Note Title 1/29/2008

homogeneous linear system

$$\dot{x} = A x \qquad (1)$$

$$\begin{pmatrix} \cdot & & & \\ & \ddots & & \\ & & \ddots & \\ & & & \cdot \end{pmatrix}$$

Transformation P

$$x = P y$$

$$\dot{x} = P \dot{y}$$

insert into (1): $P \dot{y} = A P y \quad \Big| \cdot P^{-1}$

$$\dot{y} = P^{-1} A P \cdot y = \Lambda y$$

Λ = Matrix of Eigenvalues

"Modal Canonical Form in Book

Find Matrix P

$$\Lambda = \begin{pmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{pmatrix}$$

$|A - \lambda I| = 0$ characteristic equation

$(A - \lambda_1 I) p_1 = 0$ $\quad$ p_1 = first Eigenvector

$(\quad \vdots \quad) P_2 = 0$ $\quad$ $P = (p_1, p_2, p_3 \cdots)$

Example $\dot{x} = \begin{pmatrix} 0 & 1 \\ -6 & -5 \end{pmatrix} x$

$$\det(sI - A) = \begin{vmatrix} s-0 & -1 \\ 6 & s+5 \end{vmatrix} = s^2 + 5s + 6 = 0$$

2 roots of char. eqn: $s_1 = -2$; $s_2 = -3$

$\lambda_1 = -2$

$$(A - \lambda_1 I) = \begin{pmatrix} 0+2 & 1 \\ -6 & -5+2 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -6 & -3 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = 0$$

$$2p_1 + p_2 = 0$$
$$-6p_1 - 3p_2 = 0$$

let $p_1 = 1$ $\quad p_2 = -2$

Eig vec$_1$ $= \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ $\quad$ Eig vec$_2$ $= \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$$P = \begin{pmatrix} 1 & 1 \\ -2 & -3 \end{pmatrix}$$

"Vandermonde Matrix"

$$P = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ \lambda_1 & \lambda_2 & \cdots & & \lambda_n \\ \lambda_1^2 & \lambda_2^2 & \cdots & & \lambda_n^2 \\ \lambda_1^3 & & & & \vdots \\ \vdots & \cdots & & & \end{pmatrix}$$

$$\begin{pmatrix} \dot{y}_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

2 uncoupled equations $\dot{y}_1 = -2y_1$

$\dot{y}_2 = -3y_2$

$$y_1(t) = C_1 e^{-2t} \qquad y_2(t) = C_2 e^{-3t}$$

System with External input

$$\dot{x} = Fx + Gu$$

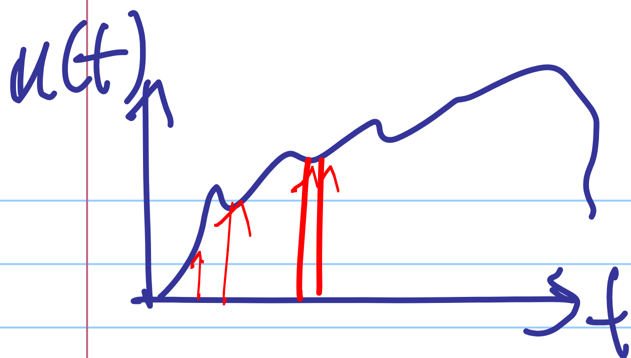
$$s\,X(s) - x(0) = F\,X(s) + G\,U(s)$$

$$X(s) = \underbrace{[sI - F]^{-1}}_{\Phi(s) = \text{"Transition Matrix"}} x(0) + [sI - F]^{-1} G\,U(s)$$

