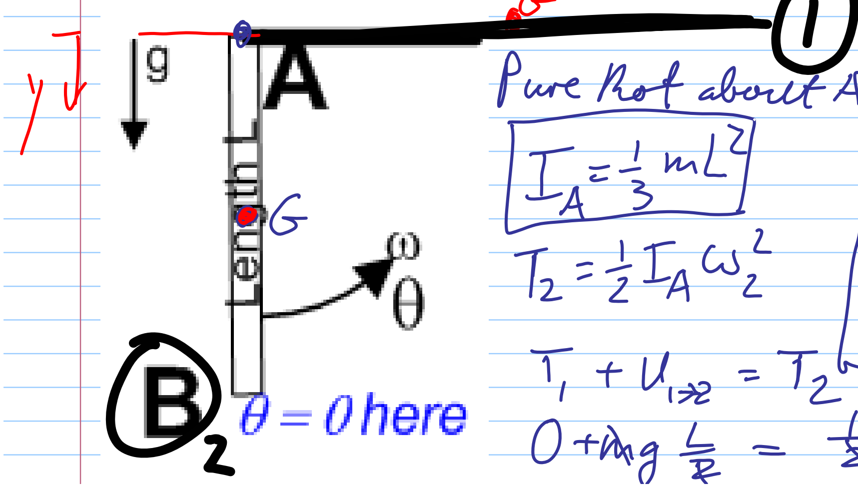
Chapter 18 Energy Methods

$dW = \vec{F} \cdot d\vec{s}$ $\quad \downarrow g \quad \downarrow y$

Note Title 11/14/2011

Pure rot. about A, release from horiz. position



$T_1 = 0$

Pure Rot about A: $I_A = I_G + m\left(\frac{L}{2}\right)^2$

$\frac{1}{12}mL^2 + \frac{1}{4}mL^2$

$$\boxed{I_A = \frac{1}{3}mL^2}$$

$$T_2 = \frac{1}{2}I_A\omega_2^2$$

$$\boxed{U_{1,2} = 0 \pm mg\frac{L}{2}}$$

$$T_1 + U_{1\to2} = T_2$$

$$0 + \cancel{m}g\frac{L}{\cancel{2}} = \frac{1}{\cancel{2}}\cdot\frac{1}{3}\cancel{m}L\omega_2^2$$

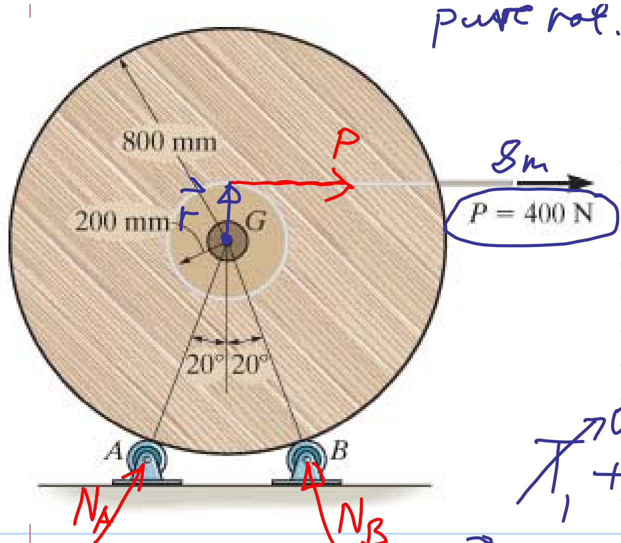
$$\omega_2^2 = \frac{3g}{L}$$

updated mid term estimates posted. (242)

T_{transl} $\frac{1}{2} m v_G^2$

T_{rot} $\frac{1}{2} I_G \cdot \omega^2$

pure rot.



