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Last First

UNIVERSITY OF NEVADA, LAS VEGAS
DEPARTMENT OF MECHANICAL ENGINEERING

EGG 207, Spring 2000, **Final Examination**

Closed Book, three pages of handwritten notes allowed.

1. (10 points) The speed of a car is increased at a constant rate from 66 km/h to 90 km/h over a distance of 180 m along a curve with 240-m radius. Determine the magnitude of the total acceleration after the car has traveled 100 m along the curve.

$V_1 = \frac{66 \cdot 10^3}{3.6 \cdot 10^3} = 18.3 \frac{m}{s}$ $\qquad V_2 = 25 \, m/s$

$v\,dv = a\,dx$

$\frac{1}{2}(V_2^2 - V_1^2) = a_t \cdot 180\,m$

$a_t = \frac{25^2 - 18.3^2\,m^2}{2 \cdot 180\,m\,s^2} = 0.8 \frac{m}{s^2}$

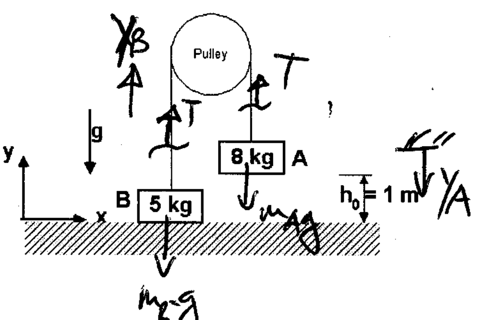
at 100 m: $\frac{1}{2}(V_{100}^2 - V_1^2) = a_t \cdot 100\,m = 80 \frac{m^2}{s^2}$

$V_{100}^2 = V_1^2 + 2 \cdot 80 = 496 \frac{m^2}{s^2}$, $V_{100} = 22.3 \frac{m}{s}$

$\vec{a} = (a_t^2 + a_n^2)^{1/2} = 0.8 \frac{m}{s^2} + r \cdot \frac{v^2}{r^2} = \sqrt{0.8^2 + \left(\frac{496}{240}\right)^2} = 2.2 \frac{m}{s^2}$

Answer: a_{total} = 2.2	(m/s² units)

2. (15 points) The two blocks with masses m_A = 8 kg and m_B = 5 kg are initially at rest. Neglecting the masses of the pulleys and all friction, determine **(a)** the acceleration of block A
(b) the velocity of block A immediately before hitting the floor at y=0.



B: $T - m_B \cdot g = m_B \cdot \ddot{y}_B$ (1) $\qquad y_B = y_A = y$ (3)

A: $m_A g - T = m_A \cdot \ddot{y}_A$ (2)

solve (1) for T and insert into (2)!

$m_A g - m_B(\ddot{y} + g) = m_A \cdot \ddot{y}$

$g(m_A - m_B) = (m_A + m_B)\ddot{y}$

(a) $\ddot{y} = \frac{9.81 \cdot 3\,\cancel{kg}}{13\,\cancel{kg}} \frac{m}{s^2} = 2.26 \frac{m}{s^2}$

$v\,dv = a\,dx$

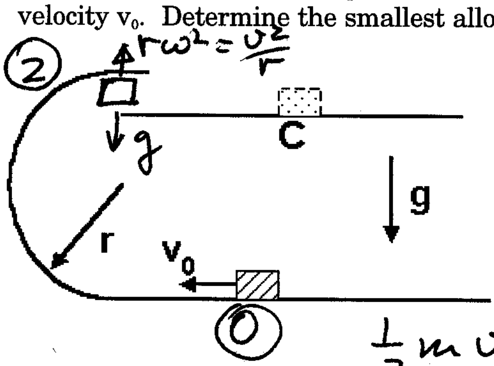
$\frac{1}{2}(v_1^2 - \cancel{v_0^2}^{\,0}) = 2.26 \frac{m}{s^2} \cdot 1\,m$

(b)

$V_1^2 = 2 \cdot 2.26 \frac{m^2}{s^2}$

a_a = 2.26	(m/s² units)
v_A (y=0) = 2.13	(m/s units)

3. (20 points) A package (point mass) with mass m is projected into a vertical return loop with radius r at velocity v_0. Determine the smallest allowable velocity so that the package can reach the horizontal surface at C. No friction.



at ②: $\frac{v^2}{r} \geq g$ for staying on track.

thus: $v_2^2 = rg$; only gravity does work:

$T_1 + U_{1\to 2} = T_2$ $\qquad U_{1\to 2} = -2rgm$

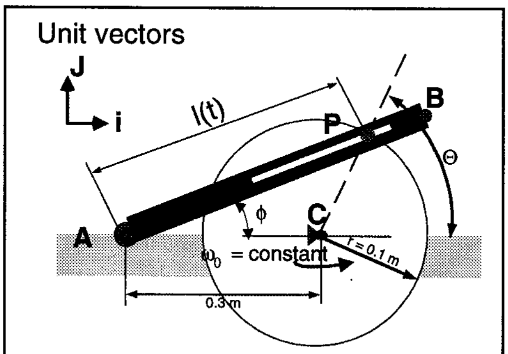
$\frac{1}{2} m v_0^2 - 2rgm = \frac{1}{2} m v_2^2$

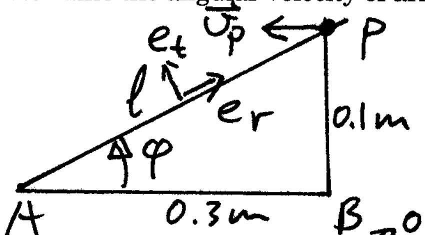
$\cancel{m} v_0^2 - 2rg \cdot 2 = \cancel{m} v_2^2 = \cancel{m} rg$

