Today:

Chapter 6 continued: Dimensions and Units

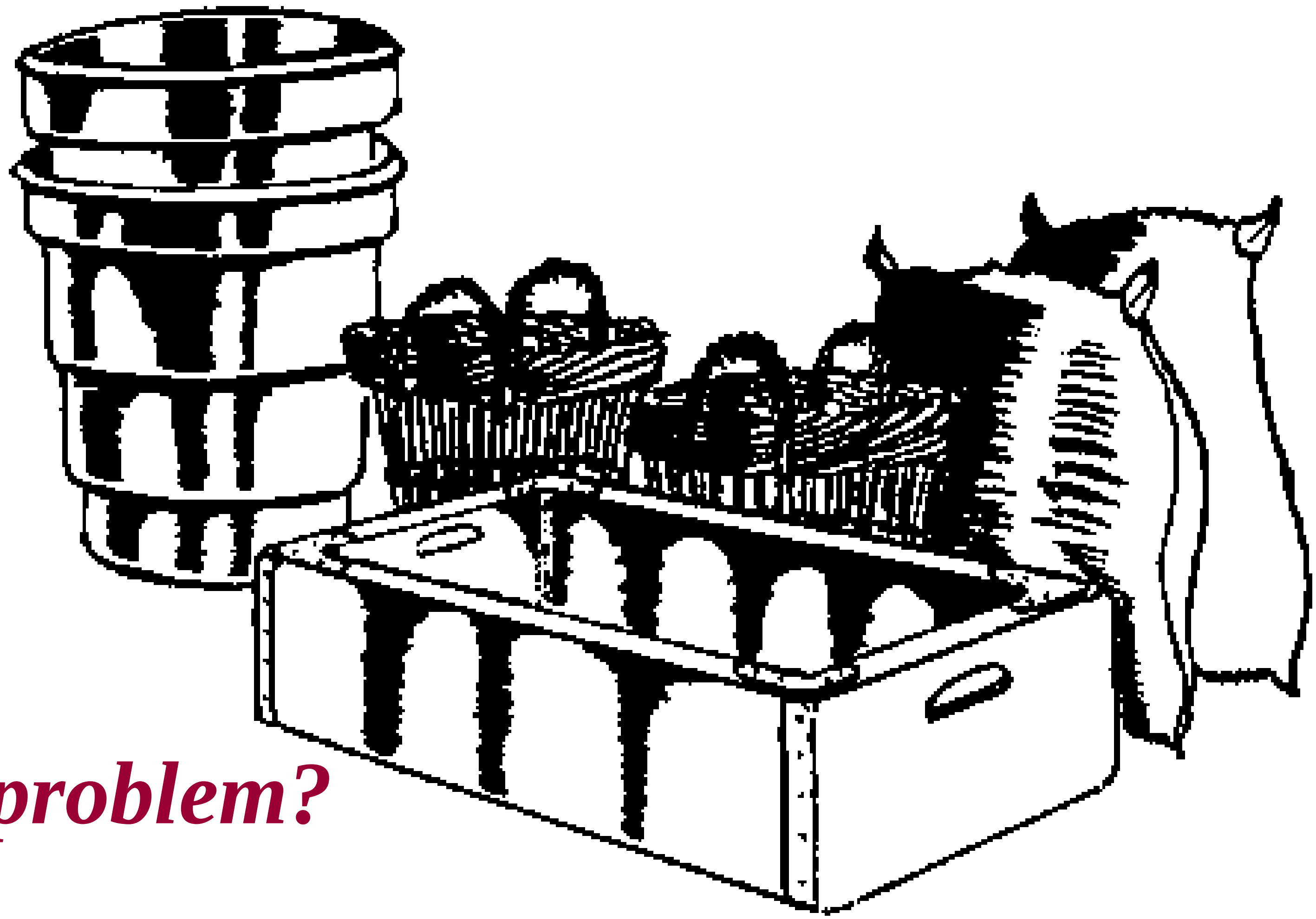
English units of measurement

A system of weights and measures used in a few nations, the only major industrial one being the United States. It actually consists of two related systems—the U.S. Customary System of units, used in the United States and dependencies, and the British Imperial System.

Units of Weight

The pound (lb) is the basic unit of weight (which is proportional to mass) ***(how?).*** Within the English units of measurement there are three different systems of weights. In the avoirdupois system, the most widely used of the three, the pound is divided into 16 ounces (oz) and the ounce into 16 drams. The ton, used to measure large masses, is equal to 2,000 lb (short ton) or 2,240 lb (long ton). In Great Britain the stone, equal to 14 lb, is also used.

Question:



Is there a problem?

“**weight** is proportional to mass.”

Answer: You need to be aware of the law governing that proportionality: Newton's Law

Force = Mass* Acceleration

Question:

Is there a problem?

"**weight** is proportional to mass."

Force = Mass* Acceleration

Acceleration is NOT a constant, mass is.

Even on earth, $g = 9.81\ m/s^2$ is NOT constant, but varies with latitude and elevation.

“**weight** is proportional to mass.”

Another problem arises from the common intermingling of the terms "mass" and "weight", as in:

"How much does a pound of mass weigh?"

Or:

"If you don't know whether it's pound-mass or pound-weight, simply say pounds."

A mass is NEVER a "weight".

Force = Mass* Acceleration

"Weight" = Force

“Weight” = Force

…because on earth all masses are exposed to gravity. So

Weight =m * g

The notion of ‘weight’ is not very useful in engineering, because many situations are not static.

Forces in SI Units

$1N = 1\ kg * 1m/s^2$

When SI units are used, the factor g_c is 1. There is no need for g_c in the SI system.

Forces in US Customary Units

Force is a Basic unit
Mass becomes a derived unit:
1 lb-mass (lbm)

1lbf = 1lbm*g or 1 lbm = 1lbf/g
Where g is approx. 32.2 ft/s^2

Conversions

Example: convert meters to miles.
Conversion factor: 1 mile = 1609 m
3200m =? miles

Answer:
3200m = 3200m* 1mi/1609m
= 3200/1609 mi = 1.989 miles

Conversions

In class exercise: convert feet to miles.

Conversion factor: 1 mile = 5280 ft

9800 ft =? miles

Conversions

In class exercise (speed): convert miles/hour to feet/second.
Conversion factor: 1 mile = 5280 ft

45 mph = ??? ft/s

In class exercise (speed): convert miles/hour to feet/second.

Conversion factor: 1 mile = 5280 ft

45 mph = ??? ft/s

Step by step procedure:

$$45mph = \frac{45mi}{1hr}$$

We can multiply by

$$1 = \frac{5280\,ft}{1mi}$$

In class exercise (speed): convert miles/hour to feet/second.

Conversion factor: 1 mile = 5280 ft

45 mph = ??? ft/s

Step 2:

Thus:

$$45mph = \frac{45mi}{1hr} * \frac{5280\,ft}{1mi}$$

In class exercise (speed): convert miles/hour to feet/second.
Conversion factor: 1 mile = 5280 ft
45 mph = ??? ft/s

Step 3:

We repeat the process for the time:

$$45mph = \frac{45mi}{1hr} * \frac{5280\,ft}{1mi}$$

Multiply by 1 = $\frac{1hr}{3600s}$

In class exercise (speed): convert miles/hour to feet/second.

Conversion factor: 1 mile = 5280 ft

45 mph = ??? ft/s

Step 4:

We get:

$$45mph = \frac{45mi}{1hr} * \frac{5280\,ft}{1mi} * \frac{1hr}{3600s}$$

We can now simplify the fractions.

The units *mi* and *hr* cancel.

In class exercise (speed): convert miles/hour to feet/second. 45 mph = ??? ft/s

Step 5:

We get: $$45mph = 45 * \frac{5280\,ft}{3600s} = 66\frac{ft}{s}$$

Plausibility check:

The resulting units ***must*** be distance/time.

The resulting numbers ***must*** be consistent with the original question.

Conversions

Pressure = Force per unit area

In class:

Convert 35,000 Pa to psi, using basic units such as feet and pounds-force.

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Convert 35,000 Pa to psi, using basic units such as feet and pounds-force.

Definitions:

$$1Pa = \frac{1N}{m^2}$$

$$1psi = \frac{1lbf}{in^2}$$

In class:

Convert 35,000 Pa to psi, using basic units such as feet and pounds-force.

Solution:

$$35000 Pa = \frac{35000 N}{m^2} * \frac{0.2248 lbf}{1N} * \frac{1 m^2}{39.37^2 in^2}$$

In class:

Convert 35,000 Pa to psi, using basic units such as feet and pounds-force.

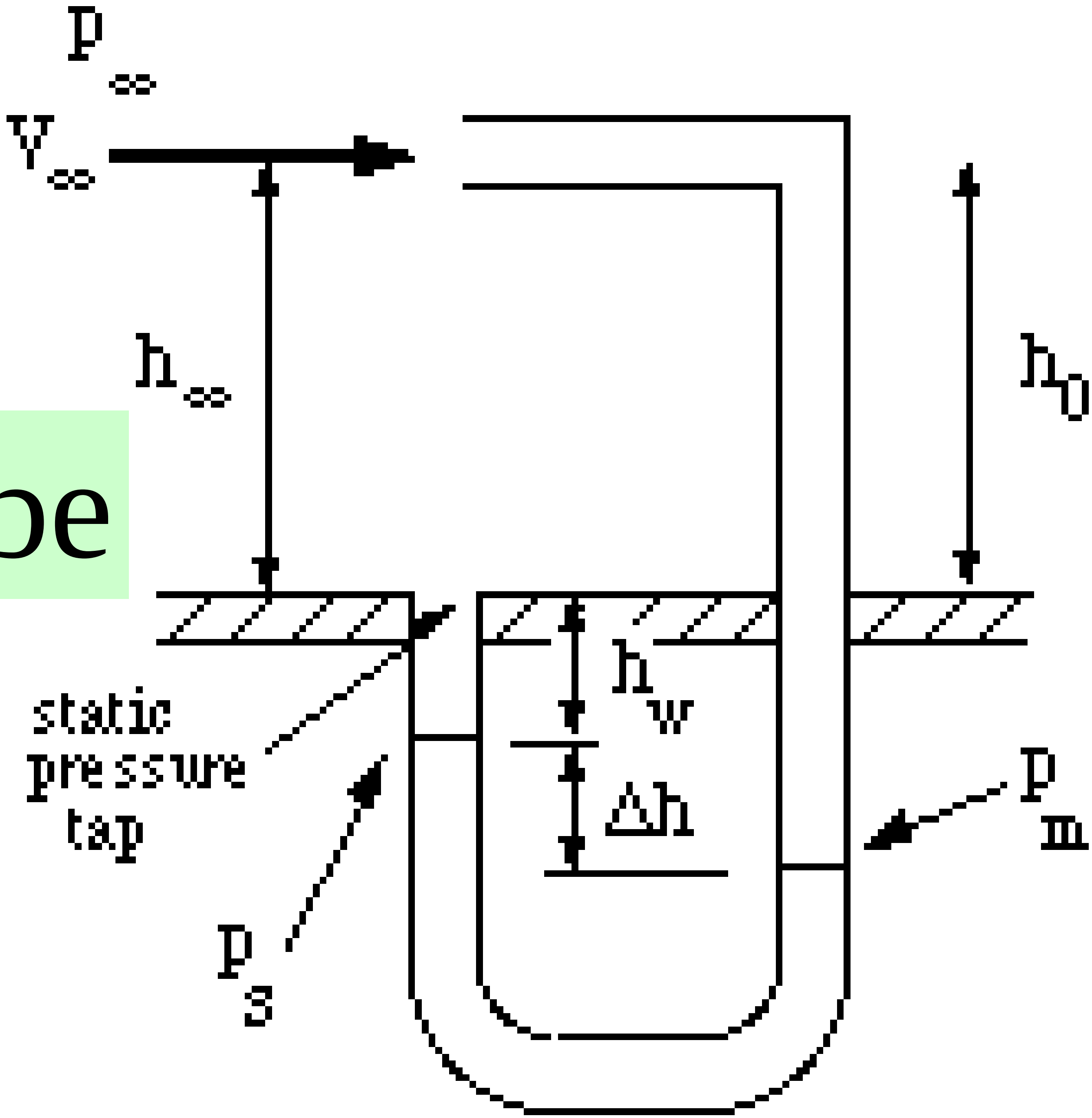
Solution:

Thus: $$35000\,Pa = 35000 * \frac{0.2248\,lbf}{39.37^2\,in^2} = 5.076\,psi$$

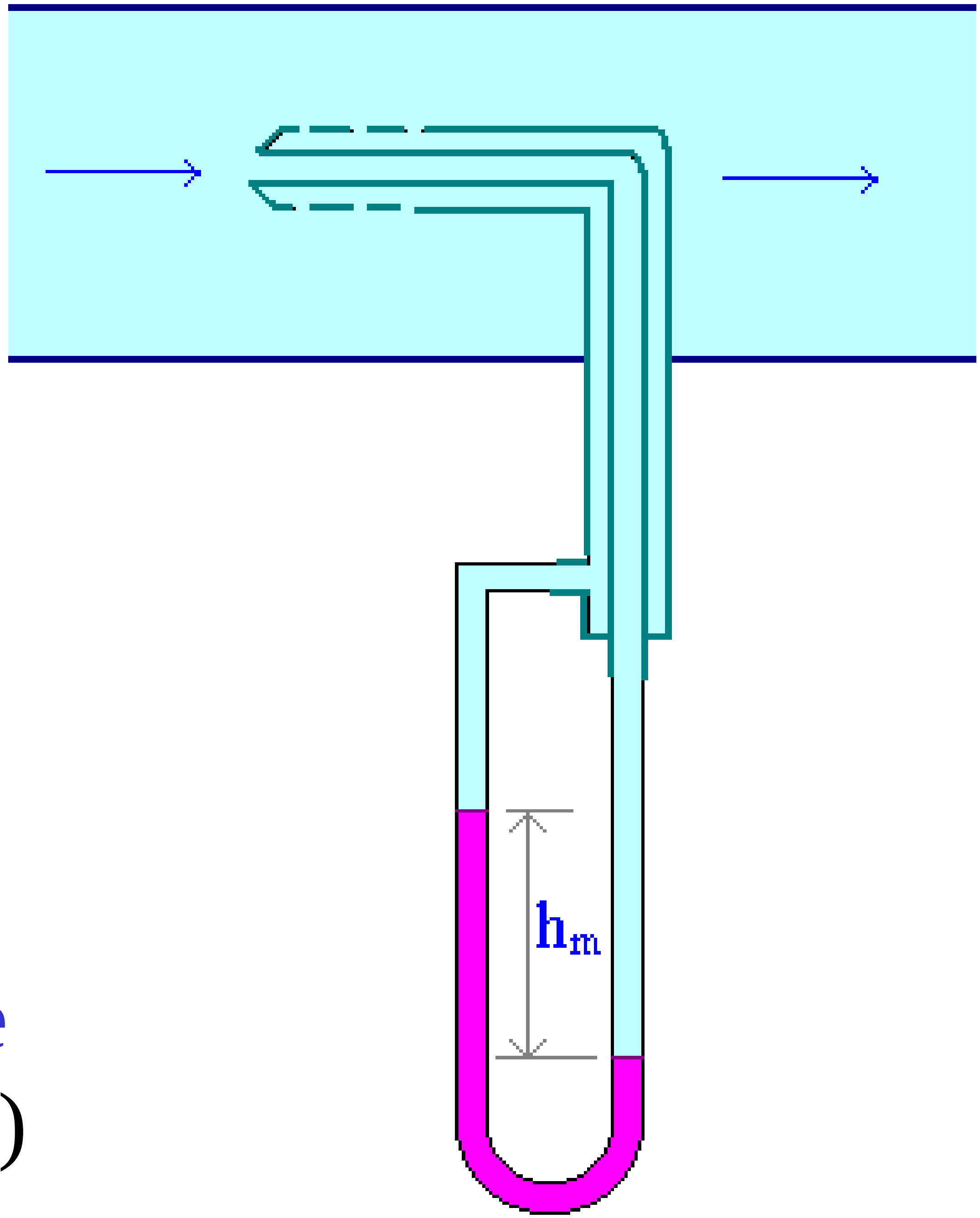
Some measurements involve multiple units and conversions

Example: Flow speed measurement with a Pitot tube

Pitot tube

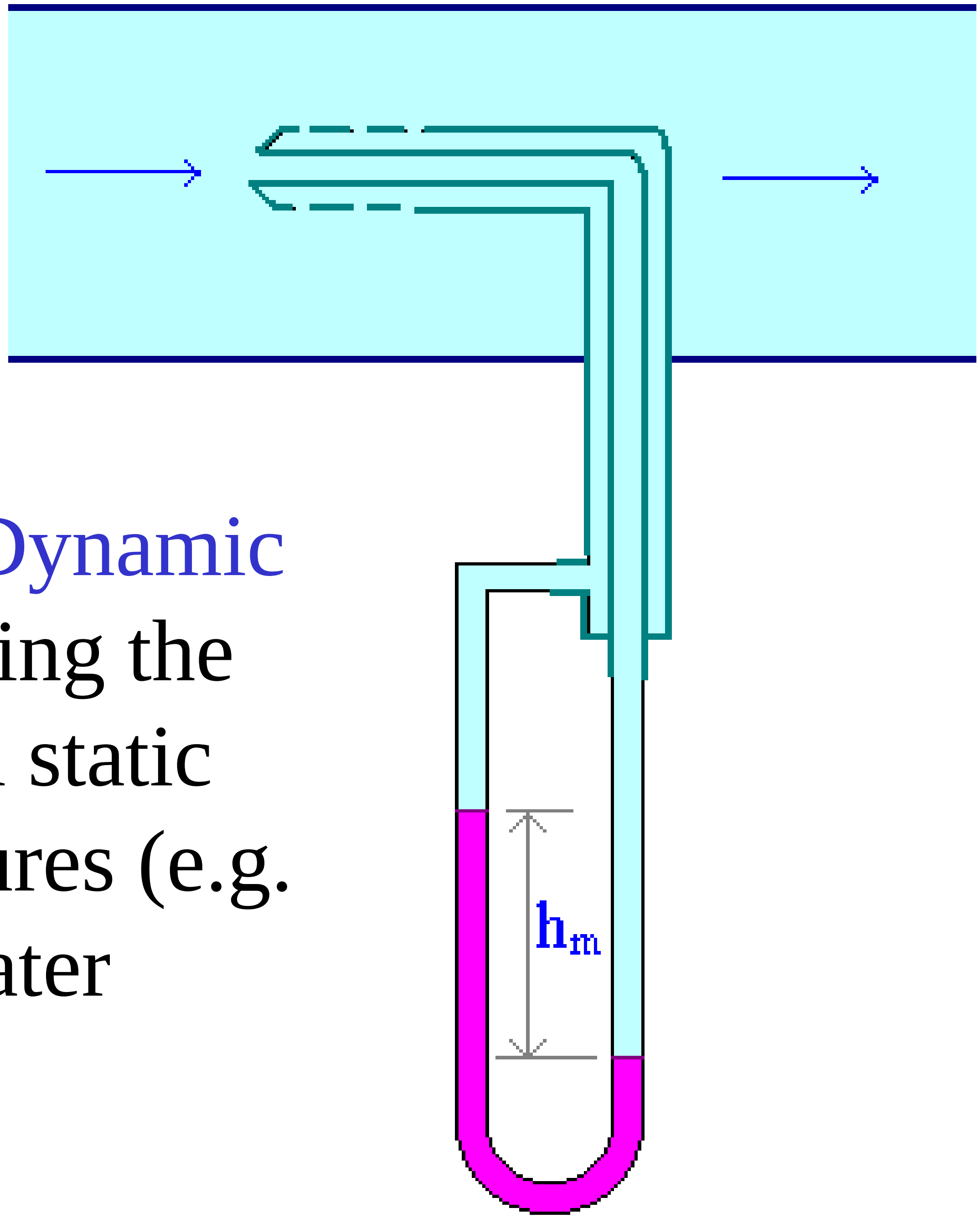


Pitot tube



According to Bernoulli's law,
Total pressure =
Static pressure +
Dynamic pressure
(from flow speed.)

Pitot tube



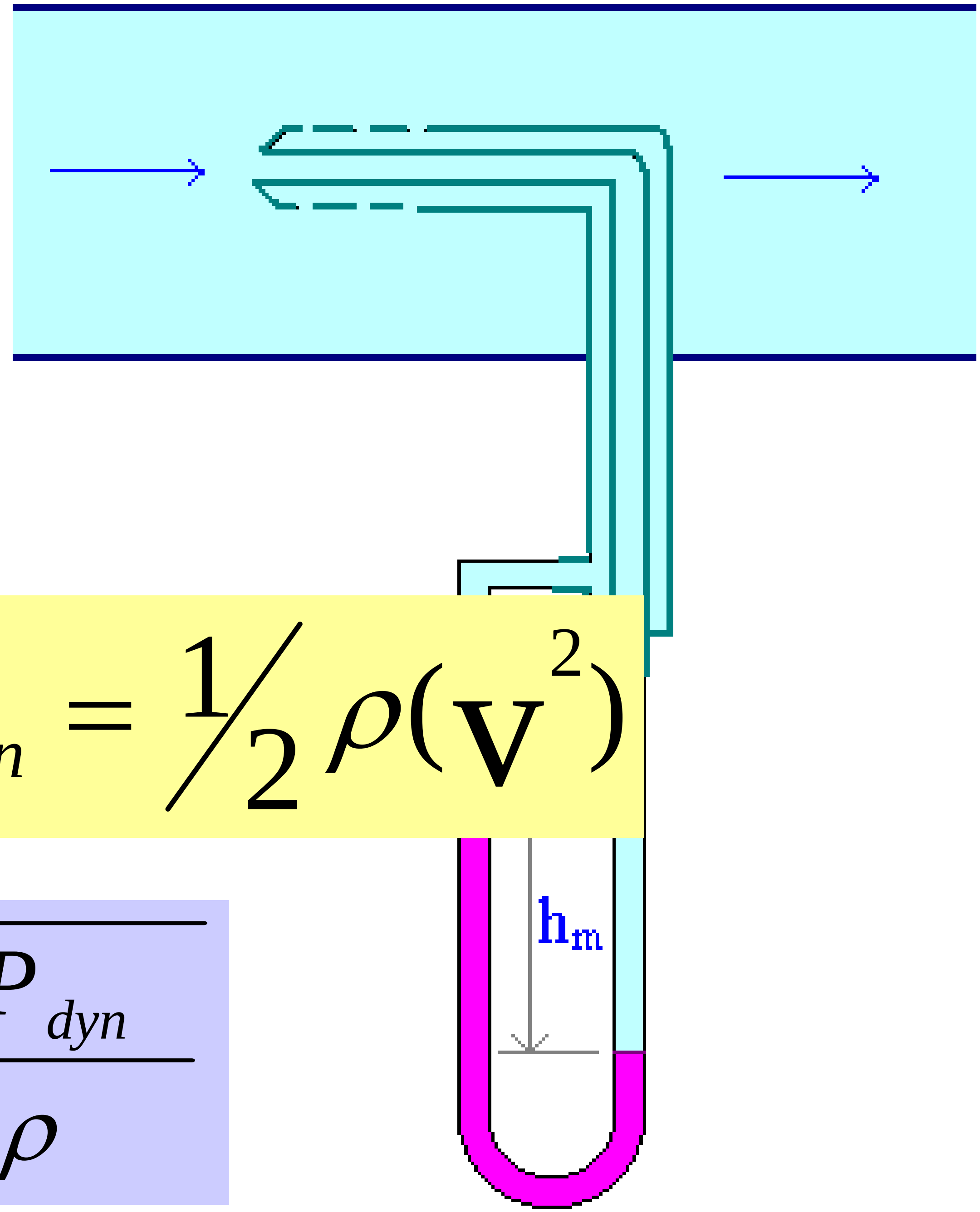
We can detect the Dynamic pressure by measuring the **difference** between static and dynamic pressures (e.g. **height h_m** of the water column at right)