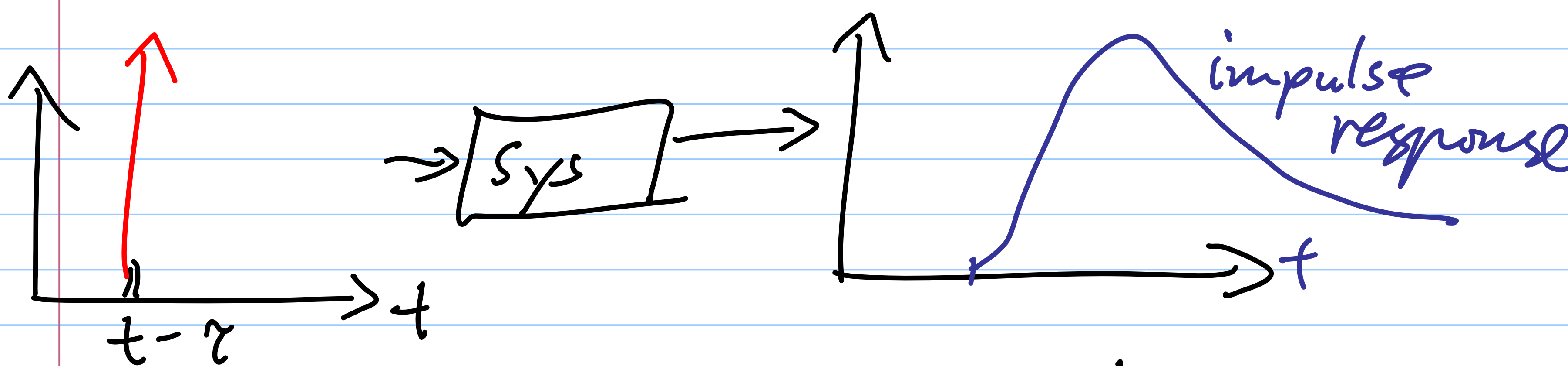
time-domain sol.

$$x(t) = \underbrace{\Phi(t)\,x(0)}_{\text{homogeneous}} + \underbrace{\int_0^t \Phi(t-\tau)\,G\,u(\tau)\,d\tau}_{\text{forced response}}$$

"Convolution"



$$\dot{x} = Fx + Gu$$
$$y = Hx$$

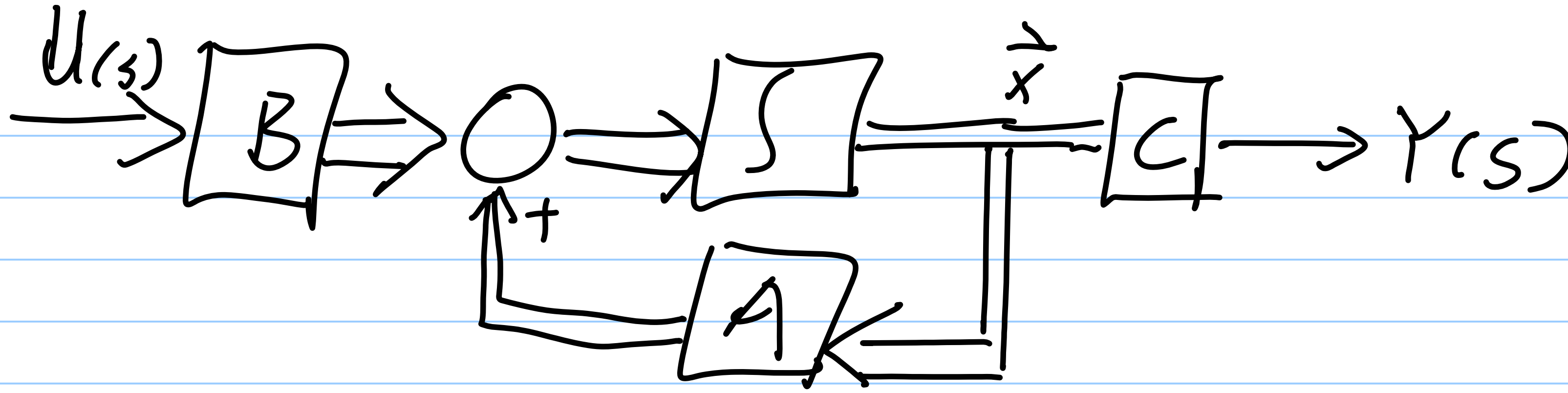


recall: $\Phi(s) = (sI - F)^{-1}$

$$\Phi(t) = \mathcal{L}^{-1}(sI - F)^{-1}$$

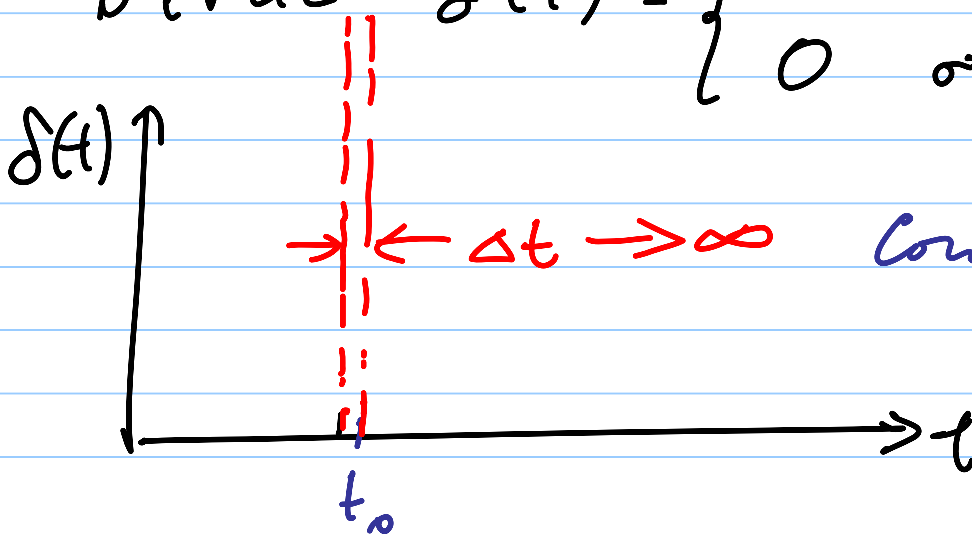
(17.48)