$v_0^2 = 5rg$

$v_0 =$ $\sqrt{5rg}$

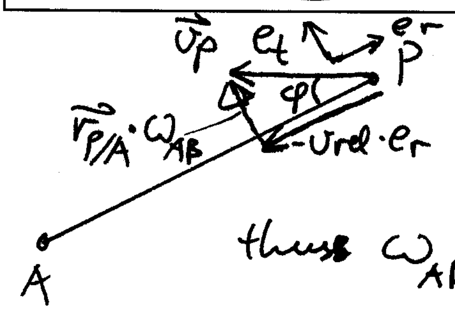
4. (20 points) The flywheel (r = 0.1 m) shown is rotating about C at constant ω_0 = 50 rad/s. Pin P slides in the slot of arm AB. Point A is fixed. AC = 0.3 m.
Determine the angular velocity of arm AB as Θ = 90 degrees.





Geometry:
$\ell = \sqrt{0.1^2 + 0.3^2} = 0.316\,m$
$\varphi = \tan^{-1}\frac{0.1}{0.3} = 18.43°$

Pin P: $\vec{v}_P = \cancel{\vec{v}_A} + \vec{\omega}_{AB} \times \vec{r}_{P/A} + \vec{v}_{rel}$ $\qquad \vec{v}_A \to 0$

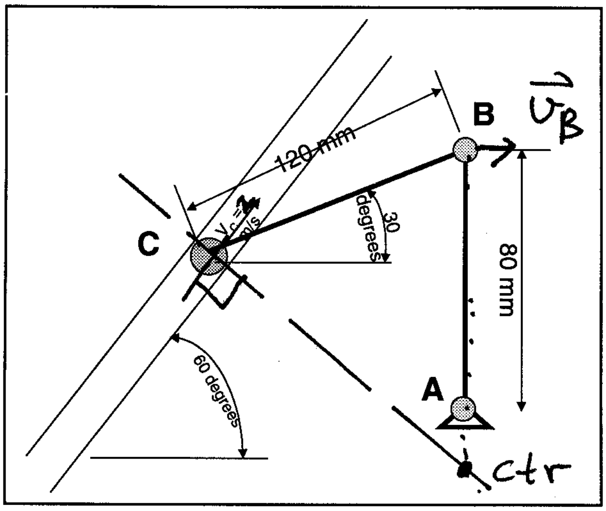


using $e_r - e_t$ frame attached to AP:
the vector sum $-v_{rel} \cdot e_r + r_{P/A} \cdot \omega_{AB}$ must result in $v_P = 0.1 \cdot 50 = -5i\,\frac{m}{s}$

thus $\omega_{AB} \cdot \ell = 5 \sin\varphi$; $\omega_{AB} = \frac{5}{\ell} = \frac{5 \cdot 0.316}{0.316} \frac{m}{s\,m} = 5 \frac{rad}{s}$

Answer $\omega_{AB} =$ 5 rad/s

5. (15 points) Roller C moves up the channel at constant velocity $v_C = 2$ m/s. For the instant shown at left, determine (a) the instantaneous center of motion of bar BC and its <u>distance from point B</u>. (construct the instantaneous center in the graph, and label it clearly)
(b) the angular velocity of bar AB



using inst. center:

$v_C = 2\frac{m}{s} = 0.12m \cdot \omega_{BC}$

$\Rightarrow \omega_{BC} = \frac{2}{0.12} = 16.67 \frac{rad}{s}$

$2\frac{m}{s}$ B $2 m/s$ C Ctr isosceles Triangle

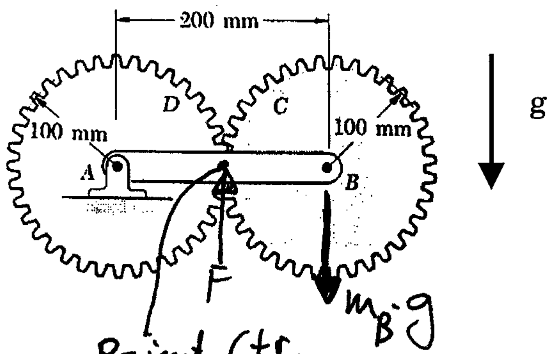
$V_B = r \cdot \omega_{BC} = 2 m/s$

(b) B $\rightarrow 2 m/s$, 0.08m, A

$\omega_{AB} = \frac{v_B}{r_{B/A}} = \frac{2}{0.08} = 25 \frac{rad}{s}$ ↻

(a) Inst. Center : construct in graph, Distance Ctr – B = 0.12	(m units)
(b) ω_{AB} = $-25k \frac{rad}{s}$ ↻	

6. (20 points) Gear D is stationary. Gear C has mass = 5 kg and radius of gyration = 75 mm. Bar AB has no mass. As the system is released from rest, determine the angular acceleration of gear C.



$\Sigma M_{(P)} = -mgr = I_C \cdot \alpha_C$

$I_C = mk^2 + mr^2 = m(k^2 + r^2)$

$\alpha_C = -\frac{\cancel{m}gr}{\cancel{m}(k^2+r^2)} = \frac{-9.81 \cdot 0.1}{0.0156} \frac{m \cdot m}{s^2\, m^2}$

$= -62.78 \frac{rad}{s^2} k$ ↻ clockwise

α_C = $62.78\ rad/s^2$ ↻