Chapter 6 Frequency Response

Note Title

11/2/2011

$$\frac{a}{b} \qquad Z = \frac{1}{\underset{Im}{j\omega} + \underset{Re}{1}} = \frac{Z_1}{Z_2}$$

Remember:

Euler

$$Z = Re + j\,Im = |Z|\,e^{j\varphi}$$

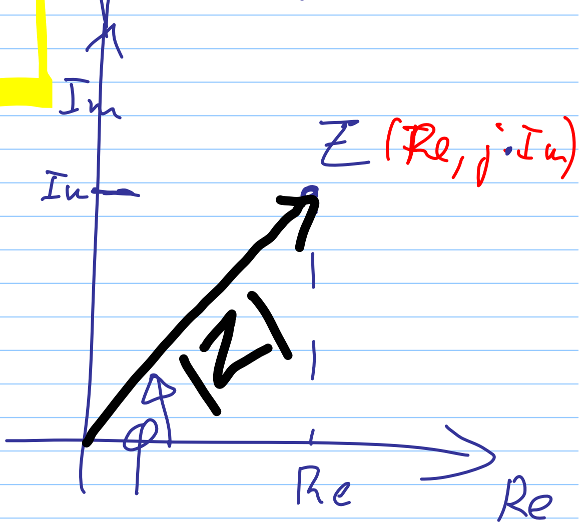
$$|Z| = \sqrt{Re^2 + Im^2} \quad \boxed{\text{no } j!}$$

$$\boxed{\frac{Z_1}{Z_2} = \frac{|Z_1|}{|Z_2|}\, e^{j(\varphi_1 - \varphi_2)}}$$

use only this format

$$\tan\varphi = \frac{Im}{Re}$$

$$\boxed{\text{no } j!} \quad \varphi = \tan^{-1}\frac{Im}{Re}$$



Mathcad

$$F(j\omega) = \frac{1}{j\omega + 1}$$

ω	$\lvert F\rvert$	$\varphi(\omega)$
0	1	0
1	0.707	$-\pi/4$
100	$1/100$	$\approx -88^\circ$

at $\omega = 100$

at $\omega = 1$

$$\boxed{F = \frac{1}{j\omega + 1}}$$

$$\lvert F\rvert = \frac{1}{\sqrt{1^2 + 1^2}} = \frac{1}{\sqrt{2}}$$

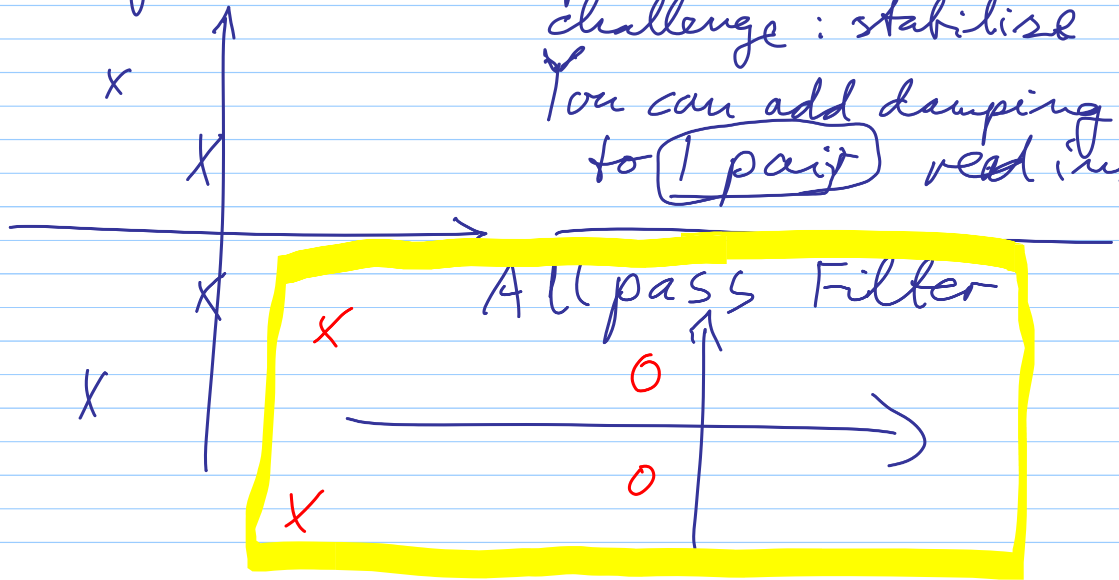
$$\varphi = 0 - \tan^{-1}\frac{1}{1} = 0 - \pi/4 = -\frac{\pi}{4}$$

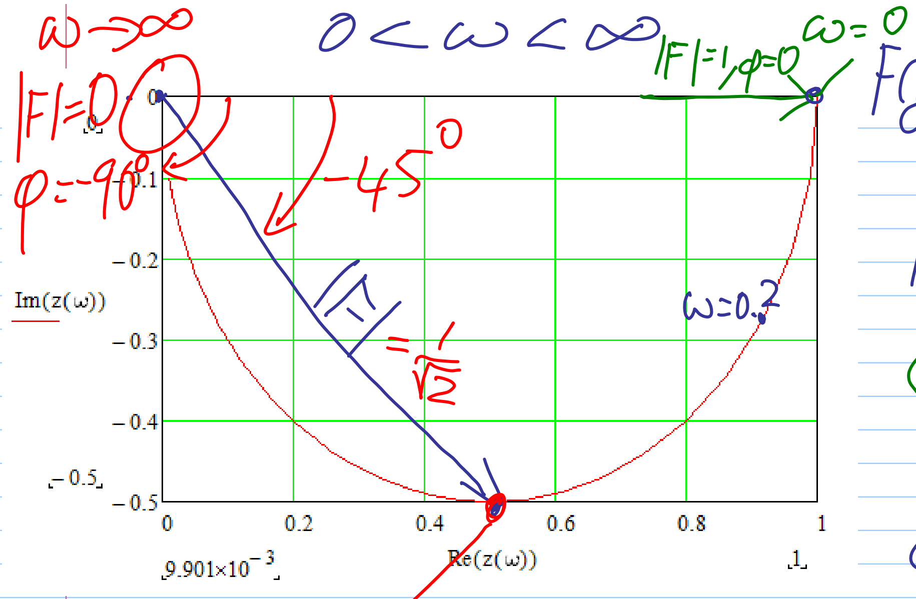
$$F \approx \frac{1}{100} \qquad \varphi = 0 - \underbrace{\tan^{-1}\frac{100}{1}}_{\approx -88^\circ} \left(\frac{Im}{Re}\right)$$

