Name:_______KEY_________

UNIVERSITY OF NEVADA, LAS VEGAS
DEPARTMENT OF MECHANICAL ENGINEERING

MEG 421 Automatic Control Fall 2004

First Test, Closed Book,

One page of handwritten notes and Table of Laplace Transforms allowed

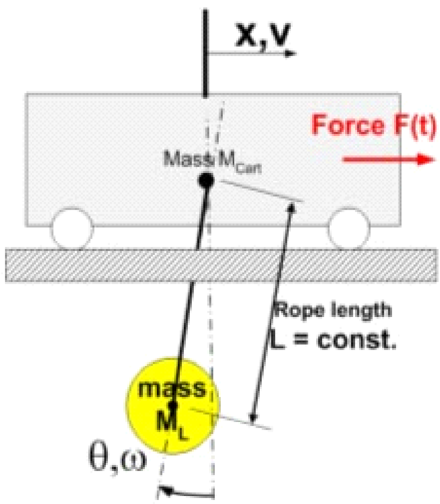


Figure 1 Overhead Crane

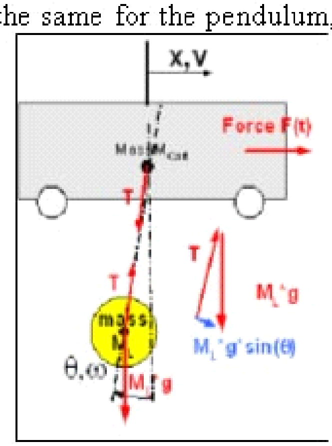
1. **(25 points)** Figure 1 shows schematically an overhead crane, consisting of a cart having mass M_{cart}, and a suspended load of mass M_L, with the rope having constant length L. No friction. The system is driven by applied force F(t) . The output variable is the angle θ of the load.

(a) Draw a free-body diagram of the system.

(b) Write the equation of motion in terms of the parameters M_{cart}, M_L and L, and the given input and output variables.

(c) Express the equation of motion as a transfer function.

Pendulum: we sum up the moments about pendulum pivot in cart. The load's moment of inertia about its pivot at the cart is M_L*L^2. The free-body analysis yields four equations: Forces of cart in x- and y-direction, and the same for the pendulum, see textbook for details of the analysis.



x: $(M_{cart}+M_L)*\ddot{x}+M_L*L*\cos(\theta)*\ddot{\theta}-M_L*L*\dot{\theta}^2\sin(\theta)=F(t)$

θ: $M_L*L^2*\ddot{\theta}=M_L*L*\ddot{x}*\cos(\theta)-M_L*g*L*\sin(\theta)$

Answer (b)

$$(M_{cart}+M_L)*\ddot{x}+M_L*L*\cos(\theta)*\ddot{\theta}-M_L*L*\dot{\theta}^2\sin(\theta)=F(t)$$

$$L*\ddot{\theta}+g*\sin(\theta)-\ddot{x}*\cos(\theta)=0$$

Answer (c) Transfer function is undefined for nonlinear systems.

2. (15 points) In a pressure control system, the output pressure, y(t) as function of the command signal, u(t) delivered by the controller is given by the DE

$$\dddot{y} + 2\ddot{y} + 3\dot{y} + 3y(t) = 1.5 * u(t) \qquad y(0) = 0$$

(a) Determine the transfer function.
(b) If u(t) is a unit step input, determine the **steady state** value of y.

In the steady state, there is no change. → All derivatives become zero. With the input at a constant u = 1.5, the st. state Equation becomes: 3y = 1.5

Answer

(a) $\frac{Y(s)}{U(s)} = \frac{1.5}{s^3 + 2s^2 + 3s + 3}$	(b) $y_{ss} = 1.5/3 = 0.5$

3. (25 points) A dynamic system with input r(t) and output y(t) is described by the DE:

$$\dot{y} + 5y = r(t)$$

a) Find the transfer function.
b) Determine y(t) when r(t) is a unit step input.
c) Graph schematically the step response of the system. Label and scale all axes.

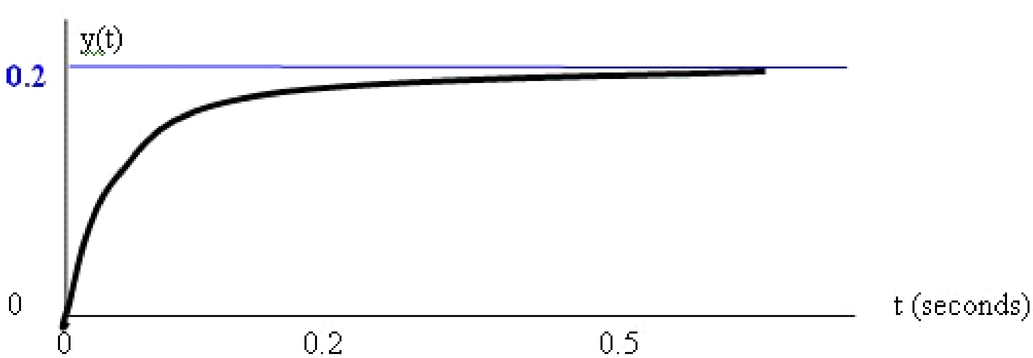
Laplace analysis: $Y(s) = \frac{1}{s} * \frac{1}{s+5} = \frac{0.2}{s} - \frac{0.2}{s+5}$