Transfer function $T_f(s) = \frac{Y}{U} = \underset{C}{H}(sI - \underset{A}{F})^{-1} \cdot \underset{B}{G} + \overset{D}{J}$



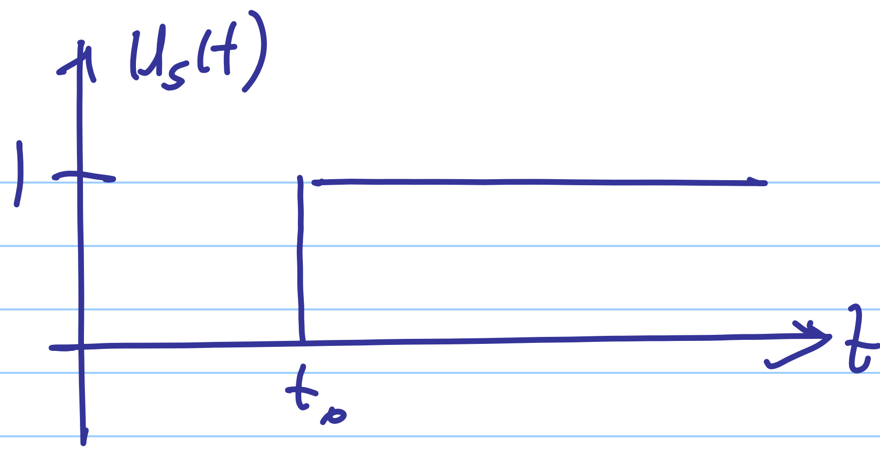
Input functions

Dirac $\delta(t) = \begin{cases} \infty & t = t_0 \\ 0 & \text{otherwise} \end{cases}$



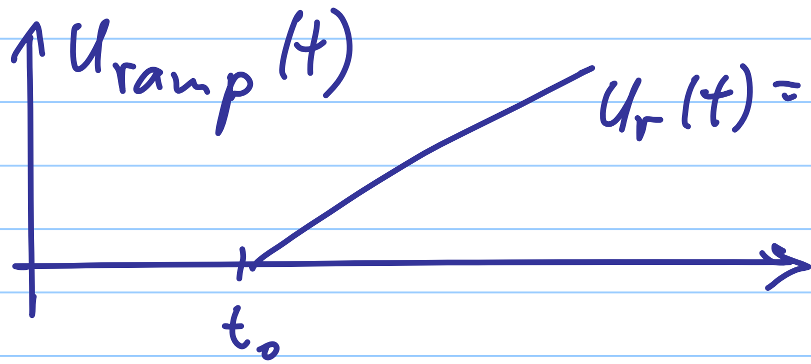
Convention: Area = 1

$\mathcal{L}(\delta(t)) = 1$



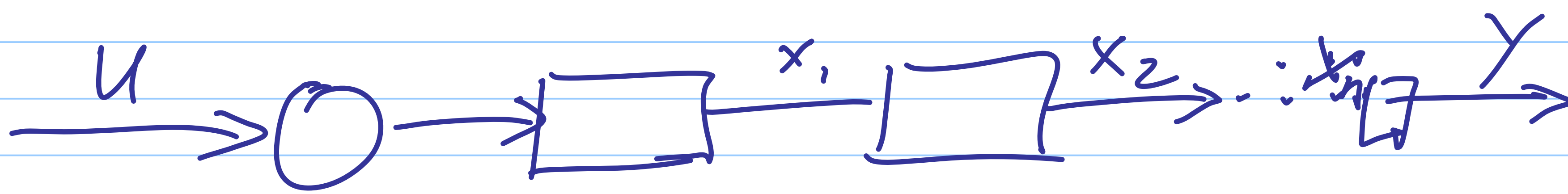
$$\int_0^\infty \delta(t)\,dt = u_s(t)$$