Project #3 report

challenge: stabilise

You can add damping to 1 pair real inst.





$$F(j\omega) = \frac{1}{j\omega+1}$$

$$|F| = \frac{1}{\sqrt{\omega^2+1}}$$

$$\boxed{\text{let } \omega = 1}$$

$$|F|(\omega=1) = \frac{1}{\sqrt{2}}$$

$$\varphi = \tan^{-1}\frac{Im}{Re}$$

here $\tau = 1$

Break Frequency

$$\varphi(\omega=1) = 0 - \tan^{-1}\frac{\omega}{1} = 0 - \frac{\pi}{4}$$

Def.: Break Freq.: $F(j\omega) = \frac{K}{j\omega\cdot\tau + 1}$

τ = time const

$$\boxed{\omega_B = \frac{1}{\tau}}$$

Division $F = \frac{K}{(\tau_1 s+1)(\tau_2 s+1)}$ $\quad 0<\omega<\infty$

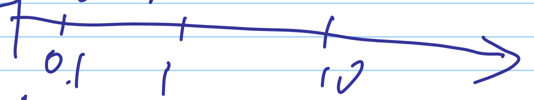
$$|F| = \frac{K}{\sqrt{(\omega\tau_1)^2+1} * \sqrt{(\omega\tau_2)^2+1}}$$

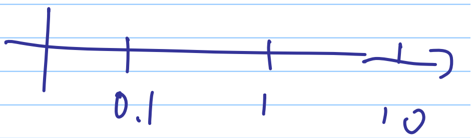
$$\log_{10}|F| = \log K - \log\sqrt{(\omega\tau_1)^2+1} - \log\sqrt{\cdot}\ \cdots$$

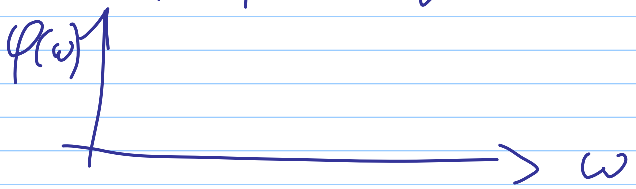
Log scaling => Bode plots

spread data

$\log|F|(\omega)$ – graphical add







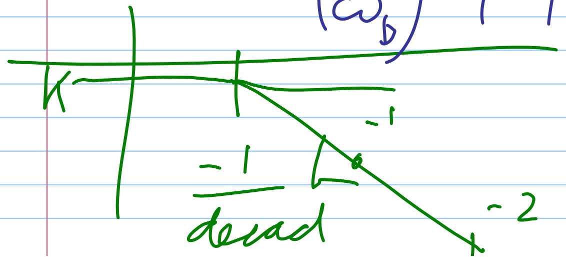
$$|F| = \frac{K}{\sqrt{(\tau\omega)^2+1}}$$

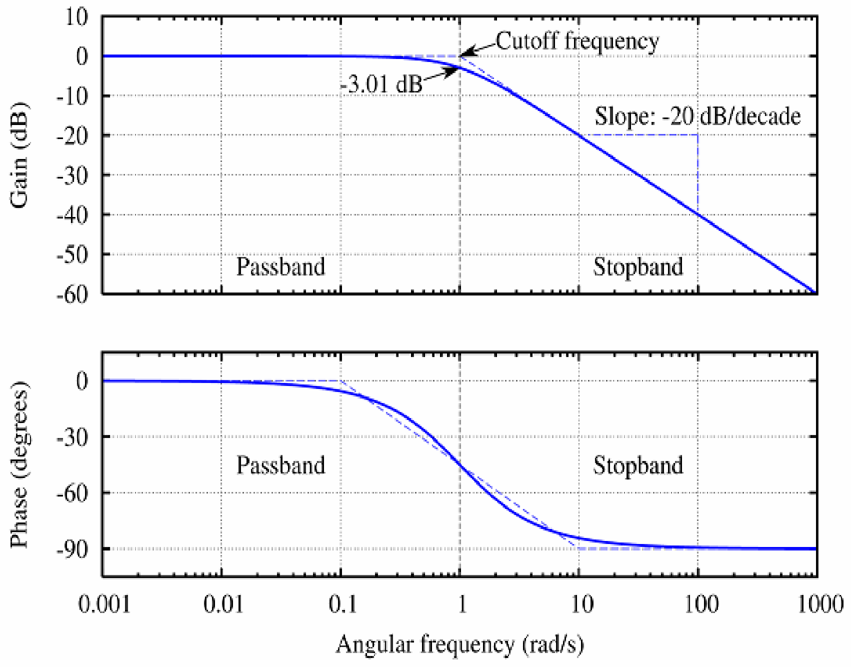
let $\tau = 1$

$\omega_b = 1$

$$= \frac{K}{\sqrt{\left(\frac{\omega}{\omega_b}\right)^2+1}}$$

ω	$\lvert F\rvert$
$1=\omega_b$	$K/\sqrt{2}$
10	$K/\sqrt{101} \approx \frac{K}{10}$
	$\log(K/10) = \log K - 1$
100	$K/100 \Rightarrow \log - 2$





