Automatic Control
GOALS: To provide advanced students in mechanical engineering with a solid background in dynamic system modeling and analysis and to enable them to analyze and design linear control systems.
FORMAT:

Lecture: 3 credits
Lab: 1 credit

You must enroll in both MEG 421 and MEG 421L
MEG 421: Automatic Control

Prerequisites by Topic:

1. Electrical Circuits
2. Mathematics for Engineers.
3. Analysis of Dynamic Systems
Automatic Control

– Most General Definition:

To Produce a Desired Result
“Trust is good, control is better.”
Definition of Automation

- “Having the capability of starting, operating, moving, etc., independently.” ¹

- “The use of machines to perform tasks that require decision making.” ²
• Technical Control Systems

Open Loop Control Systems
Open Loop Control Systems
Example: Batch Filling
Block Diagram for Feedback
(or Closed Loop) Control
England - Eighteenth Century AD
Watt Steam Engine
England - Eighteenth Century AD
The accelerating technological change of the 19th century was reflected in literature and art.

In Jacques Offenbach’s opera ‘Les contes d’Hoffmann’ the hero falls in love with Olympia, a mechanical doll. Olympia can sing and dance. She needs rewinding every 5 minutes or so.
Another famous example is Mary Shelley’s ever popular *Frankenstein* (1831).
Automation Today

The world around us
Rapid Growth of Machine Intelligence

Evolution of Computer Power/Cost

MIPS per $1000 (1997 Dollars)

Year

1900 1920 1940 1960 1980 2000 2020
Robots for Hazardous Areas
Around the House
Warm And Cuddly ...
...or Cyborg
Control Systems

Closed-loop control.

Benefits:
• System corrects “errors” (e.g. your fridge corrects for temperature variations due to door openings and other events.)
• Labor saving

Drawbacks:
• More expensive and complex.
• Need for sensors
• System can become unstable
Control System Example: Inverted Pendulum

The Problem: The cart with an inverted pendulum is "bumped" with an impulse force, $F$. Determine the dynamic equations of motion for the system, and find a controller to stabilize the system.
Control System Example: Inverted Pendulum

Force analysis and system equations

At right are the two *Free Body Diagrams* of the system.
Control System Example: Inverted Pendulum

Equation of motion for the cart:

\[ M\ddot{x} + b\dot{x} + N = F \]

Equation of motion for the pendulum:

\[ N = m\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta \]
Control System Example: Inverted Pendulum

Without control we get the velocity response shown below, i.e. the pendulum falls to one side and the system is unstable.
Control System Example: Inverted Pendulum

The equation at right describes the four “States” of the system:
Cart Pos. \( x \)
Cart Vel \( x - dt \)
Angle \( \Phi \)
Ang. Vel. \( \Phi - dt \)
Control System Example: Inverted Pendulum

Control Loop Schematic:
R = ‘reference’ = desired state
K = ‘state controller’
Control System Example: Inverted Pendulum

After some mathematical analysis, the controller stabilizes the inverted pendulum:
Control System Example: Inverted Pendulum

We can make the system respond faster, but it will oscillate more:

If we drive the controller gain too high, the system will become unstable.
Control System Example: Inverted Pendulum

There is still a small problem: The cart has wandered off too far. So we add a requirement to return to (almost) where it started:
The Airplane as Computer Peripheral
The Future of Aviation
The Future: More Automation. Manufacturing
Households and Service Industries: Repetitive Jobs will be automated.

Robotic mower for Golf courses.
(Carnegie-Mellon)
Medicine:
Robodoc (Surgical Robot for hip replacement)
Medicine
Prosthetics
Mobility
Assistance
Artificial Intelligence
With better brains and sensors, robots will interact better with humans, and perform more functions.

Sony’s ‘Aibo’
…Remember the poor poet who fell in love with the robot doll?

‘Love’ is reality for many Aibo owners who seem to think that their robot loves them.
DIANE wasn't well. Her owner, Harry Brattin, placed a white muffler around her neck. She sat quietly on a metal desk in the meeting room while the others scampered around the floor playing.

"I get very sad when one of my dogs gets ill," said Mr. Brattin, 63, a motorcycle dealer from San Diego. "When Diane's head stopped moving I felt bad. I truly felt grief."