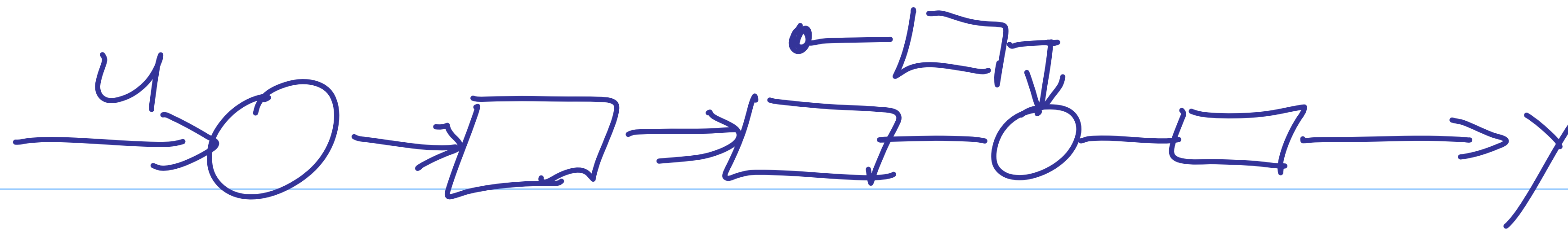
$$\mathcal{L}[u_s(t)] = \frac{1}{s}$$



$$u_r(t) = \int u_s(t)\,dt$$

$$\mathcal{L}[u_r(t)] = \frac{1}{s^2}$$



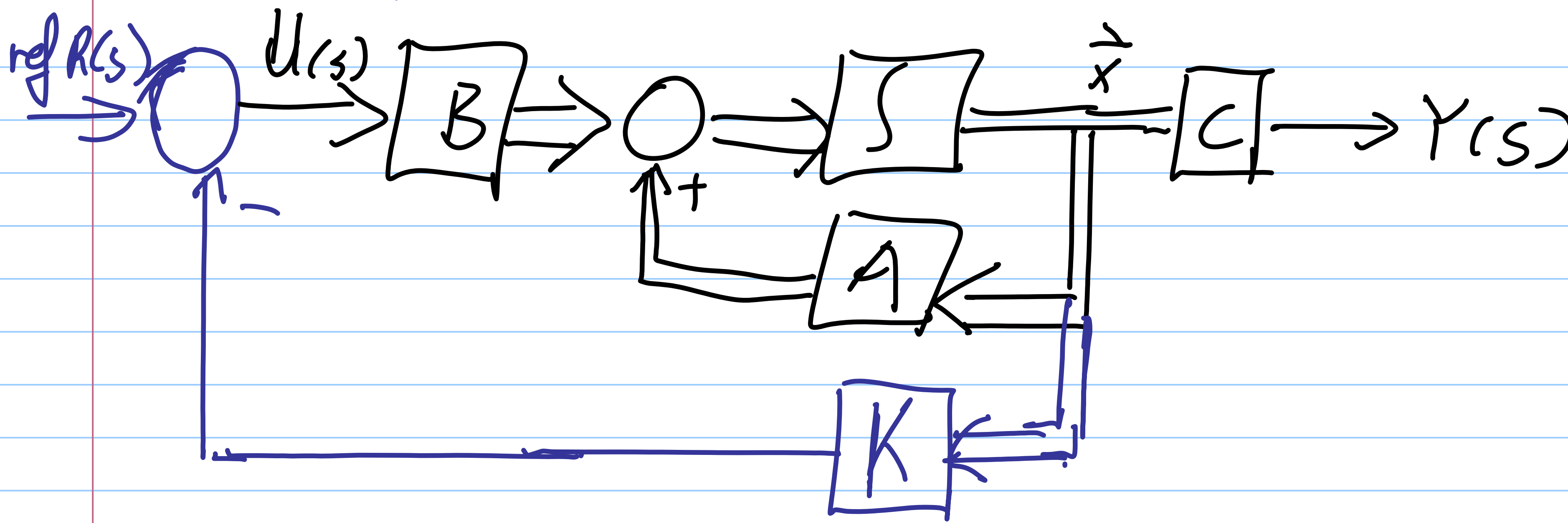


Controllability is function of state A and input B

contr. matrix $S = [B \ AB \ A^2B \ \cdots \ A^{n-1}B]$

det S must be non-zero

state feedback



K modifies each states such that closed-loop poles can be chosen arbitrarily.