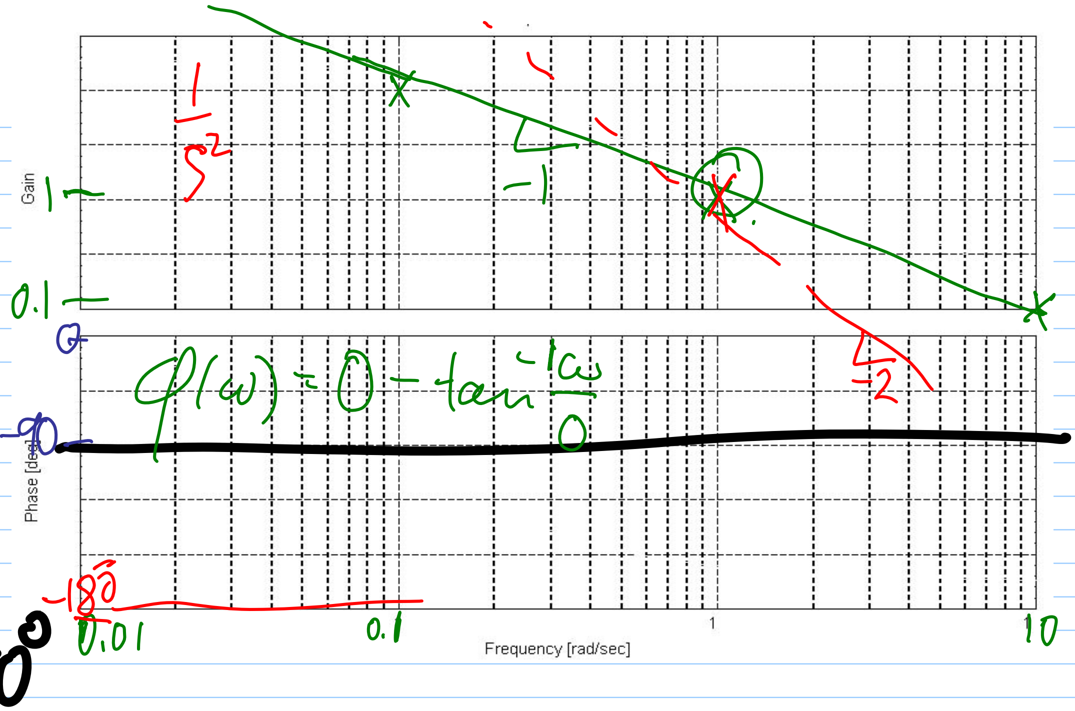
Def.:

$x_{dB} = 20\lg x$

$G(s) = \frac{1}{s}$

$|F| = \frac{1}{\omega}$

ω	$\lvert F \rvert$
1	1
10	0.1
0.1	10

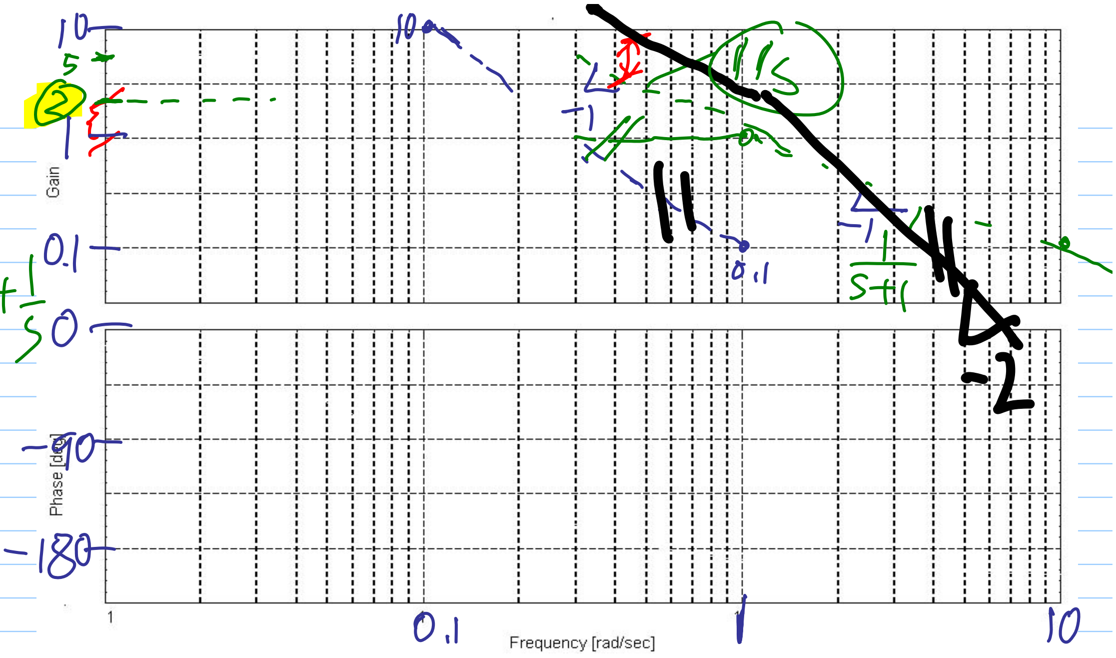


$\varphi = -90^\circ$

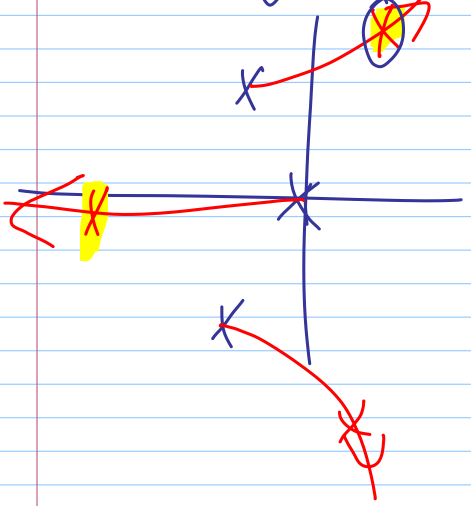
$G = \frac{2}{s(s+1)}$

add:

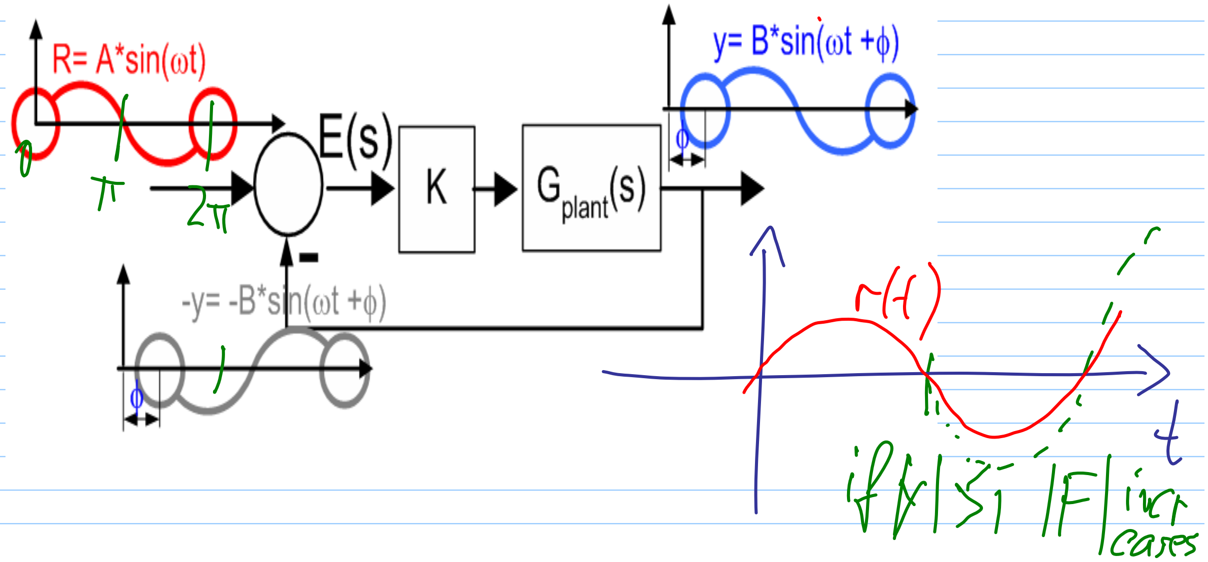
$\lg 2 + \frac{1}{s+1} + \frac{1}{s}$



Stability RLocus for chosen K ⇒ closed loop poles explicitly



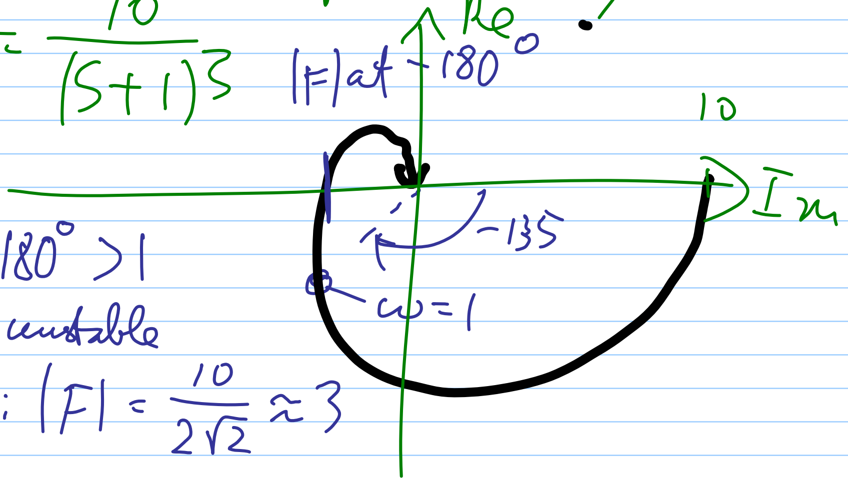
Closed Loop Stability: Nyquist Criterion