Diane is an Aibo, a computer-controlled robot made by Sony, and D.H.S. is Droopy Head Syndrome, which is caused when a clutch wears out (it's repairable by replacing the head). Weird, perhaps, but not unusual.
Exploration
A One of Salomon de Caus's theatrical sets at Hellbrunn Castle, in which movable figures were placed in mountain caves. Via a gear transmission, a water-wheel (a) drove a shaft (b) to which both ends of a rope (c) were attached. The rope ran over pulleys (d) and round a wheel (e) at the bottom of a pillar, which carried a mythological figure. Thereupon, the rope ran over more pulleys (f) and back to the shaft.
Control Systems in Entertainment
Control Systems in Entertainment
Control Systems in Entertainment
Control

Open-Loop,
Benefit:
• Simple, always stable
• Widely used in well-defined situations, e.g. Batch filling

Closed-Loop
• Maintains desired output in the presence of disturbances
• Can become unstable
Feedback Control

R(s) \rightarrow \text{Controller} \rightarrow \text{Actuator}
Feedback Control
Feedback Control

\[ \text{Controller} \rightarrow \text{Actuator} \]

\[ \text{Disturbance } W(s) \]

\[ \text{or} \]

\[ - \]

\[ \text{G}_{\text{plant}} \rightarrow \text{Y}(s) \]

\[ \text{K} \]
Each element or ‘Block’ has one input and one output variable.

Transfer Function

\[
G(s) = \frac{\text{Output}(s)}{\text{Input}(s)} \quad \text{e.g.} = \frac{Y(s)}{U(s)}
\]
For instance, the plant in the preceding block diagram can be modeled as:

\[ G_{\text{plant}}(s) = \frac{Y(s)}{U(s)} \]
Transfer Function

\[ R(s) \rightarrow G_{\text{plant}} \rightarrow Y(s) \]

Example: System with Input var. \( r(t) \), Output var. \( y(t) \):

\[ 4y + y(t) = r(t) \]

Laplace operator: \( s = \frac{d}{dt} \)

\[ 4sY(s) + Y(s) = R(s) \]

Regroup:
Transfer Function

\[ 4sY(s) + Y(s) = R(s) \]

\[ Y(s)(4s+1) = R(s) \]

\[ \frac{Y(s)}{R(s)} = \frac{1}{4s + 1} \]
Automatic Control

THE END
Chapter 2: Dynamic Models

Differential Equations in State-Variable Form
State-Variables: Example

\[
\begin{align*}
\dot{x} &= \nu \\
\dot{v} &= \nu \\
m\dot{v} + bv + kx &= u
\end{align*}
\]

- \(x\) is the variable that describes any arbitrary position of the system (also called system variable).
- \(x\) and \(\dot{x}\) are the state-variables of the system.
- Since \(\dot{x} = \nu\), the state-variables can be defined as \(x\) and \(\nu\).
State-Variable Form

Deriving differential equations in state-variable form consists of writing them as a **vector equation** as follows:

\[ \dot{X} = F X + G u \]

where \( y = H X + J u \) is the output and \( u \) is the input.
Definitions

- \( \mathbf{X} \) is the state vector. It contains \( n \) elements for an \( n^{th} \)-order system, which are the \( n \) state-variables of the system.

\[
\mathbf{X} = \begin{bmatrix}
    x_1 \\
    x_2 \\
    \vdots \\
    x_n
\end{bmatrix}
\]

- \( \dot{\mathbf{X}} \) is called state of the system:

\[
\dot{\mathbf{X}} = \begin{bmatrix}
    \dot{x}_1 \\
    \dot{x}_2 \\
    \vdots \\
    \dot{x}_n
\end{bmatrix}
\]

- The constant \( J \) is called direct transmission term.

Important: You can forget that
Deriving the State Variable Form requires to specify $F$, $G$, $H$, $J$ for a given $X$ and $u$.

$$
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\vdots \\
\dot{x}_n
\end{bmatrix} =
\begin{bmatrix}
f_{11} & f_{12} & \cdots & f_{1n} \\
f_{21} & \ddots & \vdots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
f_{n1} & \cdots & \cdots & f_{nn}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_n
\end{bmatrix} +
\begin{bmatrix}
g_1 \\
g_2 \\
\vdots \\
g_n
\end{bmatrix} u
$$

where $y =
\begin{bmatrix}
h_1 \\
h_2 \\
\vdots \\
h_n
\end{bmatrix} + J u$ is the output

and $u$ is the input
Satellite Altitude Control Example

Assumptions:

• $\omega$ is the angular velocity
• The desired system output is $\theta$

Q: Dynamic model in state-variable form?
Strategy (recommended but not required)

1. Derive the **dynamic model**.

2. Identify the **input control variable**, denoted by $u$.

3. Identify the **output variable**, denoted by $y$.

4. Define a **state vector**, $X$, having for elements the system variables and their first derivative.

5. Determine $\dot{X}$

6. Determine $F$ and $G$, in manner that $\dot{X} = FX + Gu$

7. Determine $H$ and $J$, in manner that $y = HX + Ju$
Ex 1: Dynamic Model

- Applying Newton’s law for 1-D rotational motion leads to:

\[ F_c \, d + M_D = I \ddot{\theta} \]

\[ \ddot{\theta} = \frac{F_c \, d + M_D}{I} \]

\[ \Rightarrow \quad (1) \]
Example 1 (cont’d)

