NAME:______KEY_________

UNLV Mechanical Engineering Fall 2006

Note: *Please enter your mailbox number if you wish to have the graded exam returned to your mailbox.*

MEG 421 Automatic Controls

Third Test Closed Book examination

1. (30 points) Apply the Nyquist criterion to the open-loop system $G(s) = \frac{8}{(0.1s+1)^4}$ **.**

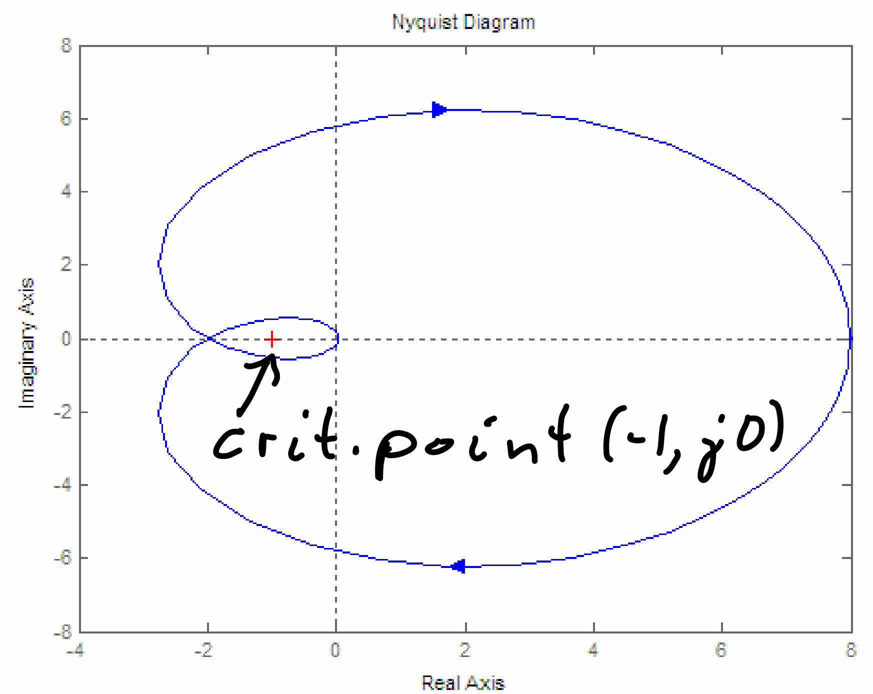
(a) Compute the entries in the table, including the system's magnitude and phase at the break frequency.

(b) Scale and label the plot below, and draw schematically the system's Nyquist plot. Three points of the Nyquist plot **must be entered accurately**: (aa) start at ω = 0, (bb) negative real axis crossing (mark the critical point clearly), and (cc) the end point as ω → ∞.

(c) Apply the Nyquist criterion to the **Nyquist plot** of the open-loop system, and determine whether the closed loop system is stable. Explain!