if $|X| \geq 1$ $|F|$ incr cases

Nyquist closed loop stability

$$G(s) = \frac{10}{(s+1)^3}$$

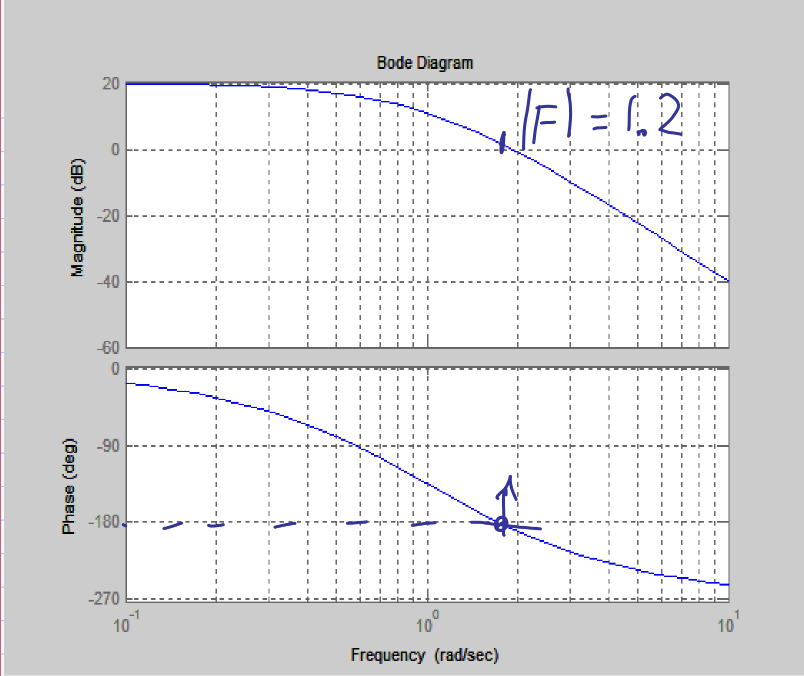


if $|F|$ at $\varphi = -180° > 1$

closed loop is unstable

let $\omega = 1$; $|F| = \frac{10}{2\sqrt{2}} \approx 3$

$= 2$



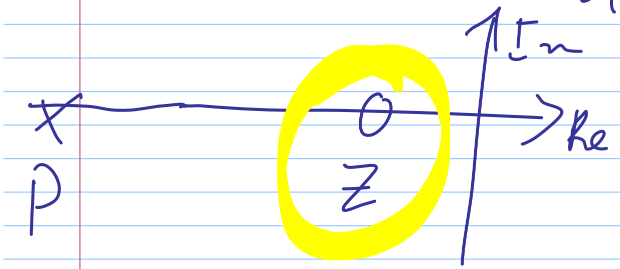
$$\frac{10}{(s+1)^3}$$

at $\varphi = -180$

$|F| \approx 1.2 > 1$

$\Rightarrow$ closed loop is unstable

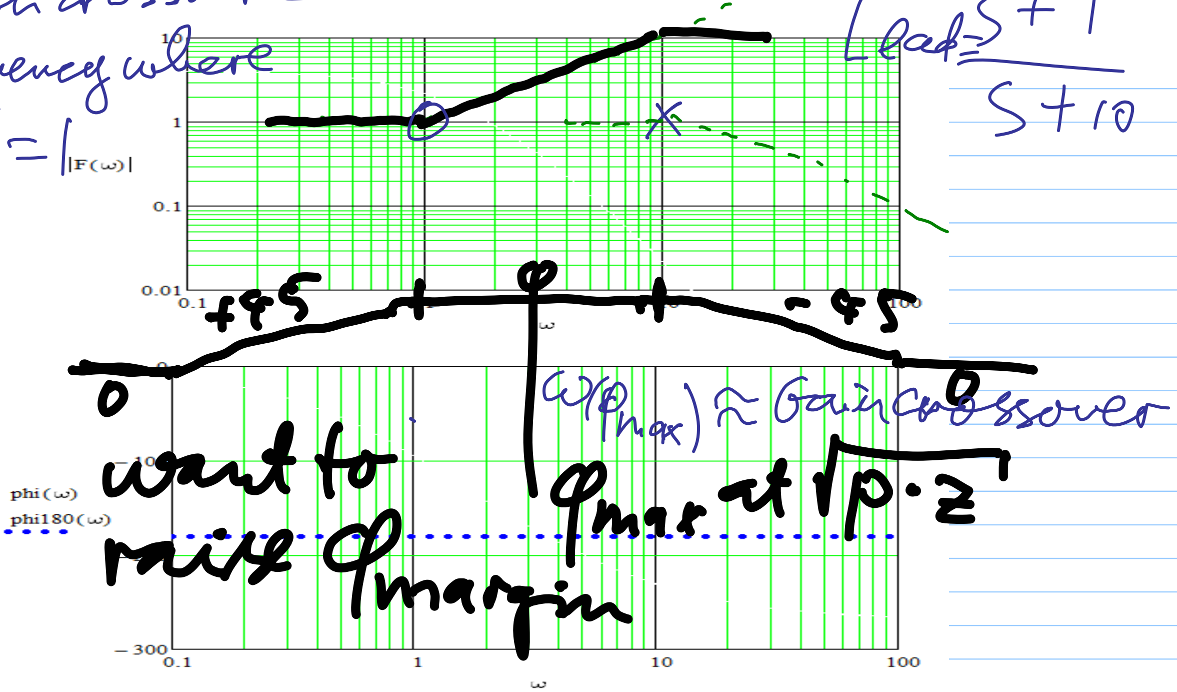
Lead $G_{lead} = \frac{s+Z}{s+P}$



$|P| >> |Z|$

Gain crossover = frequency where $|F| = 1$

Lead $= \frac{s+1}{s+10}$



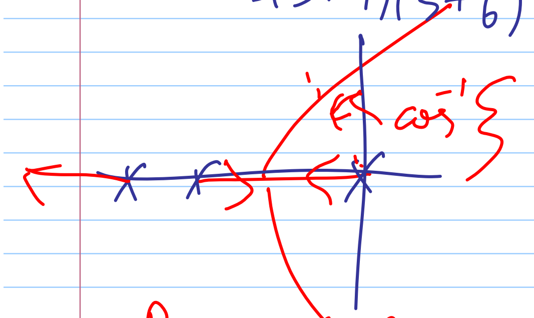
$\omega(\varphi_{max}) \approx$ Gain crossover

want to raise φmargin

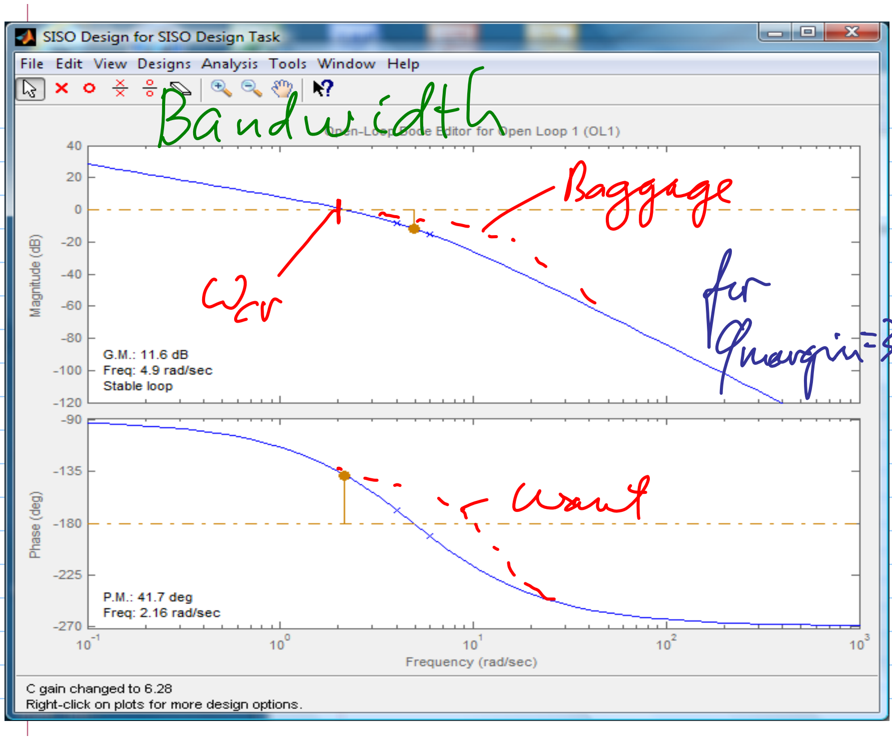
φ_{max} at $\sqrt{P \cdot Z}$

Example:

$$G(s) = \frac{10}{s(s+4)(s+6)}$$



P-control $K=9.5$ $\varphi_{margin} \approx 30°$



G.M.: 11.6 dB
Freq: 4.9 rad/sec
Stable loop

P.M.: 41.7 deg
Freq: 2.16 rad/sec

C gain changed to 6.28
Right-click on plots for more design options.