Given:

- The control input, denoted by $u$, is given by: $u = F_C d + M_D$
- The output, denoted by $y$, is the displacement angle: $y = \theta$

Assumption:

- The state vector, denoted by $X$, is defined as:

\[
X = \begin{bmatrix} \theta \\ \omega \end{bmatrix} \text{ with } \omega = \dot{\theta}
\]

Known: The dynamic model

\[
\ddot{\theta} = \frac{F_C d + M_D}{I}
\]  (1)

Required: Rewrite (1) as:

\[
\begin{bmatrix} \dot{\theta} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{1}{I} \end{bmatrix} \begin{bmatrix} \theta \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{1}{I} \end{bmatrix} u
\]

where $y = \begin{bmatrix} \theta \\ \omega \end{bmatrix}$ and $u = \begin{bmatrix} \frac{1}{I} \end{bmatrix} u$
Example 1 (cont’d)

• By definition: \( \dot{\theta} = \omega \). Thus, \( \ddot{\theta} = \dot{\omega} \)

• Expressing the dynamic model:

\[
\ddot{\theta} = \frac{F_c d + M_D}{I}
\]  

(1)

as a function of \( \omega \) and \( u \) (with \( u = F_c d + M_D \) )

yields:

\[
\dot{\omega} = \frac{u}{I}
\]
Example 1  
(cont’d)

Available equations:

- From the dynamic model:
  \[
  \begin{align*}
  \dot{\theta} &= \omega \\
  \dot{\omega} &= u/I
  \end{align*}
  \]

- The output \( y \) is defined as:
  \[ y = \theta \]

- The input \( u \) is given by:
  \[ u = F_C d + M_D \]

Equivalent form of available eq:

\[
\begin{align*}
\dot{\theta} &= 0 \times \theta + 1 \times \omega + 0 \times u \\
\dot{\omega} &= 0 \times \theta + 0 \times \omega + (1/I) \times u \\
y &= 1 \times \theta + 0 \times \omega + 0 \times u
\end{align*}
\]

where

\[
\begin{align*}
u &= F_C d + M_D \\
\begin{bmatrix}
\dot{\theta} \\
\dot{\omega}
\end{bmatrix} &=
\begin{bmatrix}
1 & 0 \\
0 & 1/I
\end{bmatrix}
\begin{bmatrix}
\theta \\
\omega
\end{bmatrix}
\begin{bmatrix}
\theta \\
\omega
\end{bmatrix}
\end{align*}
\]

where \( y \) and \( M_D \)
Example 1: Dynamic Model in State-Variable Form

By defining $X$ and $u$ as:

$$X = \begin{bmatrix} \theta \\ \omega \end{bmatrix}$$

$$u = F_c \, d + M_d$$

The state-variable form is given by:

$$F = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad G = \begin{bmatrix} 0 \\ 1/I \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad J = 0$$
Ex 1½: Recall the Satellite Altitude Control Example

Assumptions:

• $\omega$ is the angular velocity
• The system output is $\omega$

Q: Dynamic model in state-variable form?
Example 1½: Dynamic Model in State-Variable Form

By defining $X$ and $u$ as:

$$X = \begin{bmatrix} \theta \\ \omega \end{bmatrix}$$

$$u = F_C d + M_D$$

The state-variable form is given by:

$$F = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad G = \begin{bmatrix} 0 \\ 1/I \end{bmatrix}$$

$$H = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad J = 0$$

*Only change*
Analysis in Control Systems

• **Step 1**: Derive a **dynamic model**

• **Step 2**: Specify the dynamic model for software by writing it **either**
  - in **STATE-VARIABLE** form or
  - in terms of its **TRANSFER FUNCTION** (see chapter 3)
Example 2: Cruise Control Step Response

- **Q1**: Rewrite the equation of motion in state-variable form where the output is the car velocity $v$?