$T_1 = 0$

[Problem 18.4](#)

The spool of cable, originally at rest, has a mass of 200 kg and a radius of gyration of k_G=325 (mm). (a) If the spool rests on two small rollers A and B and a constant horizontal force of P=400 N is applied to the end of the cable, determine the angular vel. at 8m. No friction

$\cancel{T_1}^{\,0} + U_{1\to 2} = T_2$

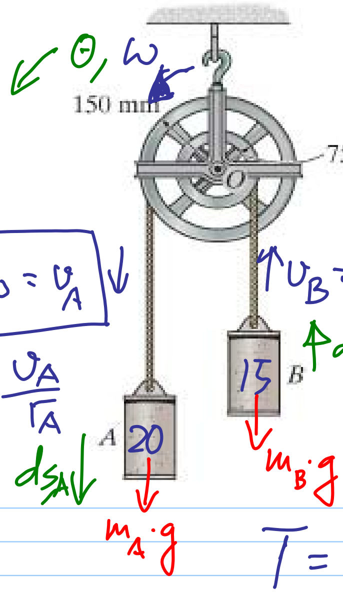
Work of P: $\vec{r} \times \vec{P}$

$400N \cdot 8m = \frac{1}{2} \cdot 200 \cdot \omega_2^2 \cdot 0.325^2$

$I_G = mk^2 = 200kg \cdot 0.325^2 \, m^2$

solve for ω_2

$\omega_2 = 17.4 \frac{rad}{s}$



Constraint

$$\boxed{r_A \cdot \omega = v_A}$$

$$v_B = r_B \cdot \omega$$

$$\omega = \frac{v_A}{r_A}$$

[Problem 18.15](#)

(a) If the system is released from rest, determine the speed of the 20kg cylinder A after A has moved downward a distance of 2m. The differential pulley has a mass of 15 kg with a radius of gyration about its center of mass of k_O= 100 mm.

$$A: dW_A = m_A \cdot g \cdot r_A \cdot d\theta$$

$$B: dW_B = -m_B g \cdot r_B \cdot d\theta$$

$$T = \frac{1}{2} I_O \omega^2 + \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$T_{Disk} = \frac{1}{2} I_O \cdot \omega^2 = \frac{1}{2} \cdot 15 \cdot 0.1^2 \cdot \omega^2 = 0.075\, \omega^2 = \frac{0.075}{0.15^2} \cdot v_A^2$$

$$T_A = \frac{1}{2} m_A v_A^2 = \frac{1}{2} \cdot 20\, v_A^2 = 10\, v_A^2 ; \quad T_B = \frac{1}{2} \cdot 15 \cdot \frac{1}{2} v_A^2$$

$$T = T_D + T_A + T_B = 15.8\, v_A^2$$

$$\text{for } s_A = 2m: \omega = \frac{v_A}{r_A} = \frac{v_A}{0.15} = 6.67\, v_A ; \quad v_B = 0.5\, v_A$$

$$\text{Work}: m_A \cdot g \cdot s_A = 20 \cdot 9.81 \cdot 2 = 392.4\, J \quad (A)$$

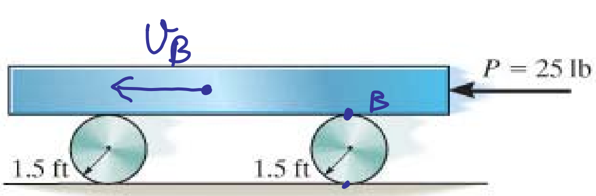
$$B: -m_B g \cdot s_B = \quad -196.2\, J$$

Energy: $0 + 392.4 - 196.2 = 15.8\, v_A^2$

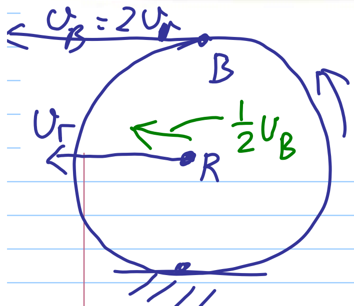
at 2 m Travel: $v_A = 3.52\ m/s \downarrow\ ;\ v_B = 1.76\ \frac{m}{s} \uparrow$