with Lead increase φ_{margin} / raise gain ~~$K=10$~~

with Lead gain $K \approx 30$

Project

mostly: good work

benefit: reality

Reports: provide complete information to judge results.

corrections due Wed 11/16

Midterm 3 : Mon 11-28

Topics: Ch. 6

Polar
Bode
Nyquist
Lead } comp design
Lag

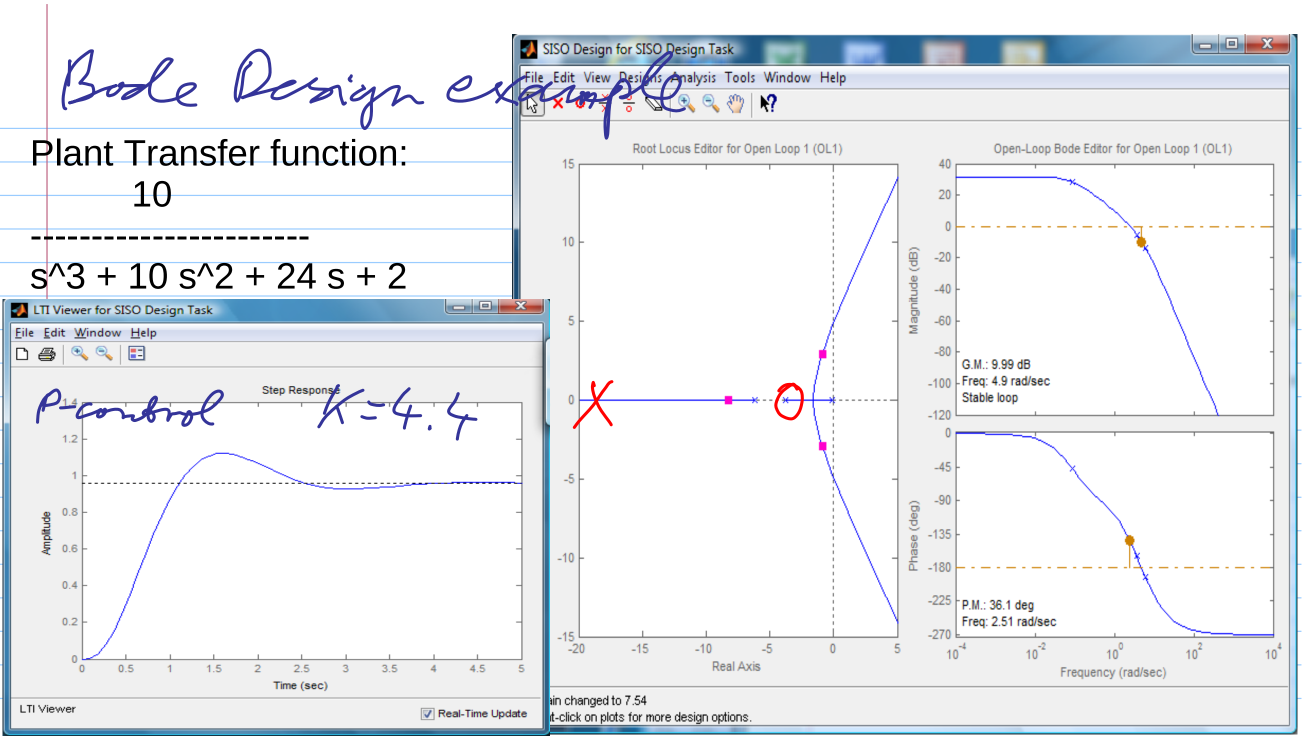
Projects
good work
complexity, systematic inquiry

Big picture: $\boxed{r \overset{?}{=} y}$

Bode Design example

Plant Transfer function:

$$\frac{10}{s^3 + 10 s^2 + 24 s + 2}$$



G.M.: 9.99 dB
Freq: 4.9 rad/sec
Stable loop

P.M.: 36.1 deg
Freq: 2.51 rad/sec

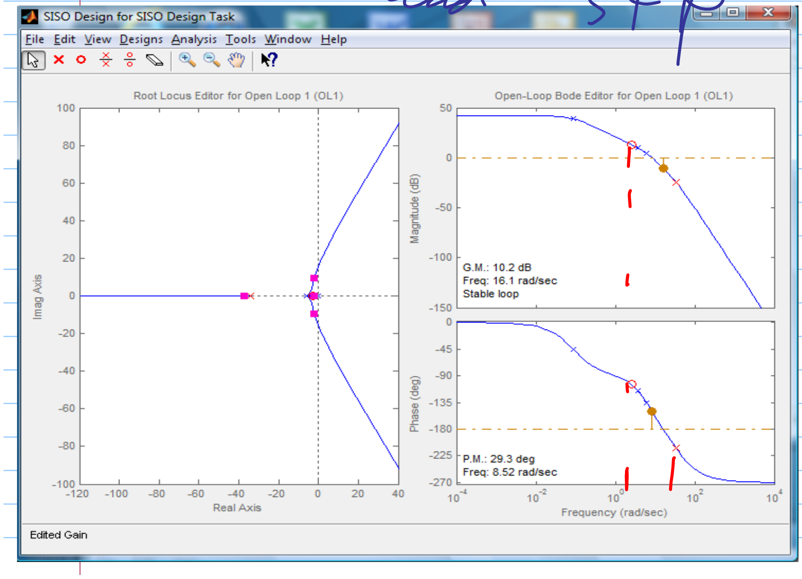
ain changed to 7.54
t-click on plots for more design options.