The Bernoulli equation states:

$$p + \frac{1}{2}\rho V^2 = constant$$

where p *is the pressure,* ρ *is the density,* V *is the velocity,*

Pitot tube



Flow speed **v** is found from the dynamic pressure **P_{dyn}** reading:

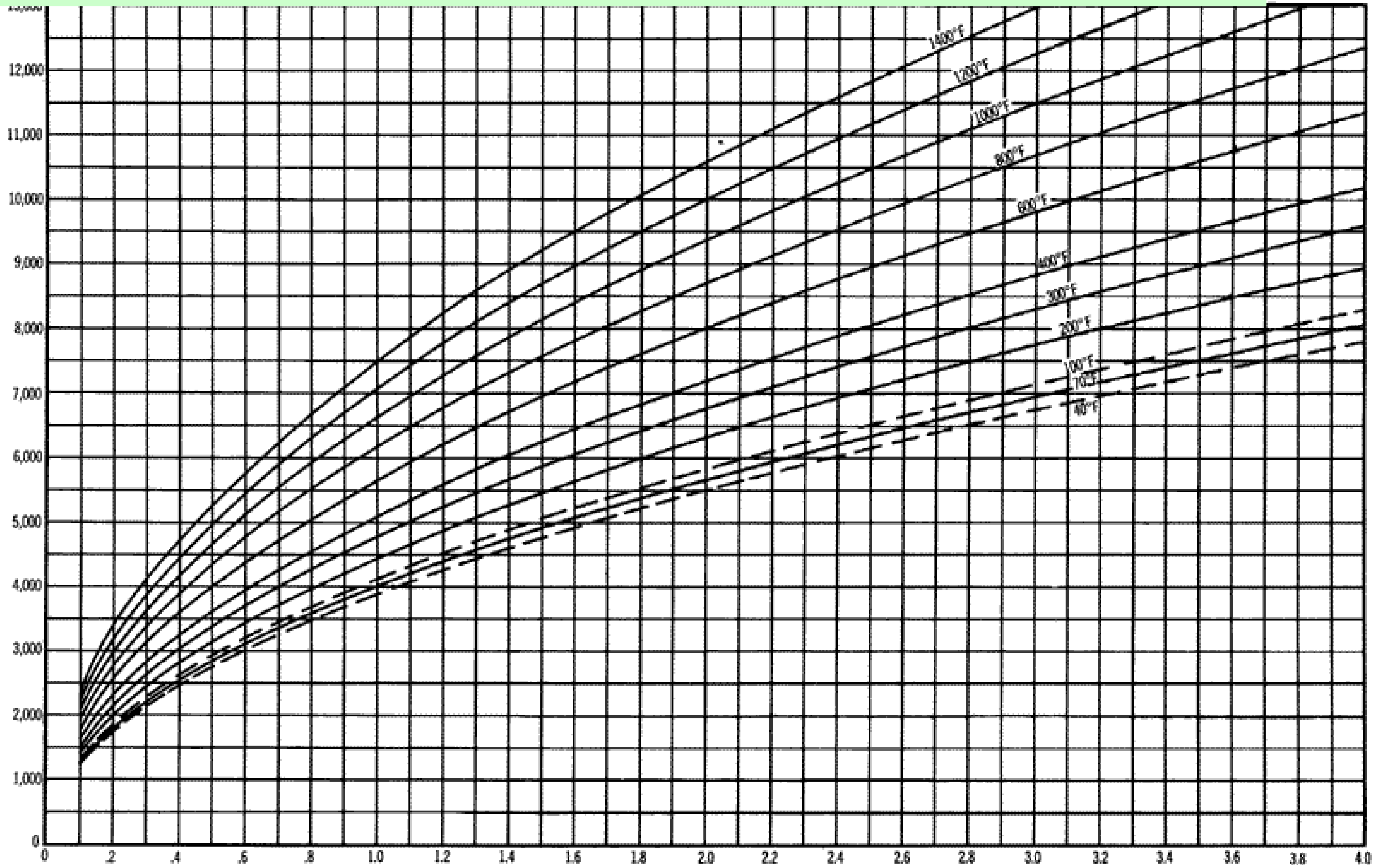
$$P_{dyn} = \tfrac{1}{2}\rho(\mathbf{v}^2)$$

or

$$v = \sqrt{\frac{2\,P_{dyn}}{\rho}}$$

Pitot tube characteristic

Air velocity (ft/minute)



Dyn. Pressure (inches of H_2O)

SI

$$v = \sqrt{\frac{2\,P_{dyn}}{\rho}}$$

In class: Pitot tube measurement analysis

Given: P_{dyn} = 500 Pa

ρ_{Air} = 1.2 kg/m^3 at 1.01*10^5 Pa

Determine the air velocity in m/s

SI

$$v = \sqrt{\frac{2\ P_{dyn}}{\rho}}$$

Inserting: P_{dyn} = 500 Pa

ρ_{Air} = 1.2 kg/m^3

gives:

$$v = \sqrt{\frac{2 * 500}{1.2} \frac{N}{m^2} \frac{m^3}{kg}}$$

SI

$$v = \sqrt{\frac{2 * 500}{1.2} \frac{N}{m^2} \frac{m^3}{kg}}$$

We substitute : 1N with 1 kgm/s^2

$$v = \sqrt{\frac{2 * 500}{1.2} \frac{N}{m^2} \frac{m^3}{kg} \frac{1\,kgm}{1\,N * s^2}}$$

Simplification gives:

$$v = \sqrt{833 \frac{m^2}{s^2}} = 28.9\, m/s$$

SI

$$v = \sqrt{\frac{2\,P_{dyn} \;*\; g_c}{\rho}}$$

US Cust.

$$v = \sqrt{\frac{2P_{dyn} * g_c}{\rho}}$$

In class: Pitot tube measurement analysis

Given: P_{dyn} = 19 lbf/ft^2

ρ_{Air} = 0.0735 lbm/ft^3 at 14.7 psia

Determine the air velocity in ft/s

US Customary example

$$v = \sqrt{\frac{2P_{dyn} * g_c}{\rho}}$$

Inserting: P_{dyn} = 19 lbf/ft^2

ρ_{Air} = 0.0735 lbm/ft^3

$$v = \sqrt{\frac{2*19lbf}{ft^2}\frac{32.2}{1}\frac{lbm*ft}{lbf*s^2}\frac{ft^3}{0.0735lbm}}$$

$$v = \sqrt{\frac{2*19lbf}{ft^2}\,\frac{32.2}{1}\,\frac{lbm*ft}{lbf*s^2}\,\frac{ft^3}{0.0735lbm}}$$

Simplification gives:

$$v = \sqrt{16648\frac{ft^2}{s^2}} = 129\, {ft}/{s}$$

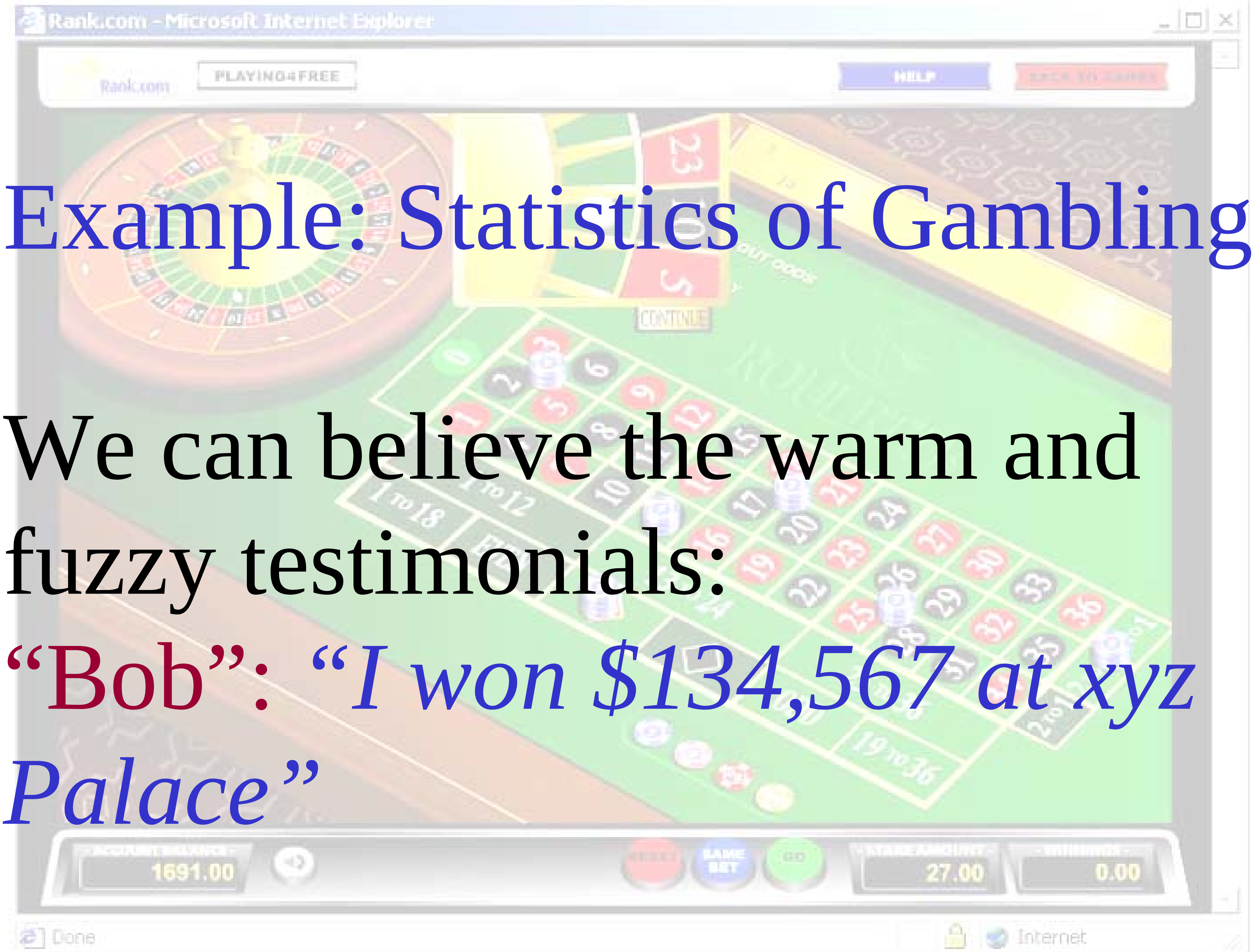
Chapter 8

Statistics

Example: Statistics of Gambling

Who wins?

