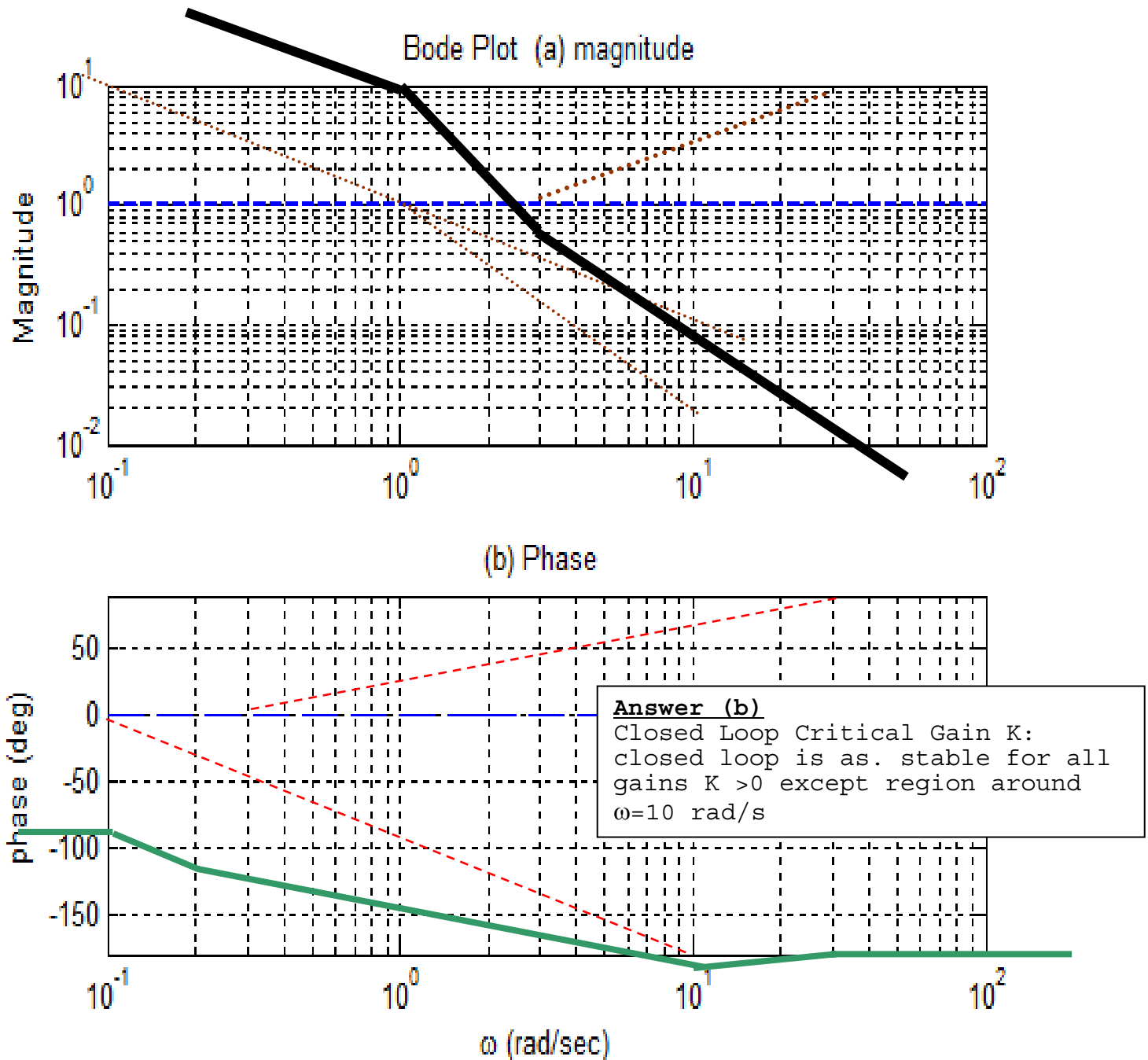


**MEG 421 Automatic Controls**  
Third Test, Closed Book examination

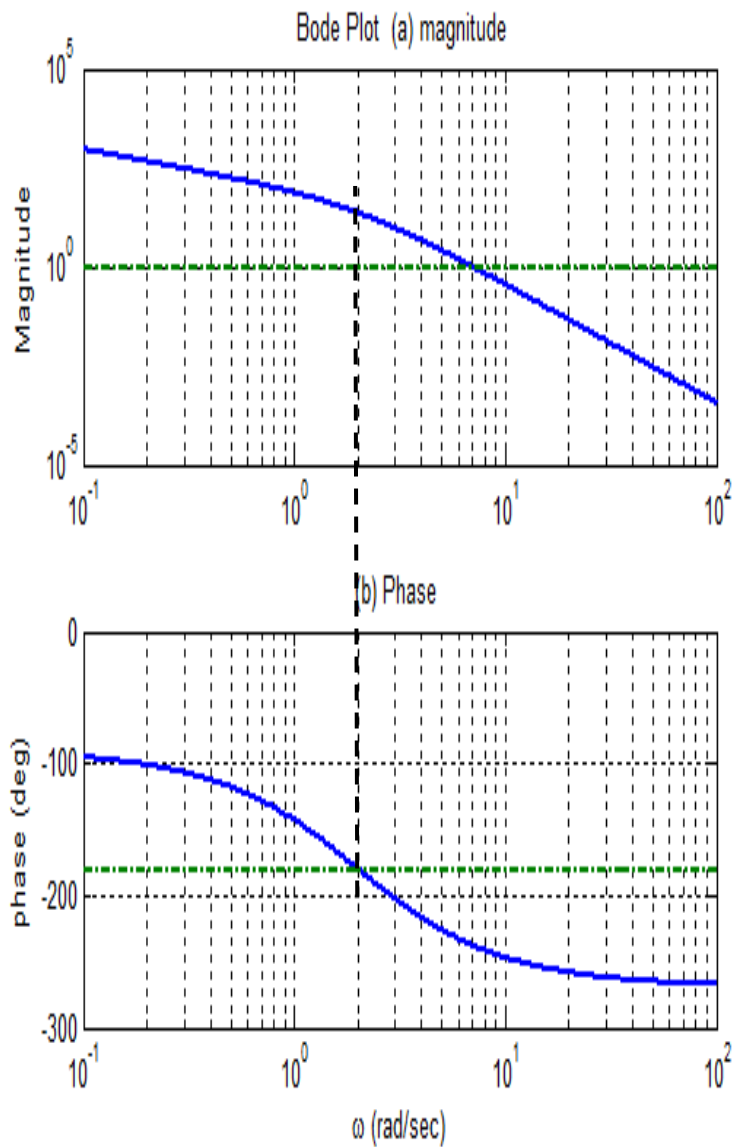
1. (30 points) (a) Construct the Bode plot of the plant  $G(s) = \frac{10K(0.3s+1)}{s(s+1)^2}$  for  $K=1$ .

Construct accurate asymptotic approximations of both magnitude and phase plots.

(b) Determine the critical gain  $K$  (stability limit) of the closed loop system, using the Nyquist criterion in the Bode plot.



2. (30 points) The Bode plot of the open-loop system  $G(s) = \frac{90}{s(0.5s+1)^2}$  is shown below at left.



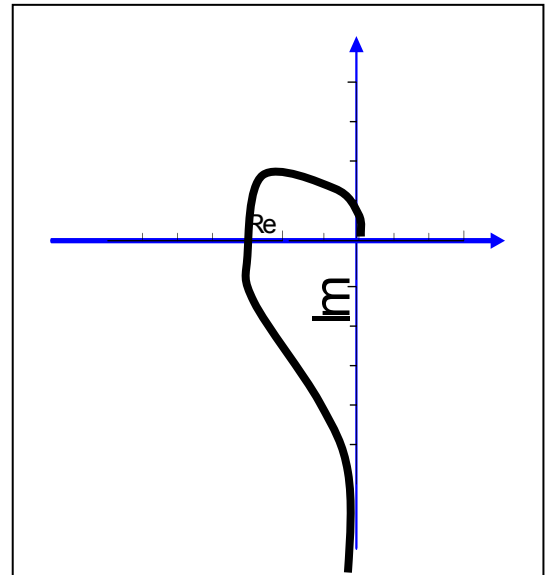
left.

(a) Draw the polar plot of the open-loop system below. Scale both polar plot axes!

(b) Applying the Nyquist criterion in one of the two plots (your choice), determine whether the closed loop system is stable. Explain!

(c) Determine the system's gain margin.

Answer a:  $|F| \rightarrow \text{inf}$  at  $\omega = 0$ . Neg. real axis crossing at  $|F| \sim 50$



Answer b: closed loop stability unstable | Reasons:  $|F| > 1$  at gain crossover

Answer c: Gain margin =  $1/K_{\text{crit}} = 0.02$

3. (10 points) A transfer function is given as

$$G_{open}(s) = \frac{36}{s(s+6)(s+4)(s+0.5)}$$