Time Domain Solution: see answer box

Answer

$\frac{Y(s)}{R(s)} = \frac{1}{s+5}$	(b) $y(t) = 0.2(1 - e^{-5t})$

c) Graph the final value is $y_{fin} = 0.2$



4. (15 points) Consider the following systems:

$$\frac{Y(s)}{R(S)} = \frac{20}{(s^2 + 1.44)(s^2 + 9)} \qquad 3$$

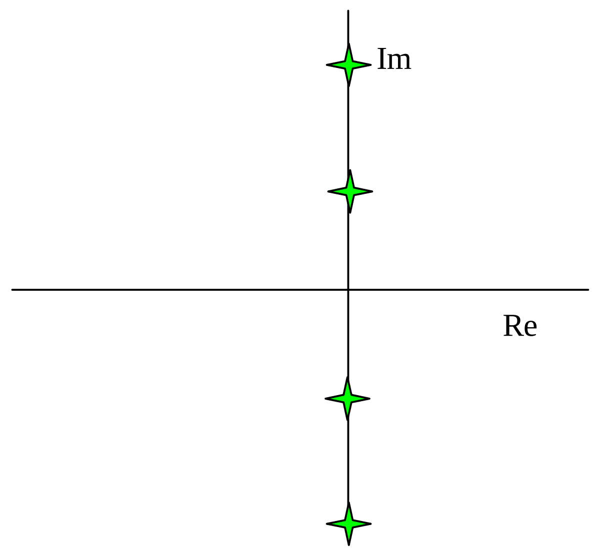
$$\frac{Y(s)}{R(S)} = \frac{20}{s^2 + 2s + 1} \qquad 4$$

For each system, find ALL poles for each system, sketch ALL poles in the complex number plane, and determine each system's stability. Scale and label each axis.

Answers

Eq. 3

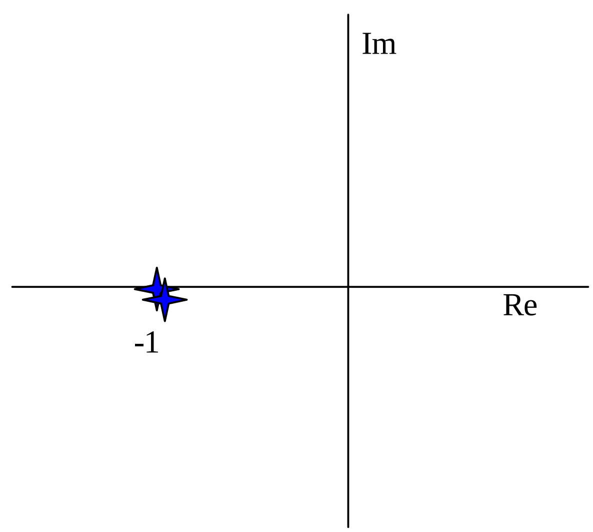
Poles: at $\pm 1.2j$ *and* $\pm 3j$



Stability: *stable, but not asymptotically stable*

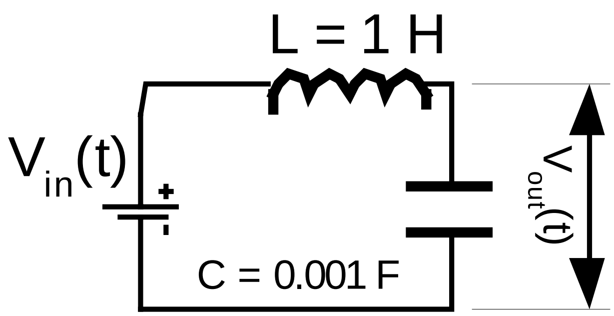
Eq. 4

Poles: *double pole at –1 on the real axis*



Stability: *asymptotically stable*

5. (20 points) Given: the electrical LC circuit shown. The input variable is the battery voltage, $V_{in}(t)$. Find the differential equation or transfer function (your choice) describing the dynamic relationship between input $V_{in}(t)$ and output $V_{out}(t)$.



The easiest approach is Kirchhoff Impedance analysis: The complex impedances are: LS and 1/(Cs). Thus

$$\frac{V_{out}}{V_{in}} = \frac{1/(Cs)}{Ls + 1/(Cs)} = \frac{1}{LCs^2 + 1}$$

Answer

Differential Equation or transfer function

$$\frac{V_{out}}{V_{in}} = \frac{1/(Cs)}{Ls + 1/(Cs)} = \frac{1}{LCs^2 + 1}$$