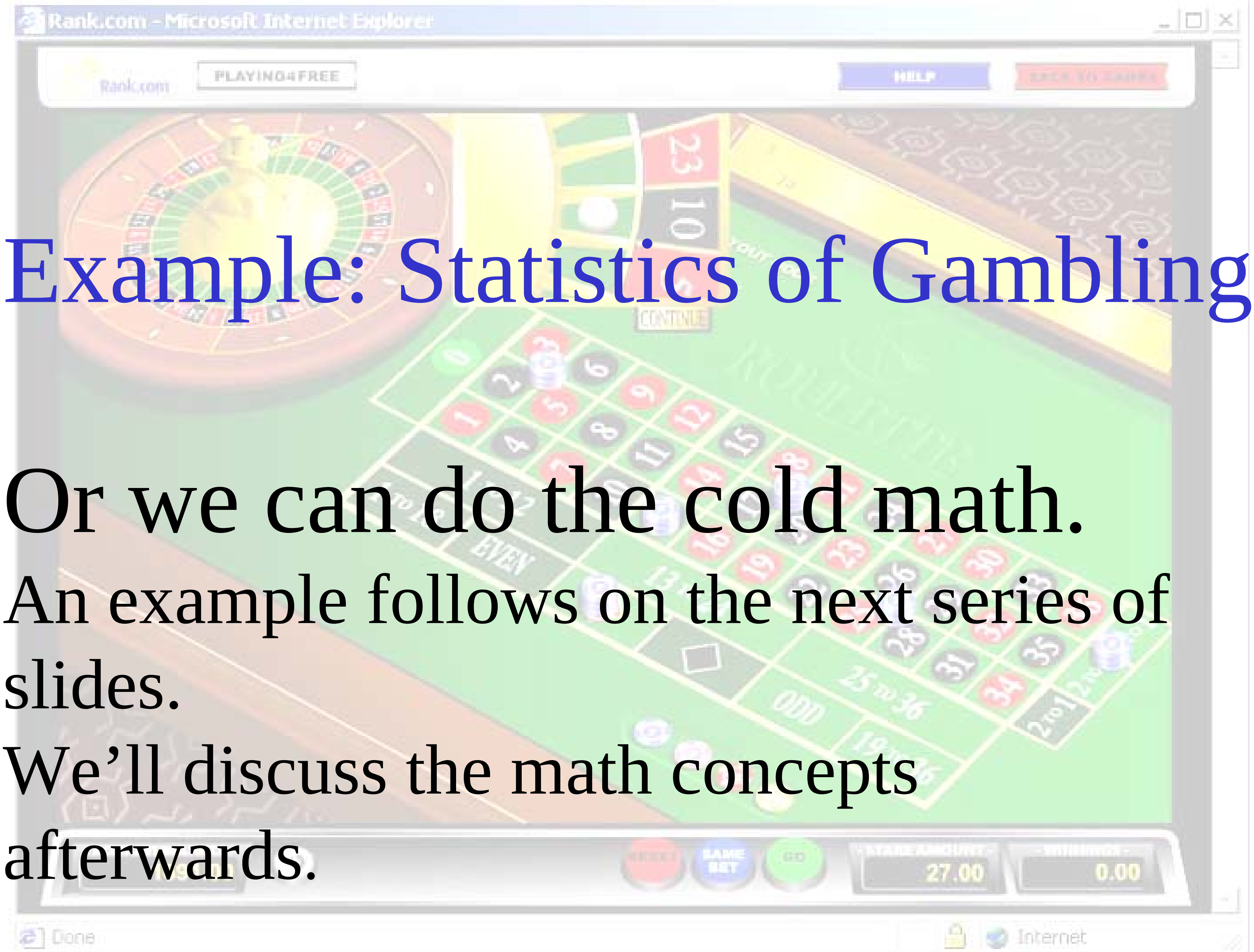
The player or the casino?



Example: Statistics of Gambling

We can believe the warm and fuzzy testimonials:

“Bob”: *“I won $134,567 at xyz Palace”*

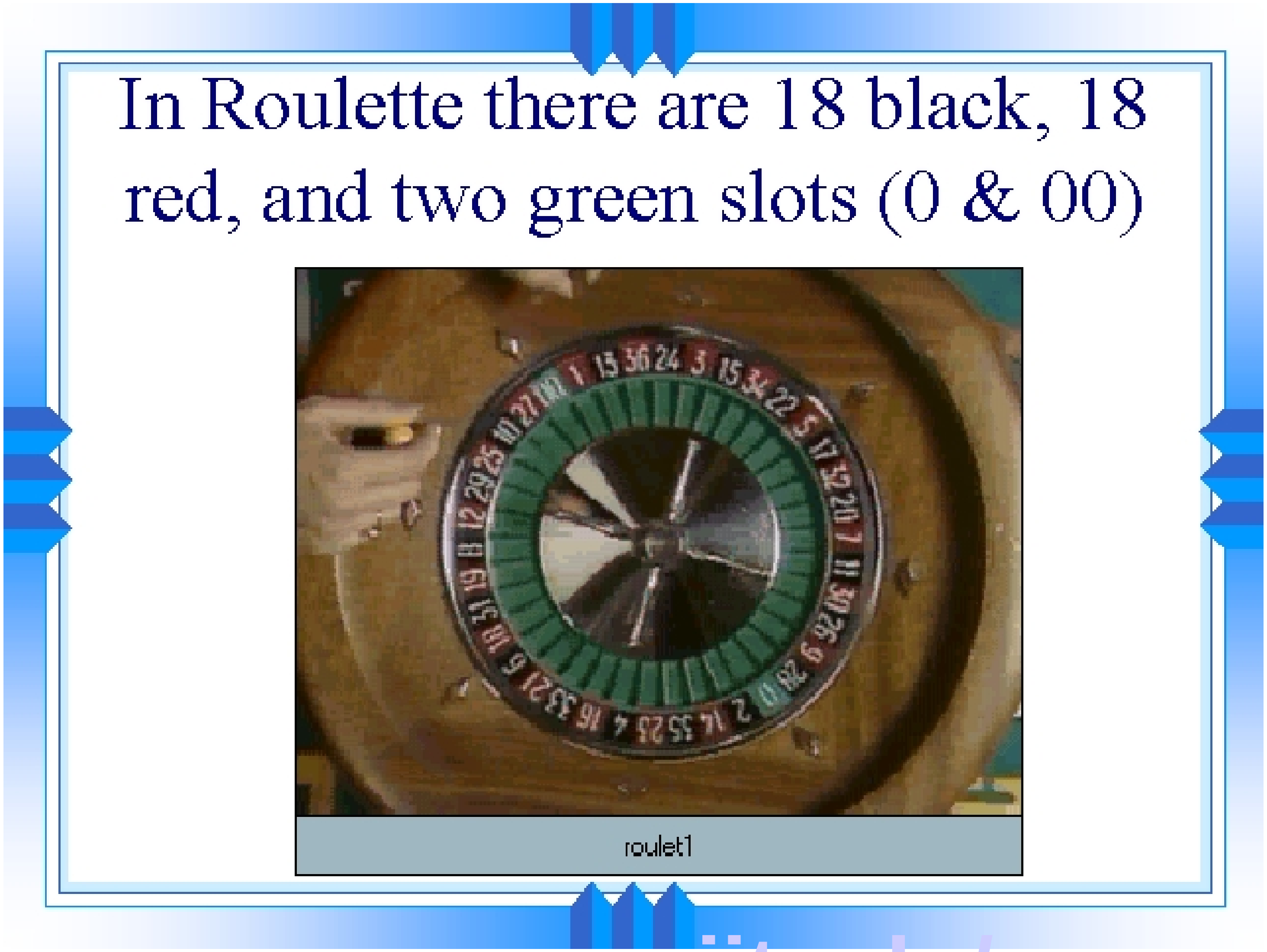


Example: Statistics of Gambling

Or we can do the cold math.

An example follows on the next series of slides.

We'll discuss the math concepts afterwards.



Source: www-ec.njit.edu/~grow/roulette/sld001.htm

As the ball goes around it is equally likely to land in any slot

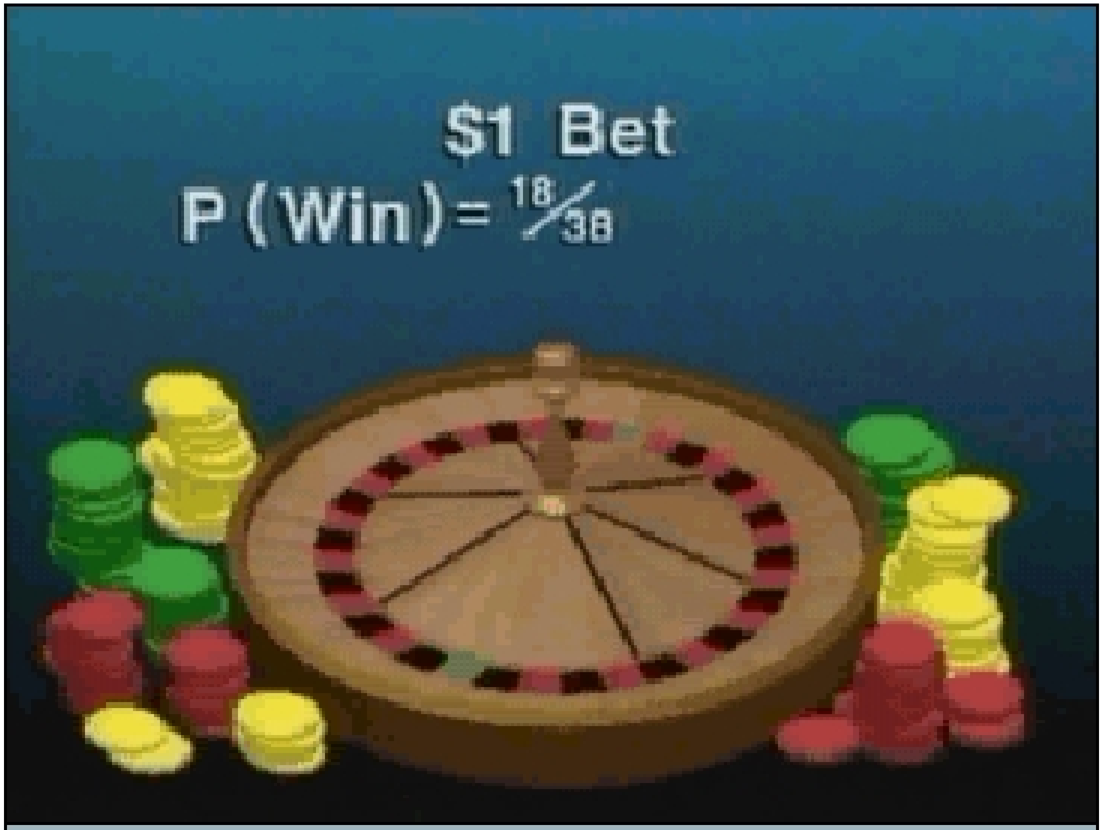
roulet1

Q: What kind of distribution will result?