[Problem 18.30](#) $T_1 = 0$

The 100-lb block is transported a short distance by using two cylindrical rollers, each having a weight of 35-lb. (a) If a horizontal force P=25 (lb) is applied to the block, determine the block's speed after it has been displaced 2 ft to the left. Start from rest,



Rollers have inertia



$\omega = \frac{v_r}{r} = \frac{v_B}{2r}$

$U_{1\to 2} = 25\ lbf \cdot 2\ ft$

$$T_2 = \underbrace{2 \cdot \frac{1}{2} m_R \cdot \left(\frac{v_B}{2}\right)^2}_{\text{Transl. of Roller}} + 2 \cdot \frac{1}{2} \cdot I \cdot \omega_R^2 + \frac{1}{2} m_{Bar} \cdot v_B^2$$

express ω in terms of v_B

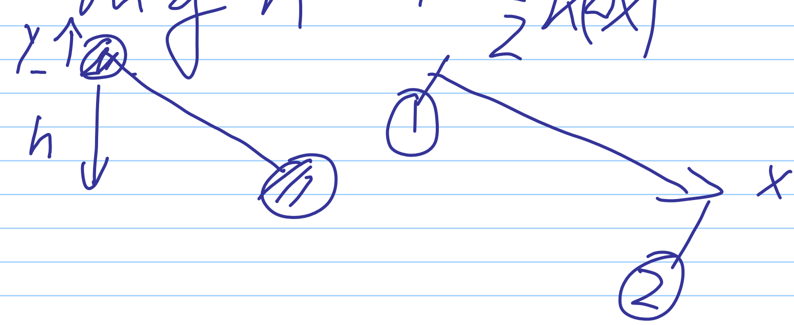
Roller: $T_{transl.} + T_{Rot}$ $\quad T_{Rot} = \frac{1}{2} \bar{I} \cdot \omega^2 = \frac{1}{2} m_R r^2 \cdot \left(\frac{v_r}{r}\right)^2$

$$= \frac{1}{2} m_R \cdot v_r^2 = \frac{1}{2} m_R \left(\frac{v_B}{2}\right)^2$$

$$v_B = 5.05 \frac{ft}{s}$$

for Monday: Read Rest of Ch. 18!

$$V = mgh + \frac{1}{2} k (\Delta x)^2$$



18.5 Conservation of Energy

3rd Midterm Wed 4/26

Topics: Ch. 17, 18
Ch. 15.1–3 Impulse

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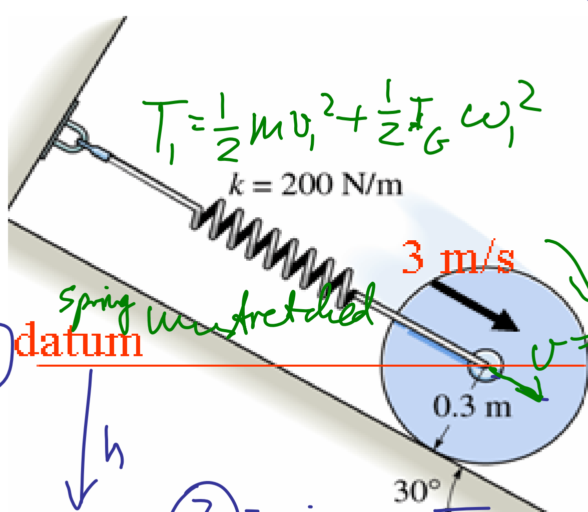
2 revs. $= 2 \cdot 2\pi \cdot r = 4 \cdot 0.3 \cdot \pi = 1.2\pi$

