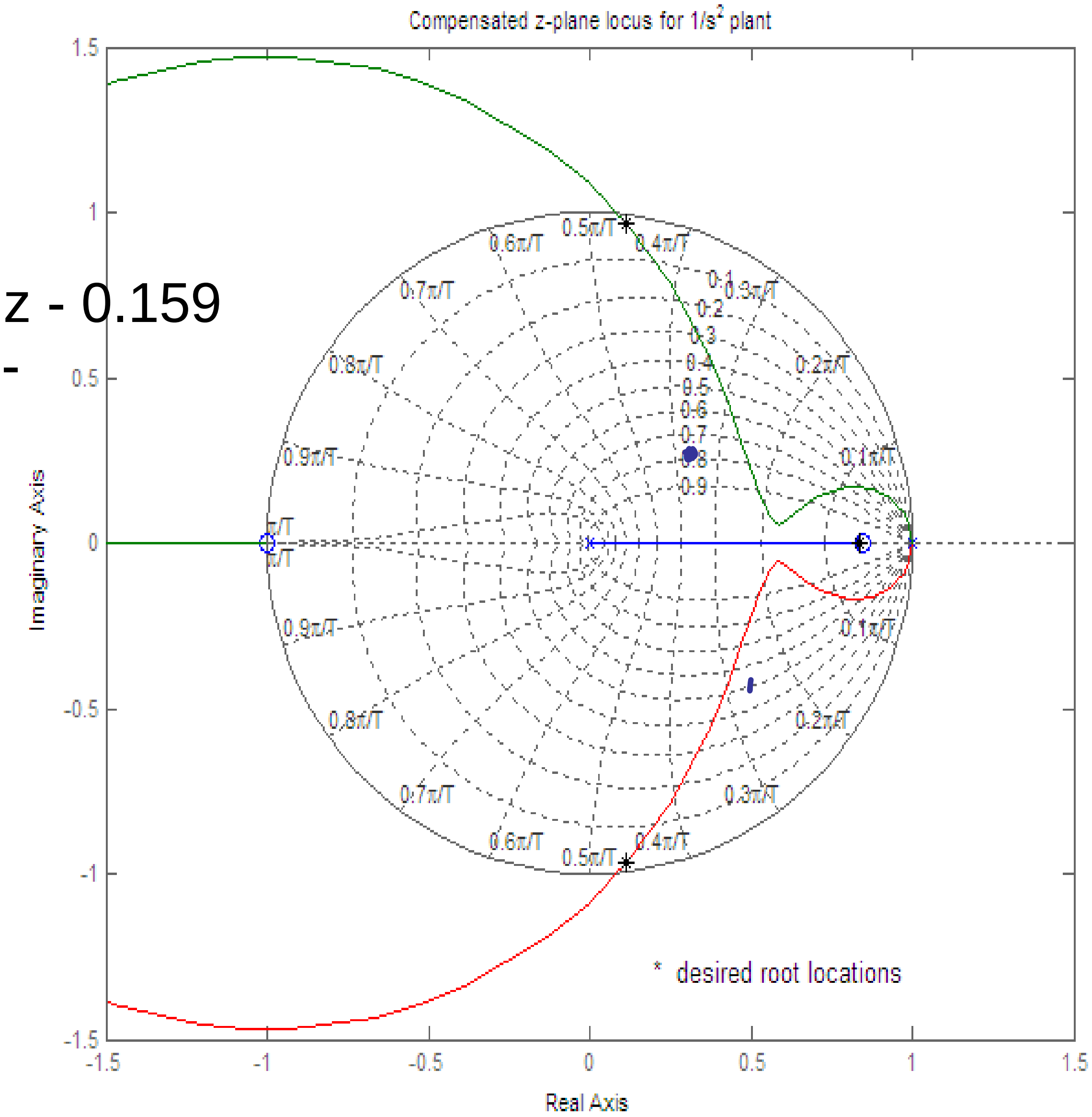
decaying oscillator

neg. half of s-plane maps inside the unit circle of the z-plane

Root locus of
Transfer function:

$$\frac{0.187 z^2 + 0.02805 z - 0.159}{z^3 - 2 z^2 + z}$$

K= 5; T =1s



Use z-Transforms for solution of discrete equations

Ex. $x(k+1) + x(k) = 0$

convert to z-Transform

$$X(z)\left[z^{k+1} + z^{k}\right] = 0 \quad \Big| \cdot z^{-k}$$

Char. eq.: $z + 1 = 0$ or $z = -1$

pole at -1

Ex. 2 $\quad y_{n+1} - 3y_n = \overset{4\cdot\text{step}}{4} \quad (y_0 = 1) \quad \Big| \cdot z^{-n}$

$$z \cdot Y(z) - z\, y(0) - 3\,Y(z) = 4 \cdot \frac{z}{z-1}$$

Solve for $Y(z)$

$$Y(z)\left[z - 3\right] - z = \frac{4z}{z-1}$$

$$Y(z) = \frac{4z}{(z-1)(z-3)} + \frac{z}{z-3}$$

$$Y(z) = \frac{z^2 + 3z}{(z-1)(z-3)} = z\,\frac{z+3}{(z-1)(z-3}$$

$$Y(z) = z\left(\frac{-2}{z-1} + \frac{3}{z-3}\right) = \frac{-2z}{z-1} + \frac{3z}{z-3}$$

$$y_n = -2 + 3 * 3^n \quad n = 1, 2 \cdots$$