If a dollar is bet on red the outcomes are win or lose a dollar

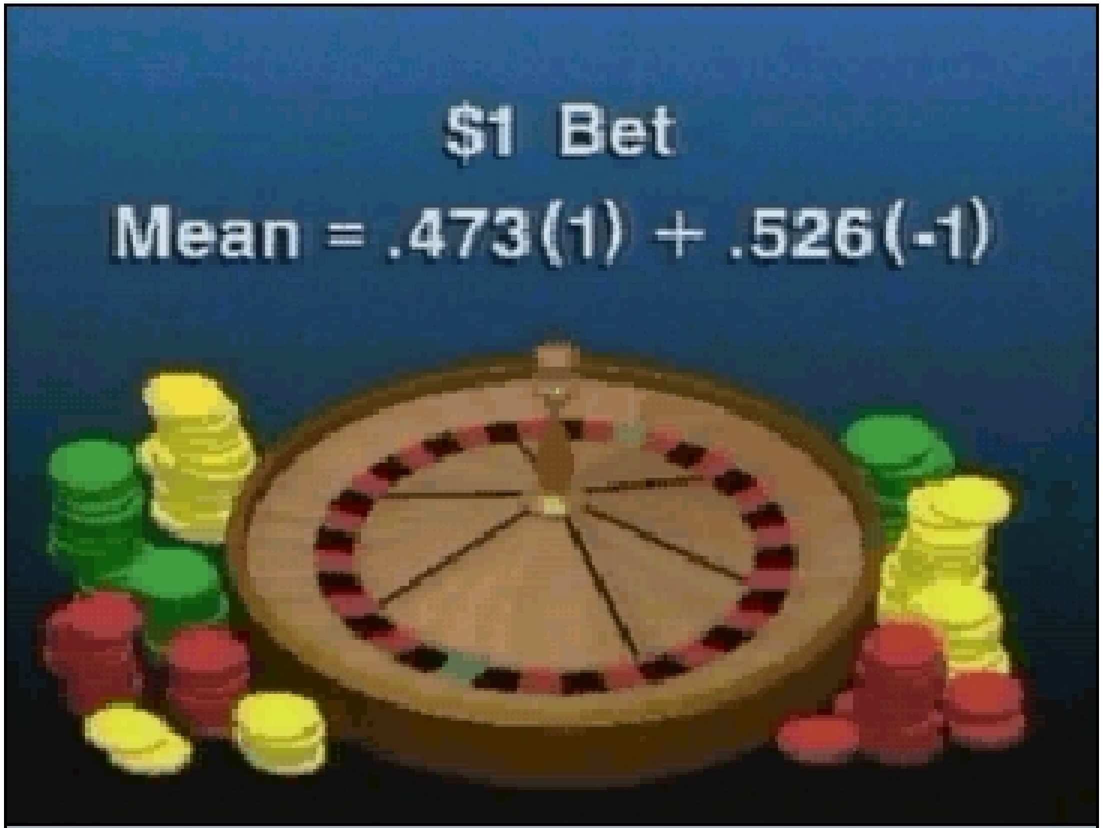
roulet1

The Probablity of winning is 18/38 or 0.473 (18 red of 38)