vertical drop $h = 1.2\pi \sin 30°$

$T_1 = \frac{1}{2} m v_1^2 + \frac{1}{2} I_G \omega_1^2$



spring unstretched

$v = 3\,m/s$

$v = r\omega \Rightarrow \omega = \frac{v}{r} = \frac{3}{0.3} = 10 \frac{rad}{s}$ clockwise

at Position 2

① ②

h

$V_1 = 0$

$T_2 = \frac{1}{2} m v_2^2 + \frac{1}{2} I_G \cdot \omega_2^2$

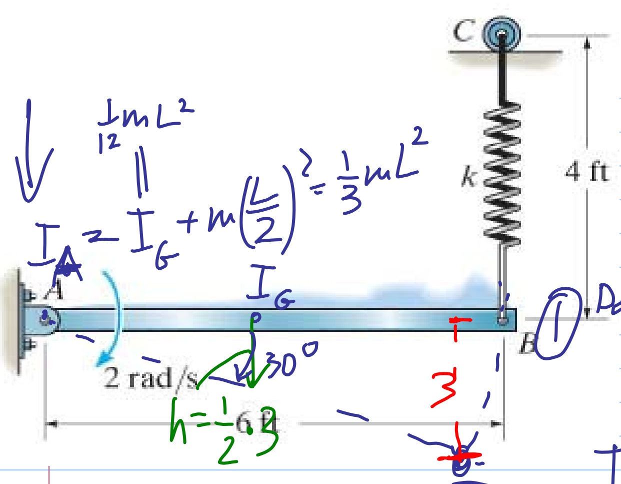
$V_2 = \frac{1}{2} k \cdot (1.2\pi)^2 - 1.2\pi \cdot \sin 30°$

$$T_1 + V_1 = T_2 + V_2$$

$$T_1 = \frac{1}{2} m v_1^2 + \frac{1}{2} I_G \omega_1^2 + 0 = \frac{1}{2} m v_2^2 + \frac{1}{2} I_G \cdot \omega_2^2 + \frac{1}{2} k \cdot (1.2\pi)^2 - 1.2\pi \cdot \sin 30°$$

substitute $v = r\omega$

can solve for ω_2



$$\frac{1}{12} m L^2 \quad I_A = I_G + m\left(\frac{L}{2}\right)^2 = \frac{1}{3} m L^2$$

I_G ; $30°$; $h = \frac{1}{2} \cdot 3$; 3 ; ① Datum ; ②

P[roblem 18.40](#)

At the instant shown, the [illegible] bar rotates clockwise at [illegible] The spring attached to its end always remains vertical due to the roller guide at C. (a) If the spring has an [illegible] and a stiffness of k=6 (lb)/(ft), determine the angular vel.

Approach: pure rot. about A $\Rightarrow I_A$

$$T_1 + V_1 = T_2 + V_2$$

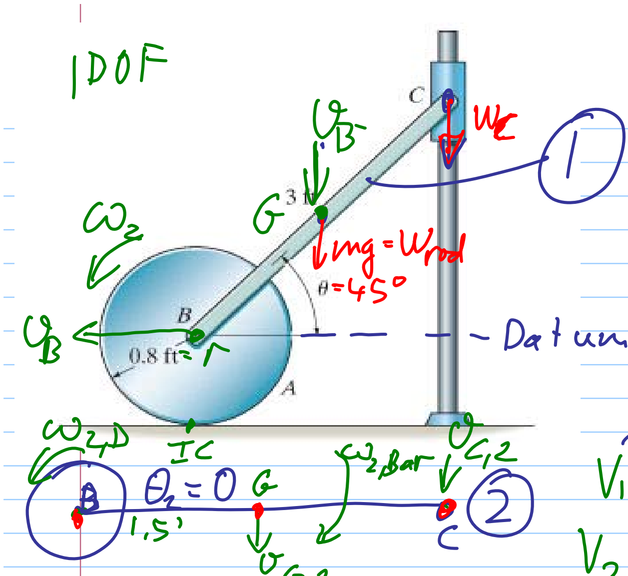
$$\frac{1}{2} I_A \cdot \omega_1^2 + 0 + \frac{1}{2} k (x - x_0)^2 = \frac{1}{2} I_A \cdot \omega_2^2 + \frac{1}{2} k (x_2 - x_0)^2 - 1.5 \cdot 50$$