Determine the system's steady state gain and time constants, so that it can be rewritten in the standard form used for frequency response analysis:

$$G_{open}(s) = \frac{K_{ss}}{s(\tau_1 s + 1)(\tau_2 s + 1)(\tau_3 s + 1)}$$

K <sub>ss</sub> = 3				
τ <sub>1</sub> =	1/6	τ <sub>2</sub> =	0.25	τ <sub>3</sub> = 2

4. (30 points) The Bode plot on the next page depicts an open-loop system G(s) where

$$G(s) = \frac{1}{(0.2s+1)(0.1s+1)(0.025s+1)} \quad \text{A lag compensator is given as} \quad G_{lag}(s) = \frac{s/z+1}{(s/0.1z+1)}$$

where z is the lag zero.

A phase margin of 45 degrees (marked in the plot) is desired for closed loop control.

- Determine the P-controlled system's gain K at the desired phase margin.
- Graphically add** a lag compensator such that the resulting system has a higher closed loop gain (at least 5 times better than P-control) at the phase margin of approximately 45 degrees. Show clearly in the Bode plot how you selected the lag zero and pole.
- Sketch the approximate compensated system (Plant \* Lead)

<b>Answers</b>		
(a) Gain K (P-control) =	3	5pts
(b) Controller gain K (Lag control) =	30	5pts
Lag transfer function =	10*(s+1.2)/(s+0.12)	5pts
(c) ω <sub>CR</sub> (Comp*Sys) =	12 rad/s	5pts

Accuracy and completeness of plot next page:

10pts

$$G(s) = \frac{1}{(0.2s+1)(0.1s+1)(0.025s+1)}$$