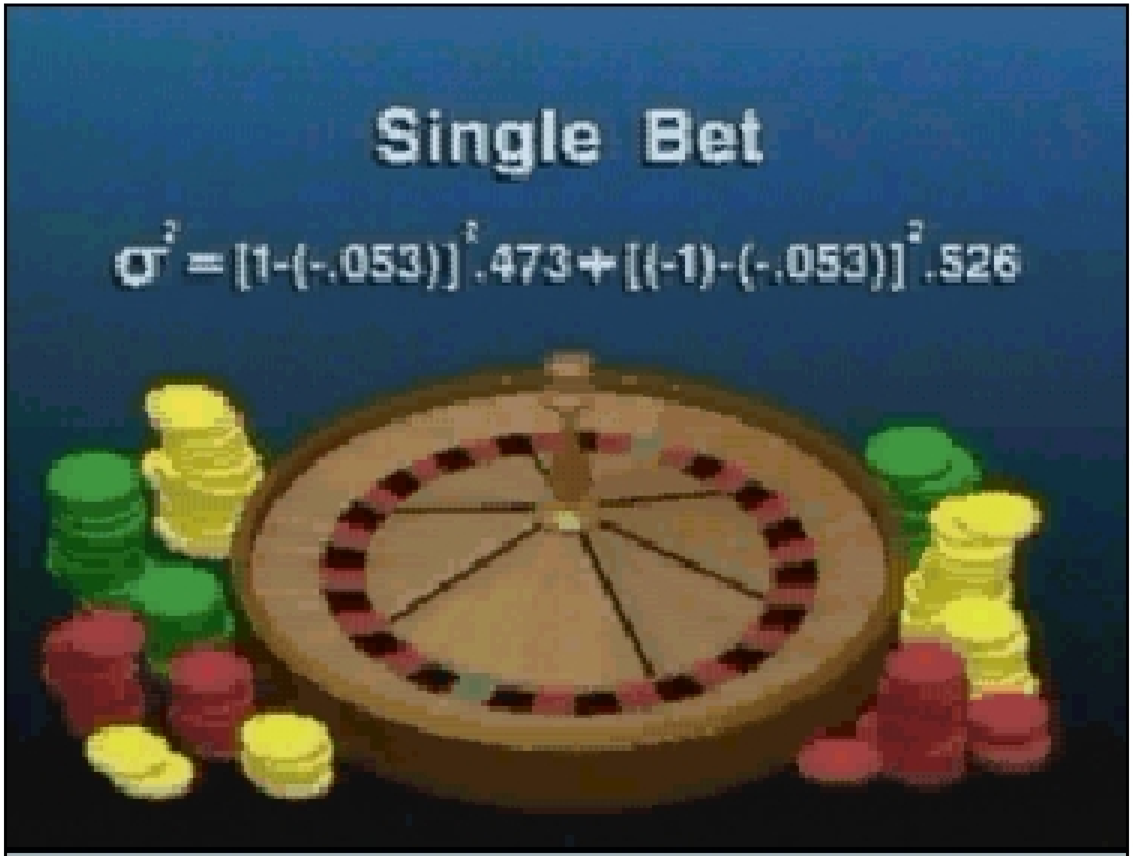
roulet1

The proability is that you will lose 5.3 cents



roulet1

The variance can be calculated

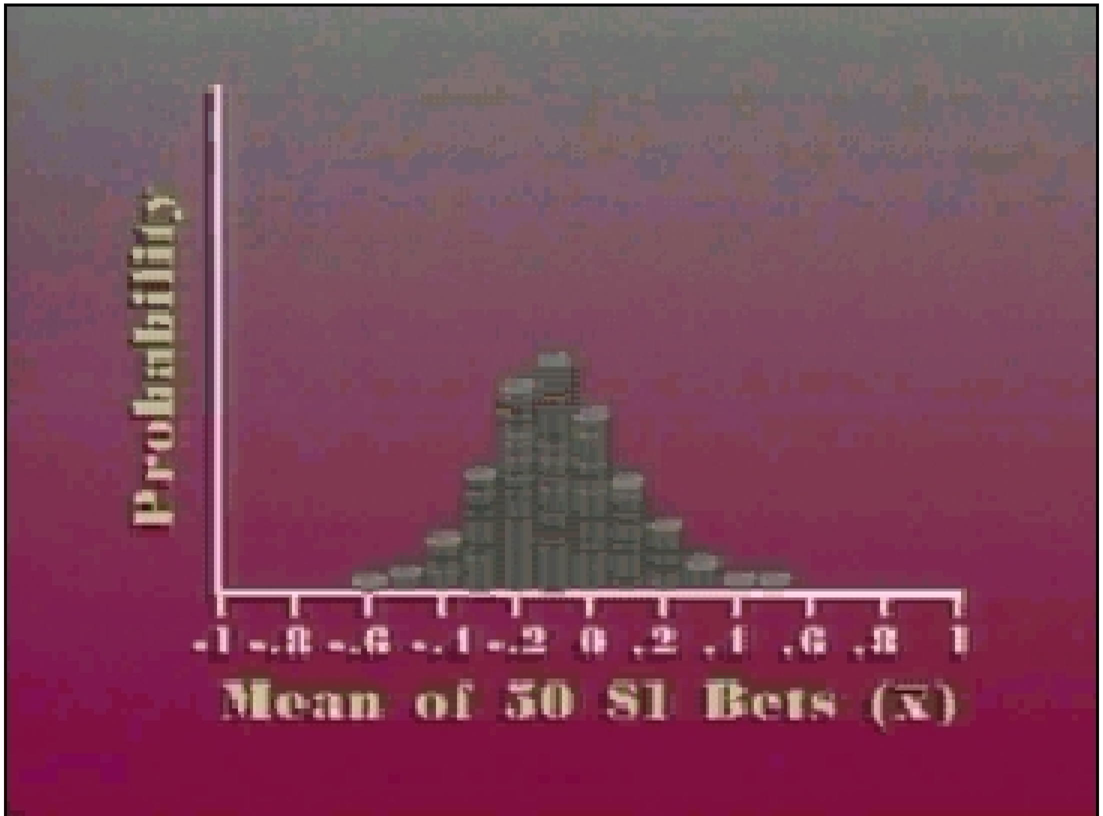


roulett

To yield a standard deviation of plus or minus 0.998

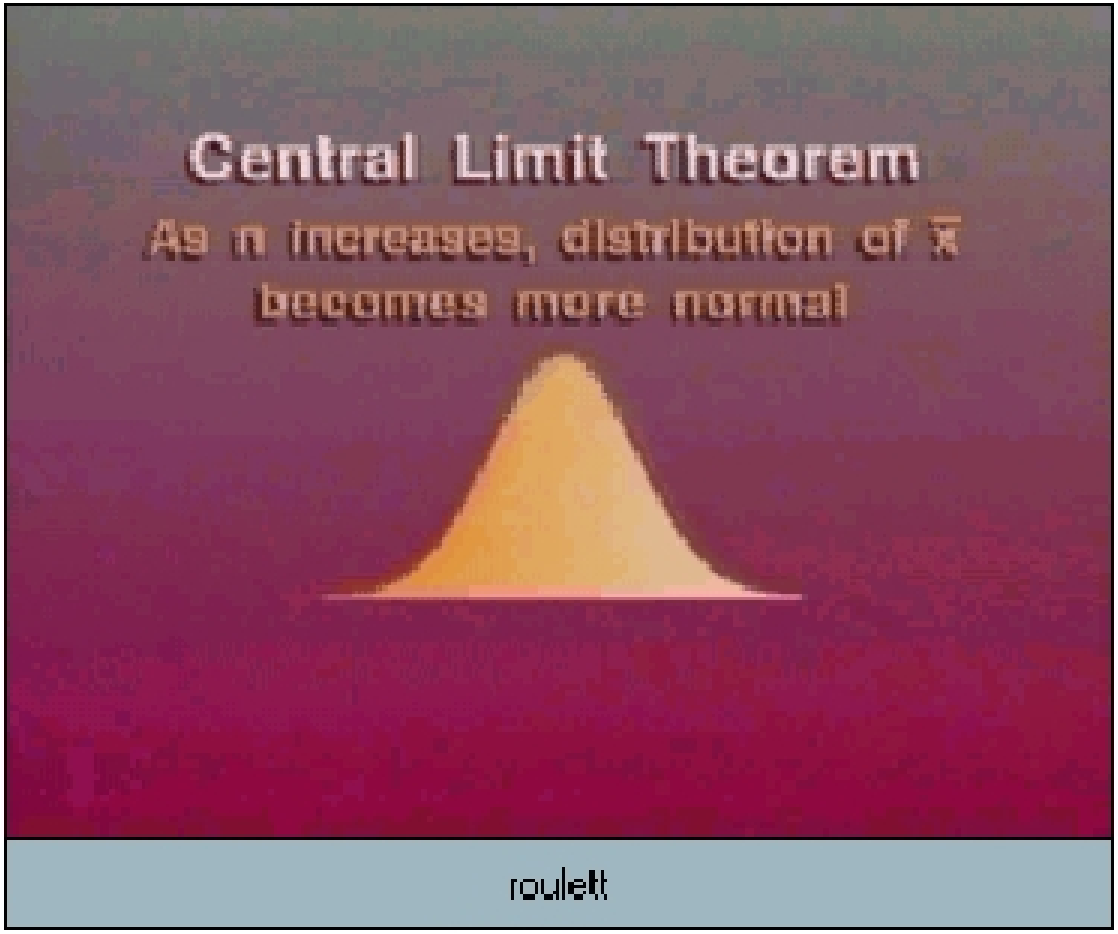
roulett

The results for 50 one dollar bets done a number of time



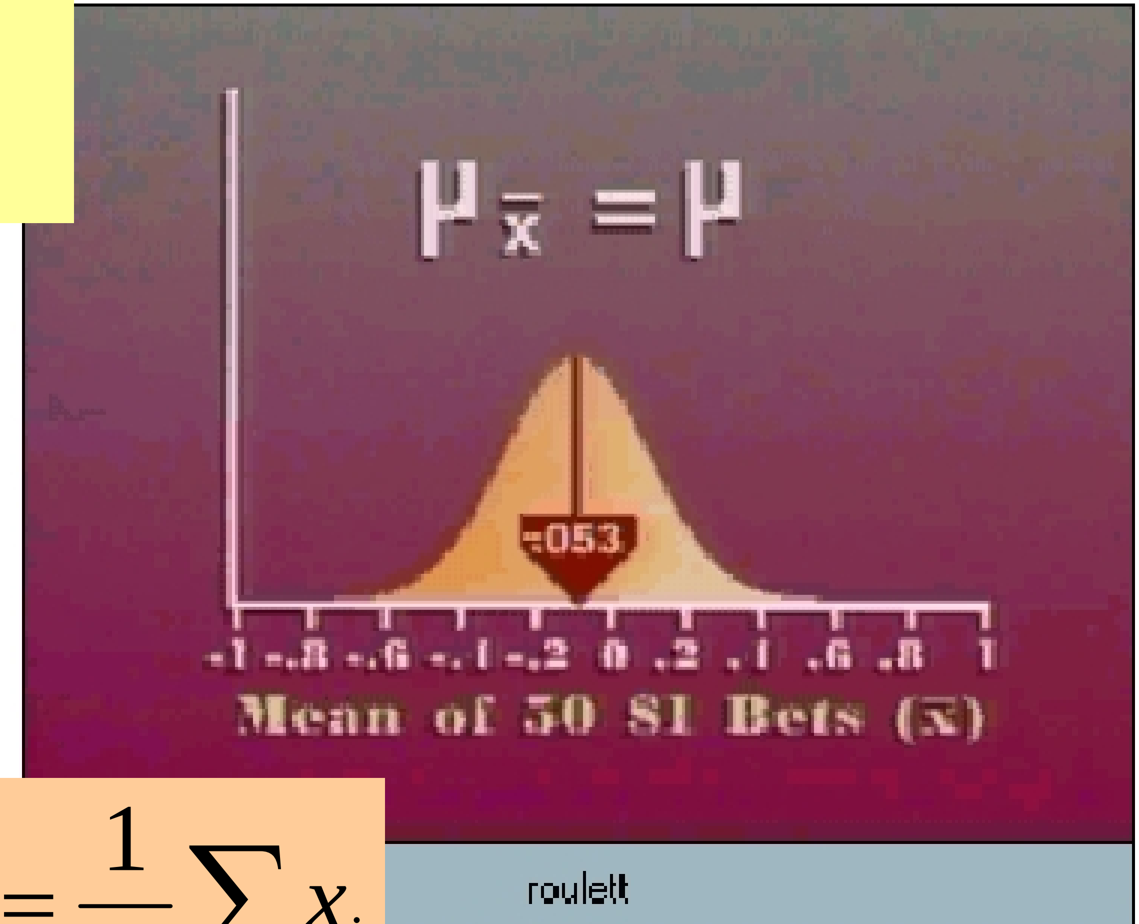
roulett

The distibution of these bets is normal



The mean of the fifty bets would be -0.53