P-control $K=4.4$

Lead: $G_{lead} = \frac{s+\boxed{z}}{s+p}$ — attract

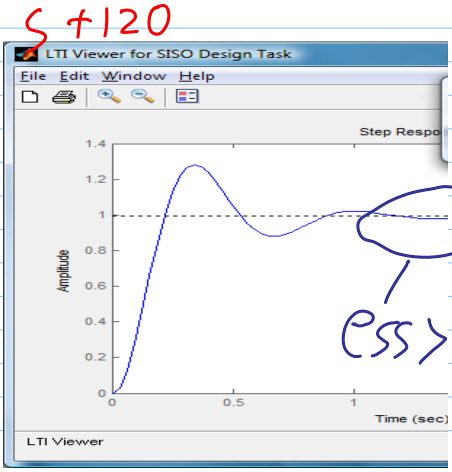
$|p| \gg |z|$

$$\frac{s+15}{s+120}$$



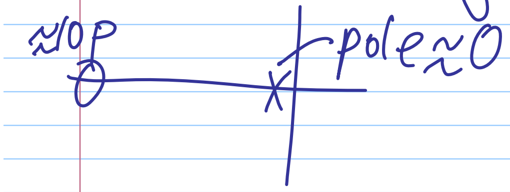
G.M.: 10.2 dB
Freq: 16.1 rad/sec
Stable loop