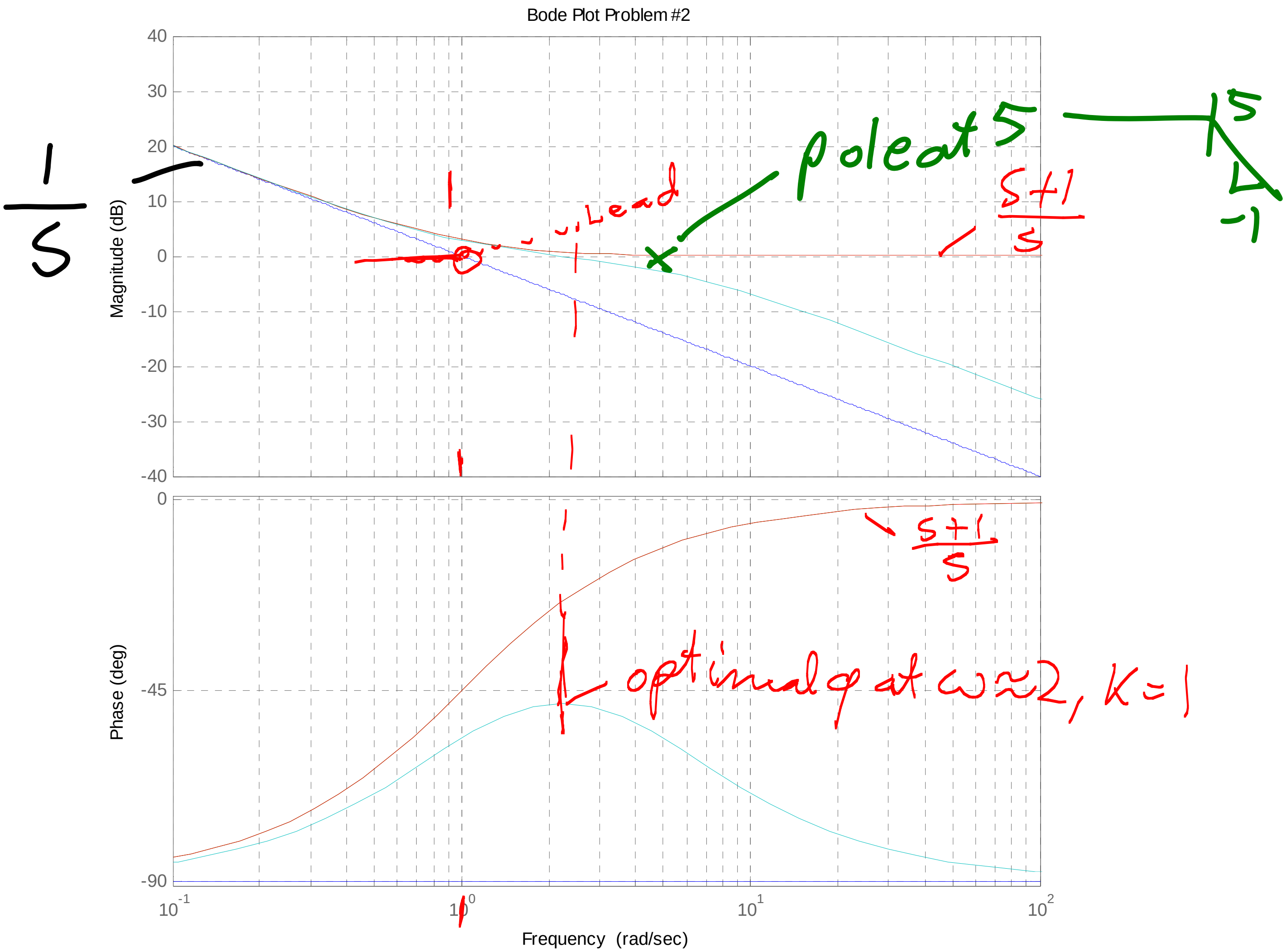
1. (a) Please complete the table below

ω	|F(jω)|	φ(jω)
0	8	0
ω_B= 10	8/4	-180
∞	0	-360



Answer 1 a: Break frequency	|F| (ω_B) = 2	Phase(ω_B) = -180 degrees

Answer 1 c : Stability: (present **Nyquist** argument) Critical point (-1,j0) is **Twice** encircled, therefore the closed loop is **NOT** asymptotically stable, and has 2 r.h.p. poles.



2.

(30 points) (a) Construct the Bode plot of the plant $G(s) = \frac{K(s+1)}{s\ *(0.2s+1)}$ for K=1.

(a) Construct accurate asymptotic approximations of both the magnitude and phase plots.

(b) Using the Nyquist criterion in the Bode plot, determine the optimal gain K for which the closed loop system will have the highest phase margin.

Max. Phase margin at ω = 0.5 rad/s. The Magnitude at ω = 0.5 rad/s is approx. 1. So the optimal gain K_{opt} =1.

<u>Answer 2(b)</u>

Closed Loop Optimal Gain K for maximum Phase Margin:

K_{opt} =1.

3. (10 points) Define mathematically the terms:

(a) Break frequency

(b) Critical Point

You may include illustrations if you wish.

(a) Def. Break frequency :   **$\omega_b = 1/\tau$, where τ = time constant**
(b) Phase Margin: **Phase Margin = 180 degrees – Phase (Gain crossover freq.)**

4. (30 points) The Bode plot on the next page depicts an open-loop system G(s) with P-controller K = 1, where

$$G(s) = \frac{5K}{(s+1)^2 * (0.1s+1)}$$

A lead compensator is proposed as

$$G_c(s) = 10\,\frac{s + zero}{s + 10*zero}$$

A phase margin of 30 degrees (marked in the plot) is desired for closed loop control.

(a) Determine the P-controlled system's gain K at the desired phase margin. Mark the P-controller gain in the Bode plot, and label it clearly.

(b) Graphically add a lead compensator such that the resulting system has a higher closed loop gain (at least 2 times better than P-control) at the phase margin of 40 degrees. Show clearly in the Bode plot the lead zero and pole.

(c) Determine the gain crossover frequency ω_{CR} of the compensated system, and the frequency at which the lead compensator has its highest phase.

(d) Sketch the approximate compensated system (Plant * Lead) in the plot on the next page.