Mean μ: Average Value:



$$Mean = \mu = \frac{1}{N}\sum_{N} x_i$$

Mean

$$\bar{x} = (\Sigma x_I) / n$$

Variance

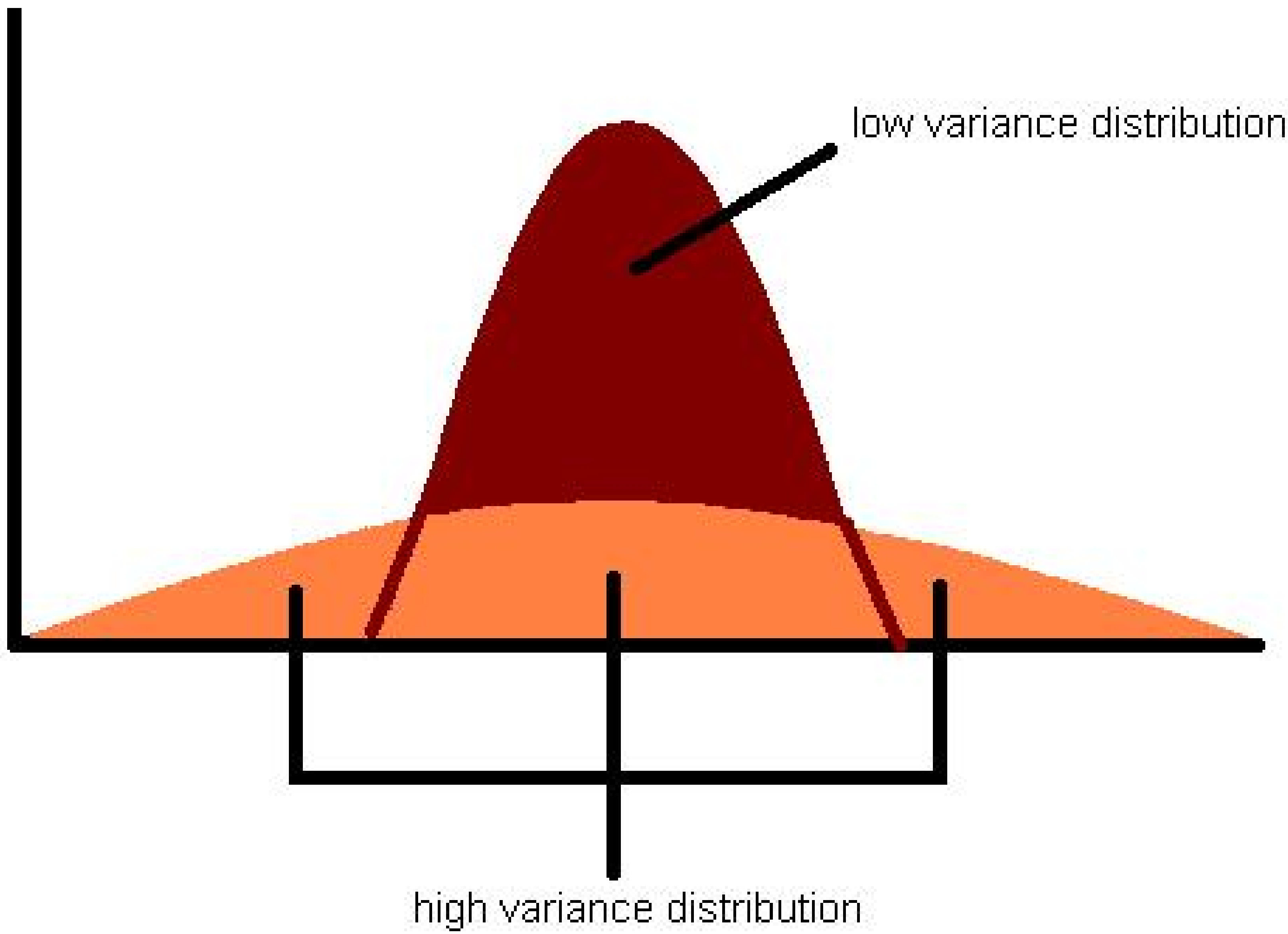
$$\sigma^2 = \sum \frac{(x_i - \bar{x})}{n-1}$$

Where σ^2 = Variance

x_I = Each item

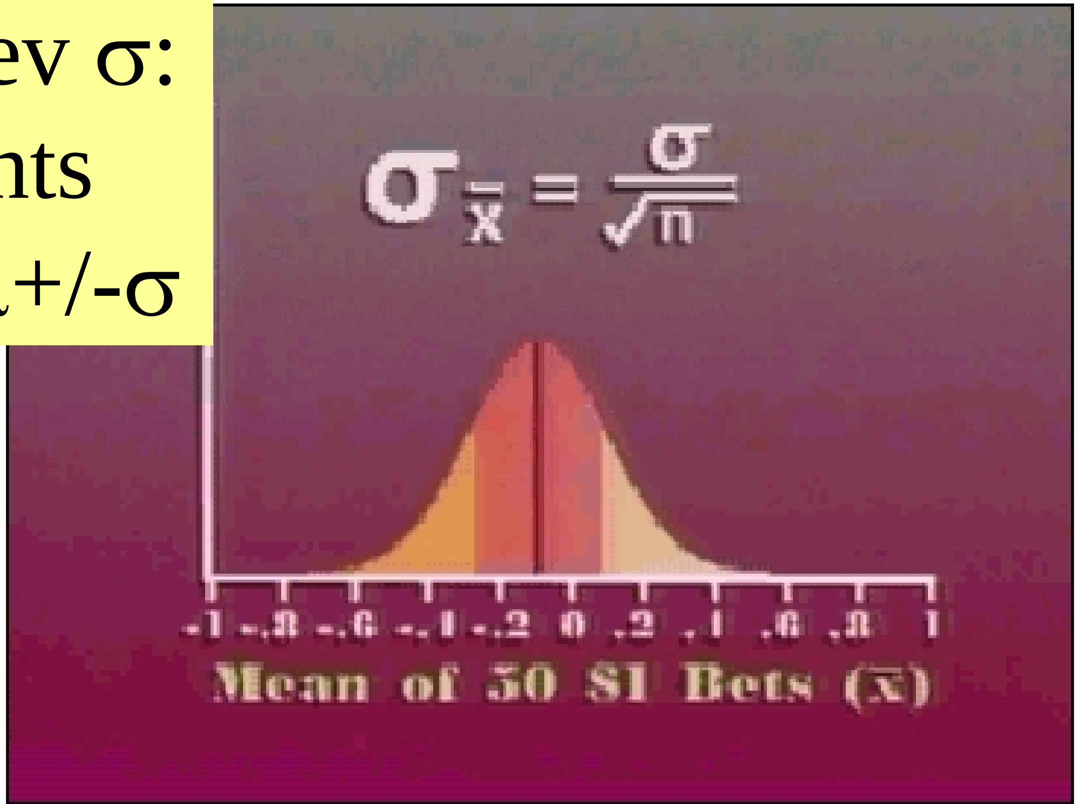
$\bar{x}$ = sample mean

n = sample size


low variance distribution
high variance distribution

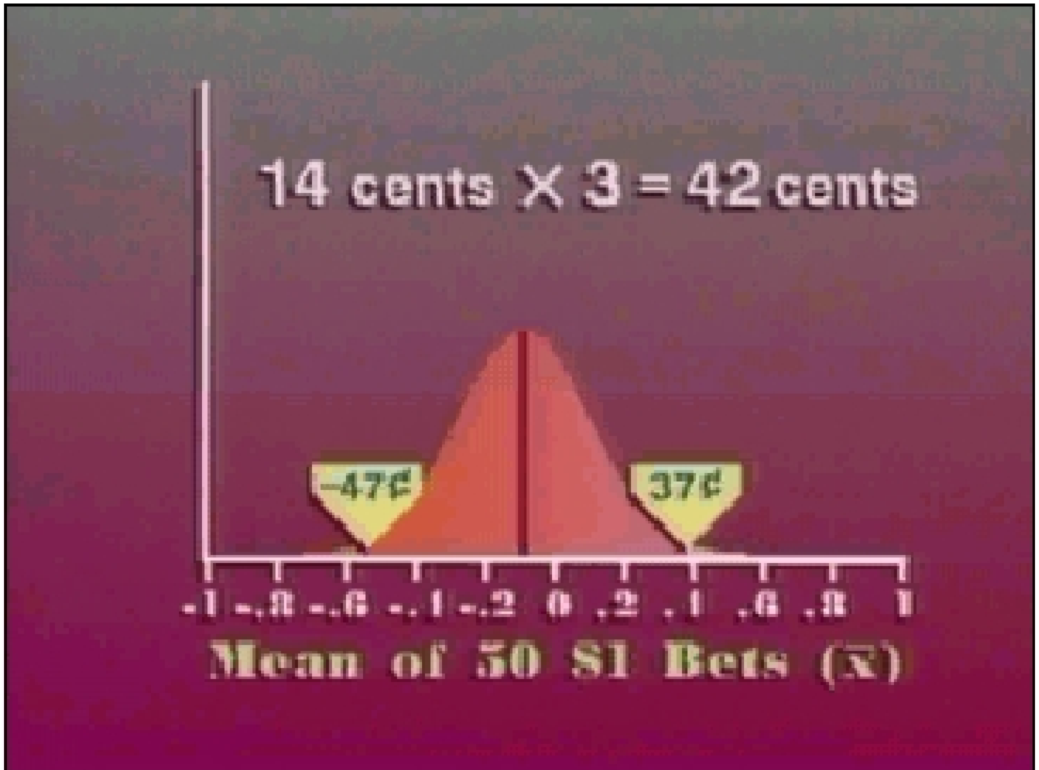
The standard deviation of the fifty bets would be