2^2 ; $(4-2)$; $(7-2)^2$; $m \cdot g$

We can solve for ω_2: $\omega_2 = -2.3k \cdot \frac{rad}{s}$

Recap: Define Datum, 1 and 2

1DOF



[Problem 18.45 •](#)

The system consists of a 20-lb disk A, 4-lb slender rod BC, and a 1-lb smooth collar C. (a) If the disk rolls without slipping, determine the velocity of the collar at the instant the rod becomes horizontal, i.e., theta=0 degree(s). The system is released from rest at theta= 45 deg.

$T_1^{\nearrow 0} + V_1 = T_2 + V_2$ (1)

$V_1 = 0 + 1\,lb \cdot 3 \cdot \sin 45° + 4\,lb \cdot 1.5 \sin 45°$

$V_2 = 0 + 0 + 0$

$T_2 = \frac{1}{2} I_B \cdot \omega_2^2 \cdot \frac{1}{2} m_C \cdot v_{C,2}^2 + \frac{1}{2} m_B \cdot v_{B,2}^2$ (2) 2 constraint equations

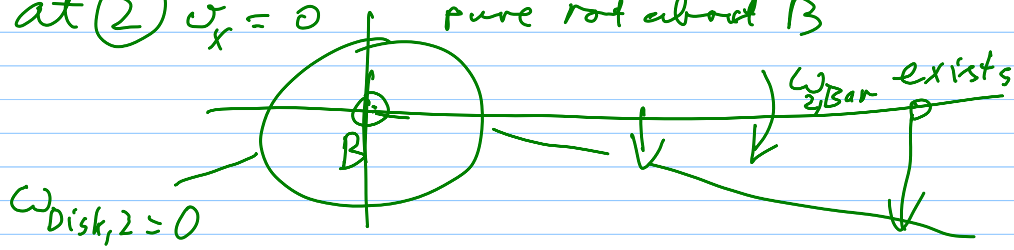
$v_B = r\omega_{Bat}$; $v_{G,2} = 1.5 \cdot \omega_2^{(3)} \Rightarrow \omega_{2,Bar} = \frac{v_G}{1.5}$;

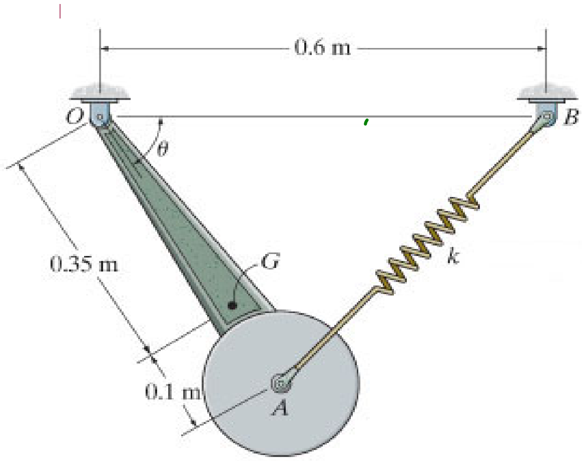
$$\boxed{v_{C,2} = 3\omega_2 \quad (4)}$$

insert (3) and (4) into energy eq.

$v_C = 13.3 \frac{ft}{s}$

at (2) $v_x = 0$ pure rot about B





Problem 18.51

The 30-kg pendulum has its mass center at G and a radius of gyration about point G of k_G = 300 mm. (a) If it is released from rest when theta=0 degree(s), determine its angular velocity at the instant theta = 90 degree(s). Spring AB has a stiffness of k = 300 N/m and is unstretched at theta = 0.