- **Q2**: Use MATLAB to find the step response of the velocity of the car? Assume that the input jumps from being $u(t) = 0$ N at time $t = 0$ sec to a constant $u(t) = 500$ N thereafter.
Reminder: Strategy

1. Derive the **dynamic model**.
2. Identify the **input control variable**, denoted by $u$.
3. Identify the **output variable**, denoted by $y$.
4. Define a **state vector**, $X$, having for elements the system variables and their first derivative.
5. Determine
6. Determine $F$ and $G$, in manner that $\dot{X} = FX + Gu$
7. Determine $H$ and $J$, in manner that $y = HX + Ju$
Ex 2, Q1: Dynamic Model

- Applying Newton’s law for translational motion yields:

\[-b\ddot{x} + u = m\ddot{x}\]

\[\Rightarrow \dot{x} = -\frac{b}{m}\dot{x} + \frac{u}{m}\]

(2)
Example 2, Question 1 (cont’d)

Given:
• The input ( = external force applied to the system) is denoted by \( u \)
• The output, denoted by \( y \), is the car’s velocity: \( y = v \)

Assumption:
• The state-vector is defined as: \( X = \begin{bmatrix} x \\ v \end{bmatrix} \) with \( v = \dot{x} \) and \( u = \)

Known: The dynamic model
\[
\dot{x} = -\frac{b}{m} \dot{x} + \frac{u}{m} \tag{2}
\]

Required: Rewrite (2) as:
\[
\begin{bmatrix} \dot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} x \\ v \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix} + \begin{bmatrix} u \end{bmatrix}
\]

where \( y = \begin{bmatrix} x \\ v \end{bmatrix} + \times u \)
Example 1 (cont’d)

• By definition: $\dot{x} = v$. As a result, $\ddot{x} = \dot{y}$

• Expressing the dynamic model:

$$\ddot{x} = -\frac{b}{m}\dot{x} + \frac{u}{m} \quad (2)$$

as a function of $v$ and $u$ leads to:

$$\dot{v} = -\frac{b}{m}v + \frac{u}{m}$$
Ex 2, Q1 (cont’d)

Available equations:

• From the dynamic model:

\[
\begin{align*}
\dot{x} &= v \\
\dot{v} &= -\frac{b}{m}v + \frac{u}{m}
\end{align*}
\]

• The output \( y \) is defined as:

\[ y = v \]

• The input is the step function \( u \)

Equivalent form of available eq:

\[
\begin{align*}
\dot{x} &= 0 \times x + 1 \times v + 0 \times u \\
\dot{v} &= 0 \times x + \left( -\frac{b}{m} \right) \times v + \left( \frac{1}{m} \right) \times u \\
y &= 0 \times x + 1 \times v + 0 \times u
\end{align*}
\]

where

Therefore:

\[
\begin{bmatrix}
\dot{x} \\
\dot{v}
\end{bmatrix} =
\begin{bmatrix}
X \\
F
\end{bmatrix} +
\begin{bmatrix}
x \\
v
\end{bmatrix} u
\]

where \( y = \begin{bmatrix} x \\ v \end{bmatrix} + \begin{bmatrix} H \\ J \end{bmatrix} \times u \)
Example 2, Question 1: Dynamic Model in State-Variable Form

By defining \( X \) as:

\[
X = \begin{bmatrix} x \\ v \end{bmatrix}
\]

The state-variable form results as:

\[
F = \begin{bmatrix} 0 & 1 \\ 0 & -b/m \end{bmatrix}, \quad G = \begin{bmatrix} 0 \\ 1/m \end{bmatrix},
\]

\[
H = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad J = 0
\]
Example 2, Question 2: Step Response using MATLAB?

Assumptions: \( m = 1000 \text{ kg} \) and \( b = 50 \text{ N}.\text{sec/m} \).

\[
F = \begin{bmatrix} 0 & 1 \\ 0 & -0.05 \end{bmatrix} \quad G = \begin{bmatrix} 0 \\ 0.001 \end{bmatrix}
\]

\[
H = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad J = 0
\]
Ex 2, Q2: Step Response with MATLAB?

- The **step function** in MATLAB calculates the time response of a linear system to a **unit step input**.

- In the problem at hand, the input \( u \) is a step function of amplitude 500 N:
  \[
  u = 500 \times \text{unity step function}.
  \]

- Because the system is linear (\( X = FX + Gu \)):
  \[
  G \times u = (500 \times G) \times \text{unity step function}
  \]

\[
G \times \text{Step 0 to 500 N} \rightarrow 500 \times G \times \text{Step 0 to 1 N}
\]
MATLAB Statements

F = [0 1;0 -0.05];
G = [0;0.001];
H = [0 1];
J = 0;
sys = ss(F, 500*G, H, J);
t = 0:0.2:100;
y = step(sys,t);
plot (t,y)

% defines state variable matrices
% defines system by its state-space matrices
% setup time vector ( \( dt = 0.2 \) sec)
% computes the response to a unity step response
% plots output (i.e., step response)
Response of the car velocity to a step input $u$ of amplitude $500 \, N$