Standard Dev σ:
67% of events
are within μ+/-σ



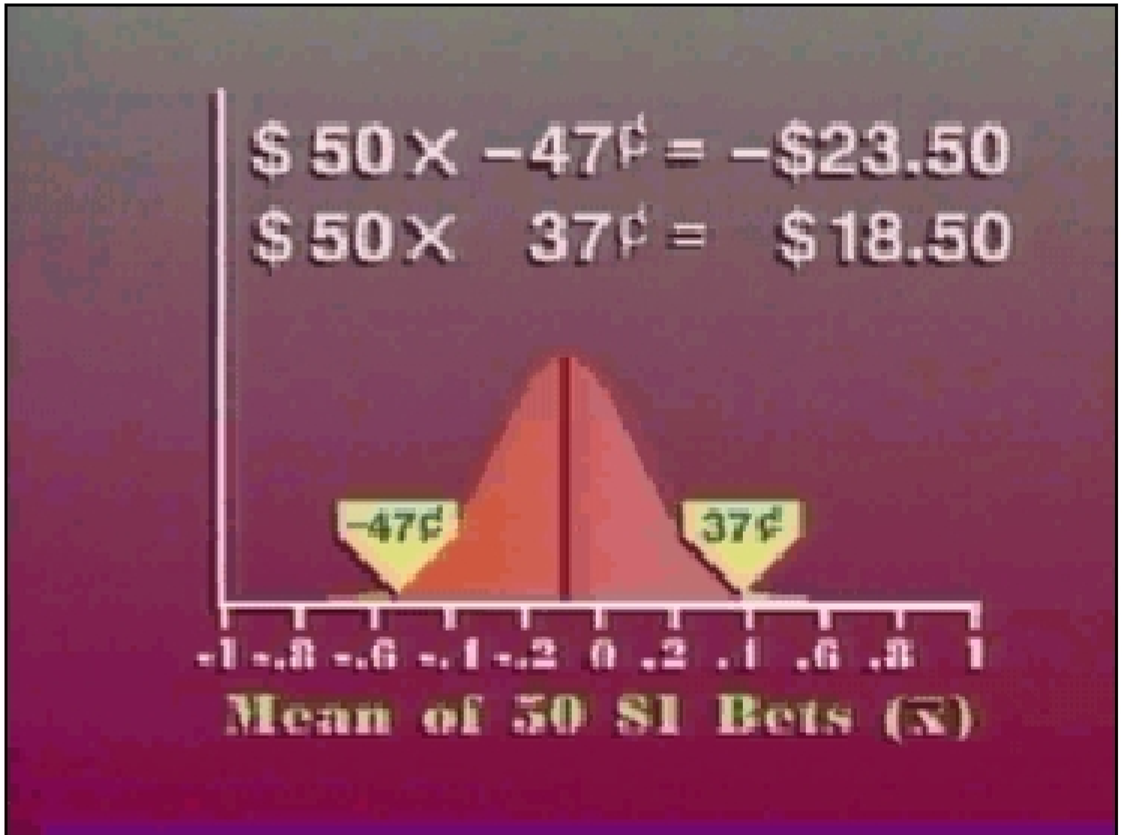
roulett

Most results will be between a lose of 47 cents to a gain of 37



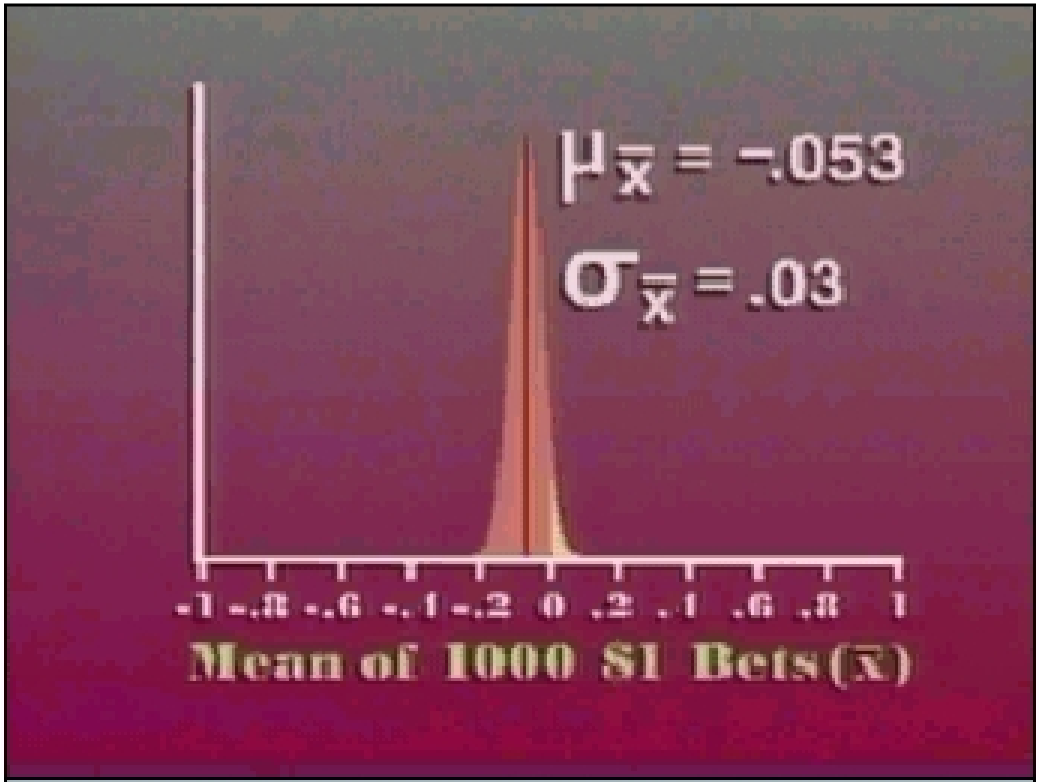
roulett

So the final results of fifty $1 bets will as follows



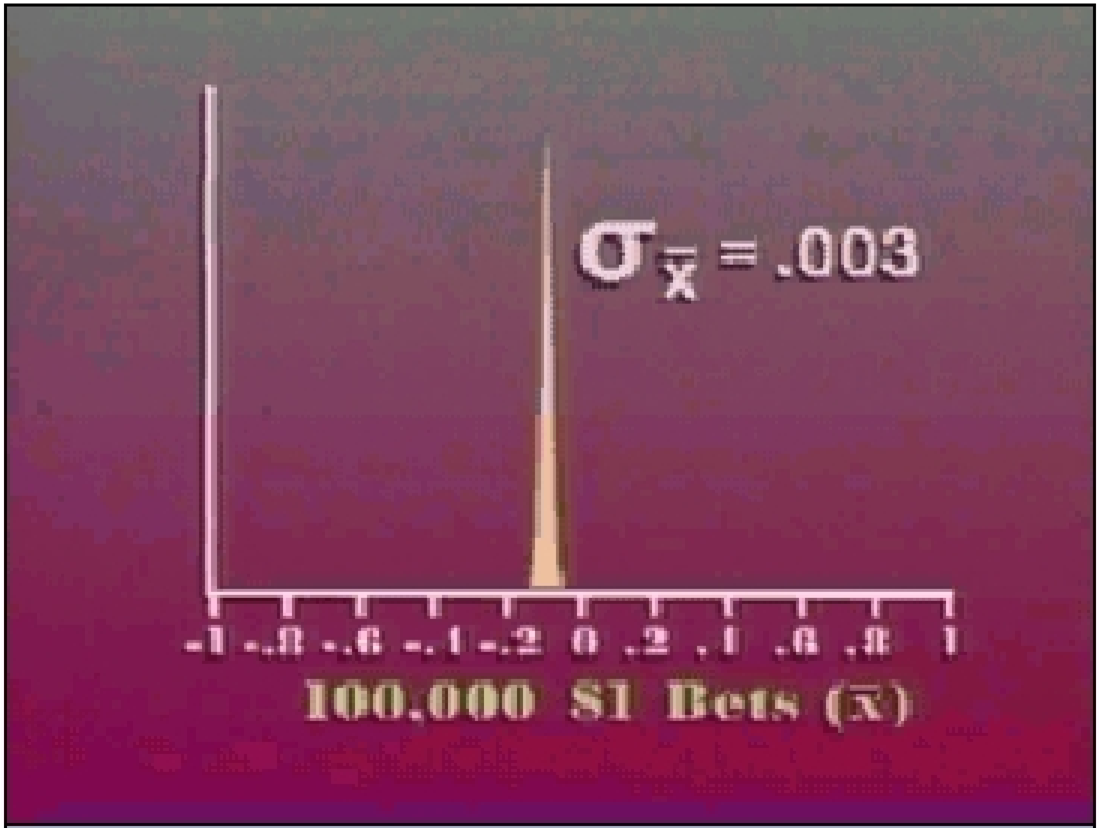
roulett

If 1000 $1 bets were made most would lose money



roulett

If 100,000 bets are made there is almost no possibility of winning



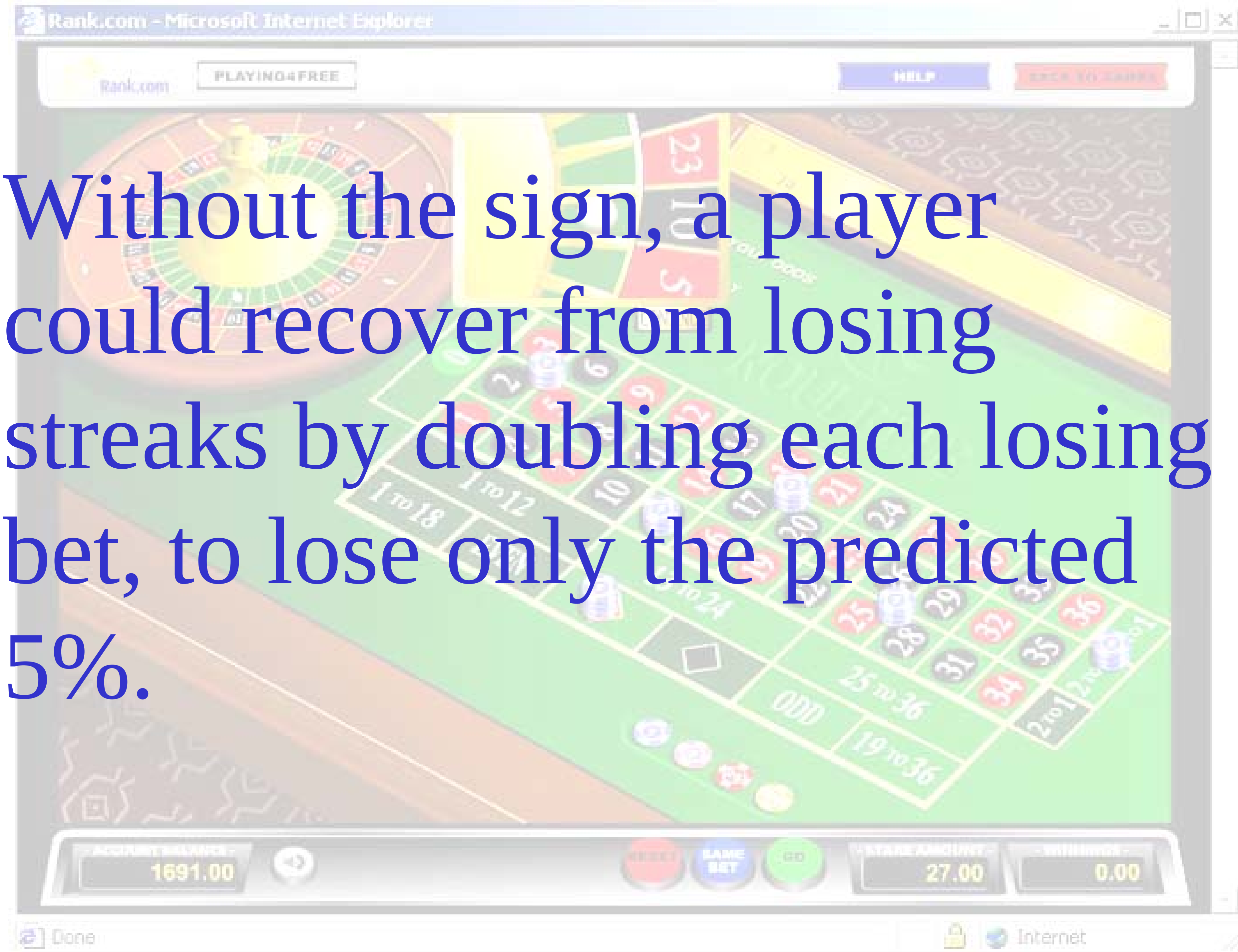
roulett

The preceding analysis did not consider the fact that the casino limits the player's capital.

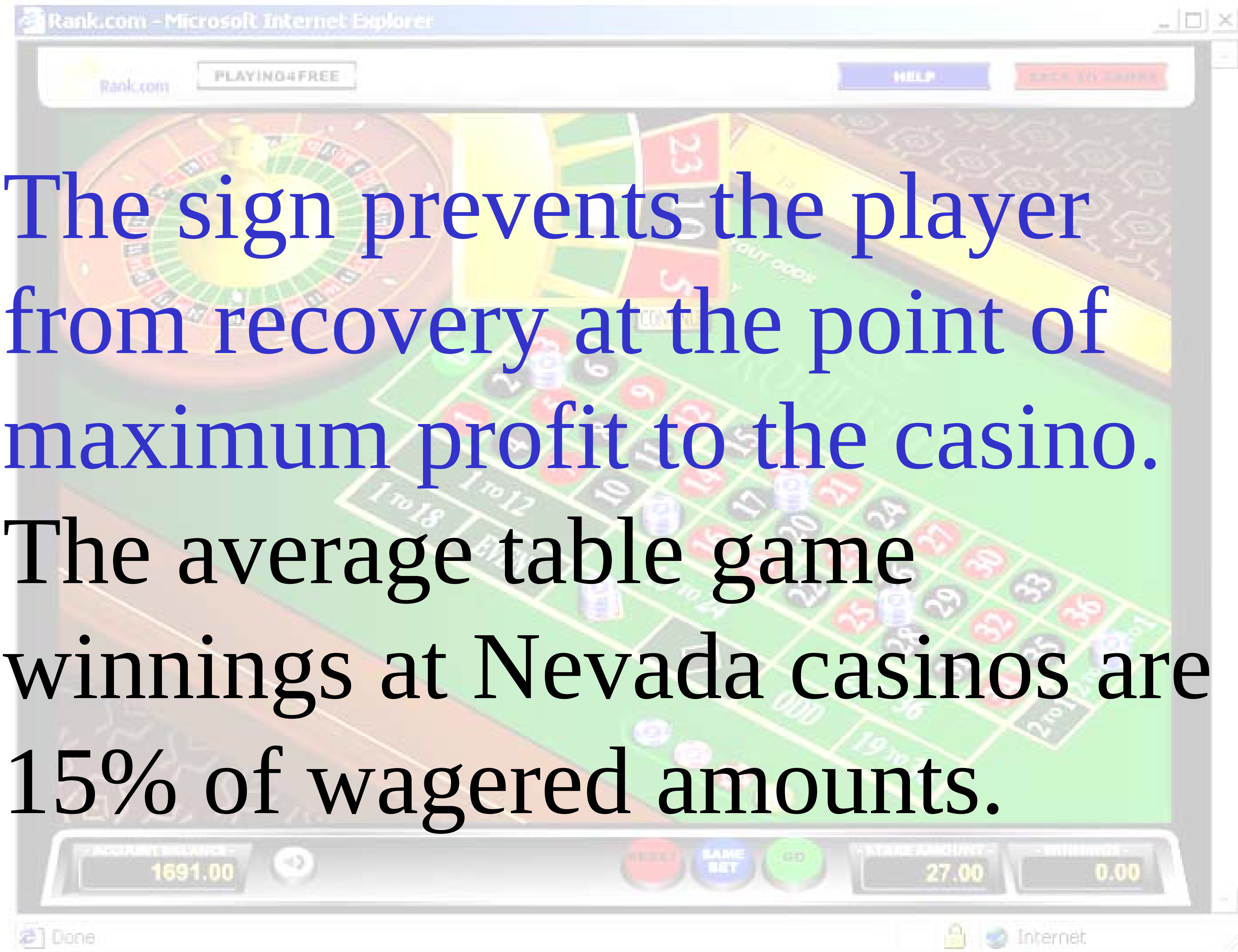
All casino tables carry a sign

e.g. "minimum bet $5

maximum bet $500"



Without the sign, a player could recover from losing streaks by doubling each losing bet, to lose only the predicted 5%.



The sign prevents the player from recovery at the point of maximum profit to the casino.

The average table game winnings at Nevada casinos are 15% of wagered amounts.

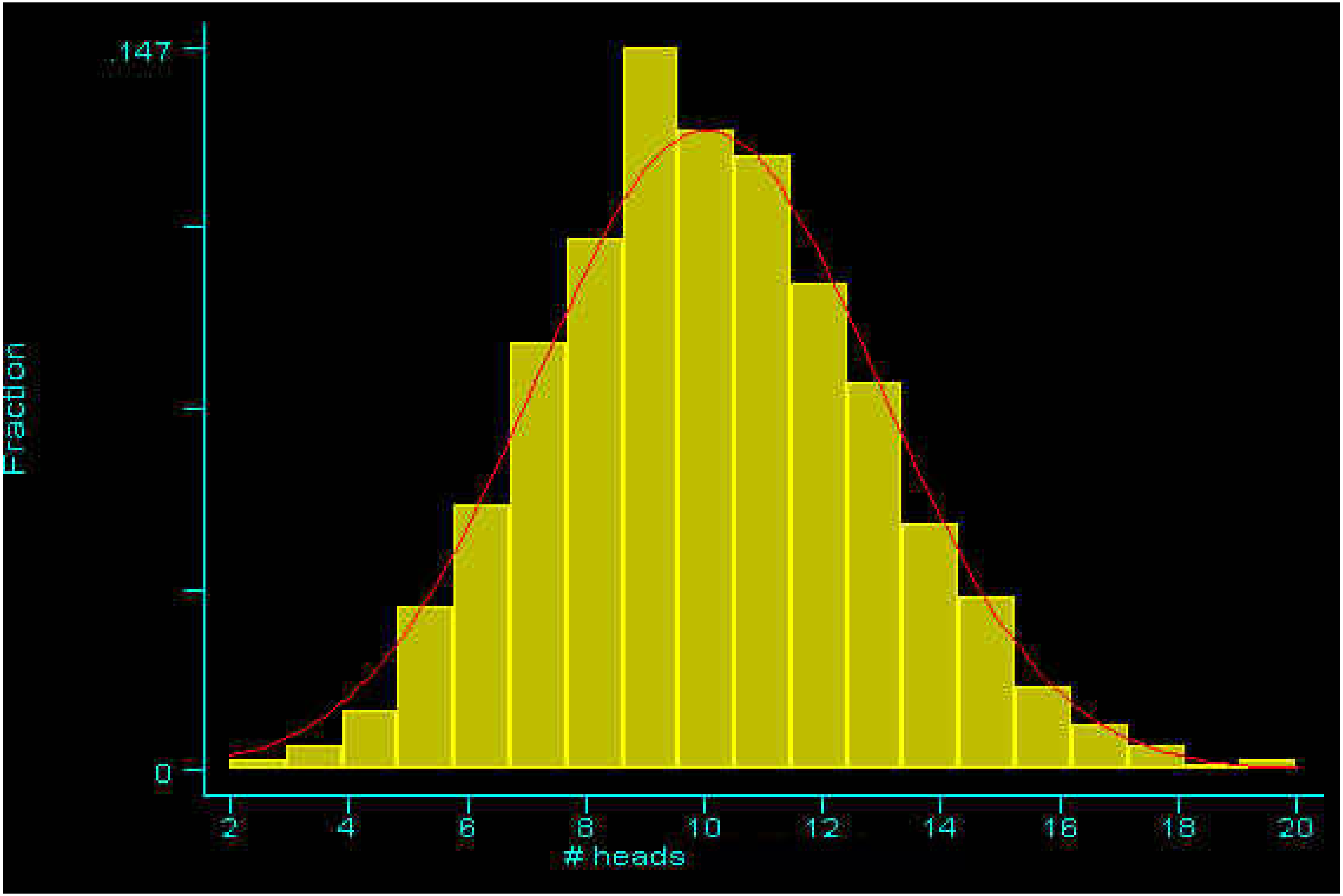
Statistical Analysis Example

Playing heads or tails.
We toss 20 coins each time, and count the numbers of heads.
1000 plays show the following results:

# heads out of 20 tossed	Freq.	Percent	Cum.
2	2	0.20	0.20
3	5	0.50	0.70
4	12	1.20	1.90
5	33	3.30	5.20
6	54	5.40	10.60
7	87	8.70	19.30
8	108	10.80	30.10
9	147	14.70	44.80
10	130	13.00	57.80
11	125	12.50	70.30
12	99	9.90	80.20
13	79	7.90	88.10
14	50	5.00	93.10
15	35	3.50	96.60
16	17	1.70	98.30
17	9	0.90	99.20
18	5	0.50	99.70
19	1	0.10	99.80
20	2	0.20	100.00
Total	1000	100.00	

We can plot the number of heads counted in each of the 20 classes vs. the frequency of occurrence.

This plot is called a histogram.



Histogram

The histograms of random distributions exhibit the familiar Gaussian “bell curve” shape.
Many distributions are **“biased”** by deterministic factors. Statisticians can detect such biases and point to their causes.
Some examples follow.