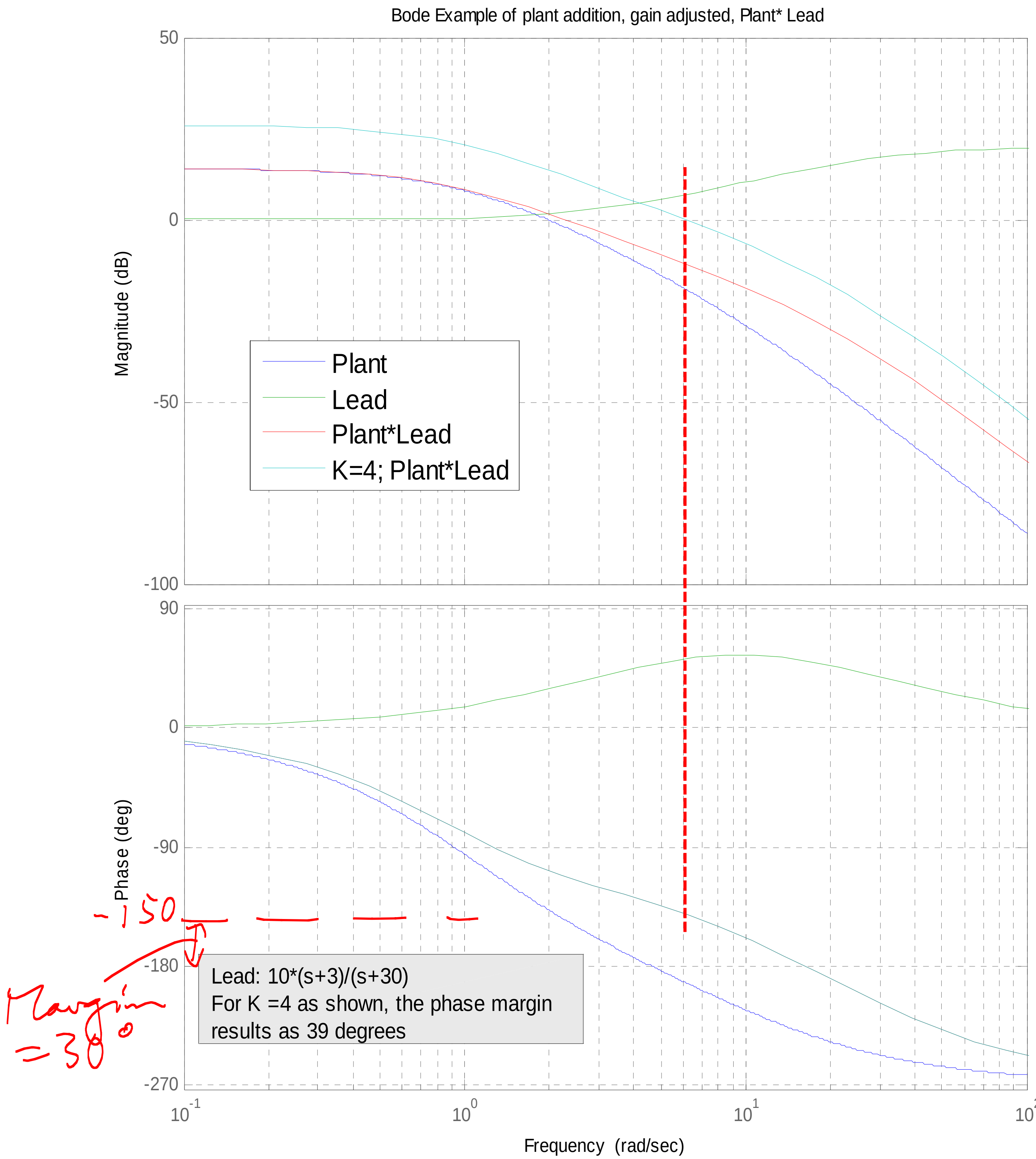
Answers Problem 4

(a) Gain K (P-control) = 1/0/7 or 1.4

(b) Controller gain K (Lead control) = 4

Lead transfer function = $\frac{10*(s+3)}{s+30}$

(c) ω_{CR} (Comp*Sys) = 6 rad/s $\omega_{max.phase}$ (Lead) at approx. 9 rad/s


Bode Example of plant addition, gain adjusted, Plant* Lead
Magnitude (dB)
50
0
-50
-100
Plant
Lead
Plant*Lead
K=4; Plant*Lead
Phase (deg)
90
0
-90
-150
-180
-270
Margin
=30°
Lead: 10*(s+3)/(s+30)
For K =4 as shown, the phase margin
results as 39 degrees
10^-1
10^0
10^1
10^2
Frequency (rad/sec)