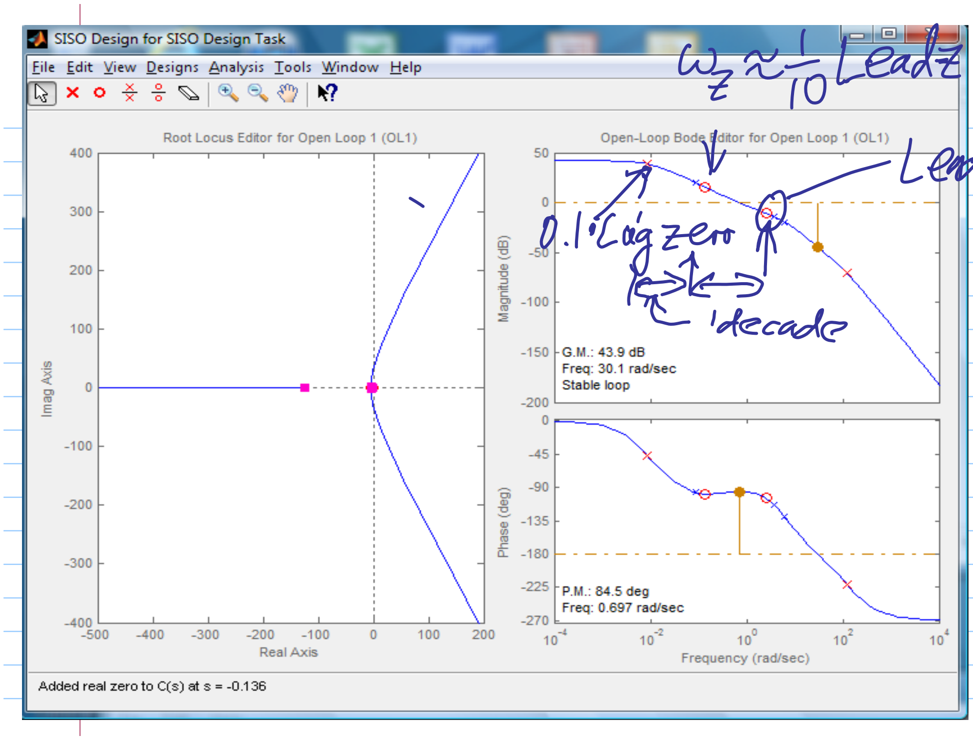
P.M.: 29.3 deg
Freq: 8.52 rad/sec



$e_{ss} > 0$

Can add Lag $\frac{s+z}{s+p}$ $|p| \ll |z|$





G.M.: 43.9 dB
Freq: 30.1 rad/sec
Stable loop

P.M.: 84.5 deg
Freq: 0.697 rad/sec

Added real zero to C(s) at s = -0.136

$\omega_z \approx \frac{1}{10}$ Lead z Rule

Lead zero

0.1·Lag zero

1decade

$$G = \frac{8}{(0.1s+1)^2\boxed{(10s+1)}}$$

$\omega_b = 10$

$\varphi(\omega=10) = -90^0$

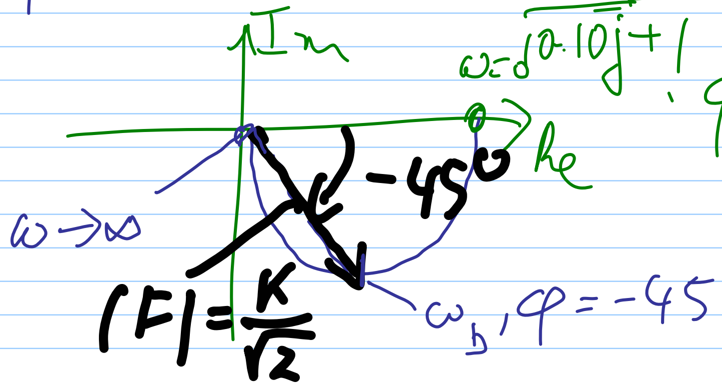
at $\boxed{\frac{1}{0.1 \cdot 10+1}}$

$\varphi = -45^0$

$$\frac{1}{0.10j+1}$$

$\omega = 10$

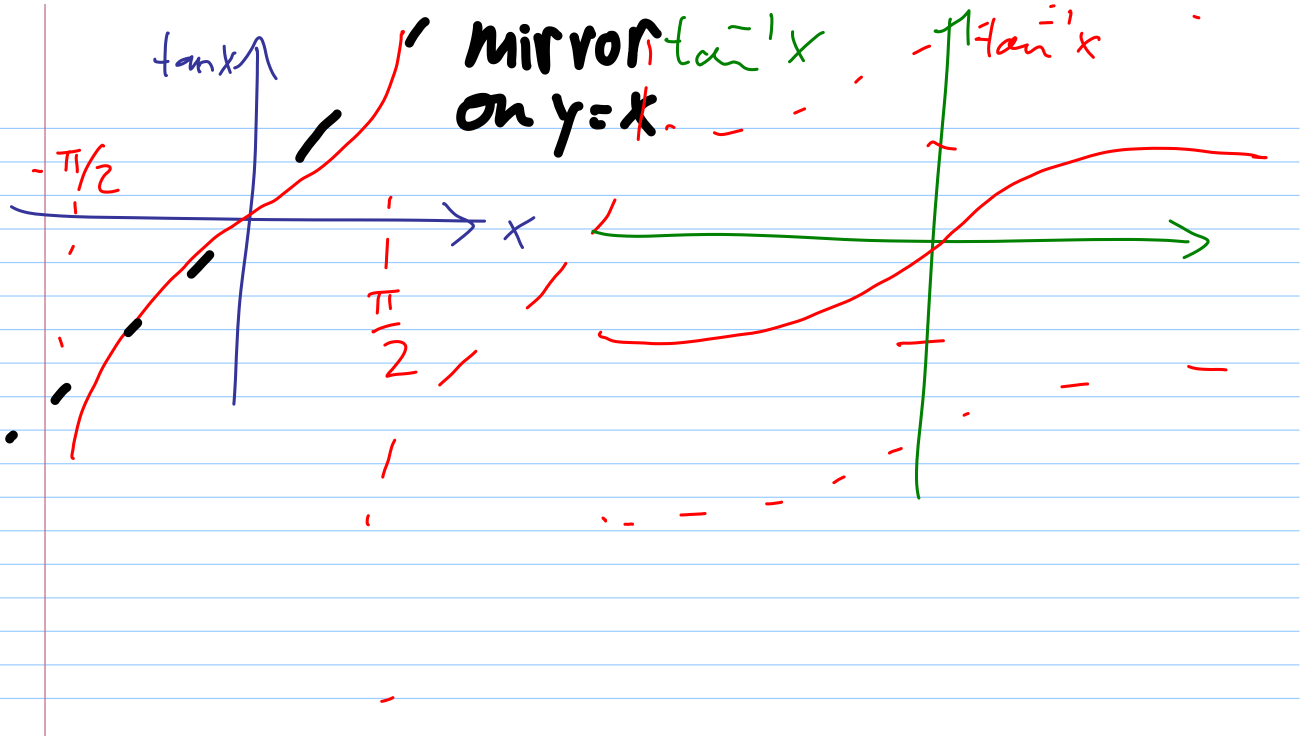
$\varphi = -\tan^{-1}\frac{100}{1} \approx -90$



$$G = \frac{10}{(s+1)(0.2s+1)}$$

$$\varphi(\omega) = -\overbrace{\tan^{-1}\frac{\omega}{1}}^{\varphi_1} - \tan^{-1}\overbrace{\frac{0.2\omega}{1}}^{\varphi_2}$$

$$F(j\omega) = \frac{10}{|s+1|\cdot|0.2s+1|}e^{j(0-\varphi_1-\varphi_2)}$$



$$G = \frac{5}{s(6s+1)}$$

-90^0

$-90^0 \to \omega \to \infty$

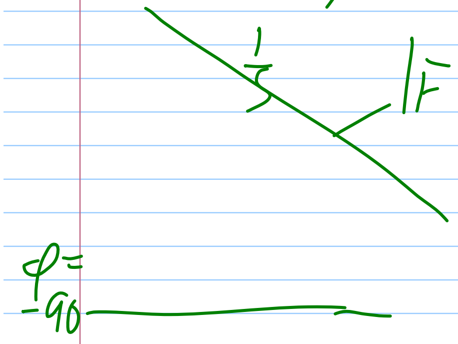
0 at $\omega = 0$

$\varphi(\text{Int}) = -90$

$$\frac{5}{j\omega}$$

$\varphi = 0 - \tan^{-1}\frac{\omega}{0}$

$\boxed{-\tan^{-1}\infty} = -90^0$



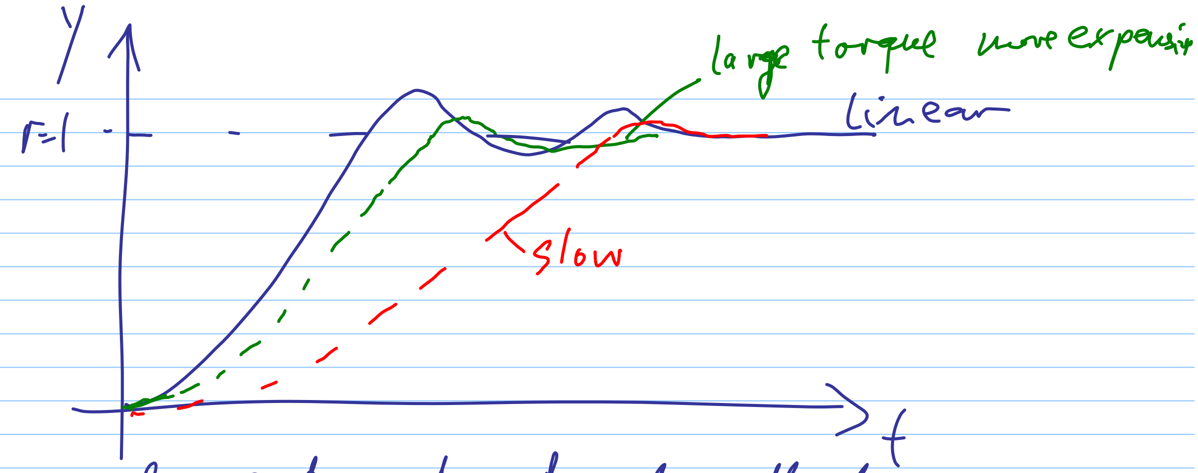
Project last report due 11/23

- review and adjust overall design
- validate actuator selection

all actuators must reach the required S.S. output value

show R in all time domain plots

create two alternate designs



several reports not yet submitted.

please submit a.s.a.p. all reports required