Name: **KEY**

# UNIVERSITY OF NEVADA, LAS VEGAS
# DEPARTMENT OF MECHANICAL ENGINEERING

MEG 421 Automatic Control Fall 2011

Second Test, Closed Book One page of handwritten notes, Handout Root Locus, and Table of Laplace Transforms allowed.

**1. (25 points)** A closed-loop feedback system (controller Comp, Plant G and feedback H) is shown in Fig. 1 below.

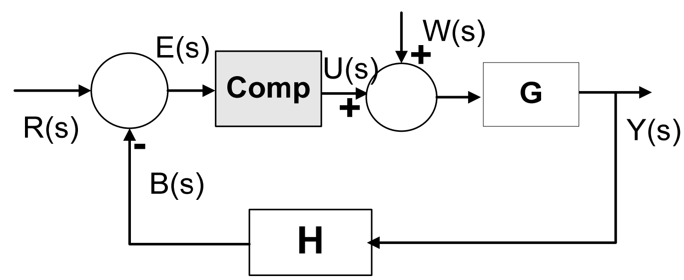



**Figure 1** Feedback System

**(a)** For G=7, Comp = 3, H=2, r = 0 and w = **unit ramp** input signal, determine the closed loop system's steady state error $e_{ss}$ **.**

**(b)** For $G = \frac{8}{(s+1)(s+4)}$, H=2, w =0, Comp(s) = 3(s +2), and r(t) = unit step input, determine the closed loop system's steady state error $e_{ss}$ **.**

**(c)** For $G = \frac{8}{(s+1)(s+4)}$ , $Comp(s) = \frac{1}{s}$ w=0, and H =1, determine the closed loop system's steady state error to a unit ramp input r(t).

Answer (a) Y/W = 7/ (1+7*3*2) for W= $1/s^2$, we get Y = 7/42*$1/s^2$ also: E = -Y, thus the error becomes infinite

(b) $Y(s) = \frac{24(s+2)}{(s+1)(s+4)+48(s+2)} * \frac{1}{s}$ ess = r - yss = 1 - 24*2/(4+96) = 0.52

(c) $Kv = \lim_{s=0} s \frac{8}{s(s+1)(s+4)} = 2$

Answers

| | |
|---|---|
| (a) $e_{ss,ramp} \rightarrow$ inf<br>*7pts*<br>-----------------------------------<br>(b) (b) $e_{ss,step}$ = 0.52<br>*7pts* | (c) $e_{ss,ramp}$ = ½<br>*8 pts* |

**2. (40 points)** The open-loop transfer functions of two systems are given as:

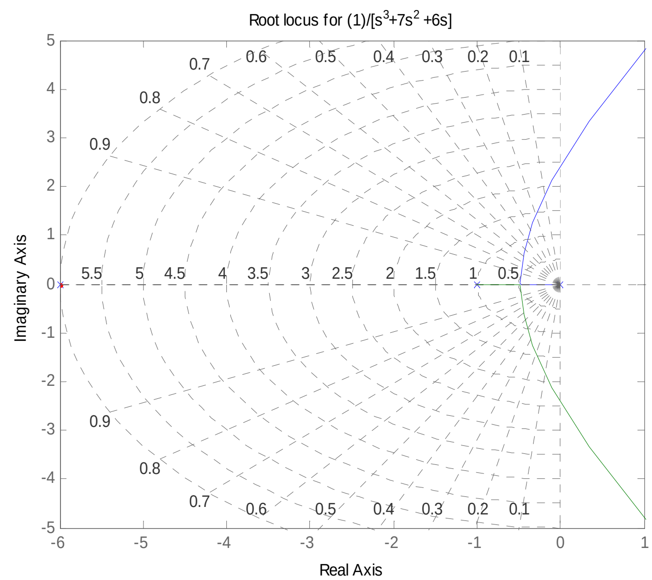
$$(a)\quad G(s) = \frac{K}{s(s+6)(s+1)} \qquad\qquad (b)\quad G(s) = \frac{Ks(s-1)}{(s+6)(s+4)}$$

For each system, draw all branches and asymptotes of the root locus, and indicate in each branch the directions of the R.L for increasing K. Determine all imaginary axis crossings if they exist. Determine the range of K, if any, for which the closed loop system is asymptotically stable. **Label and scale all axes (5 pts each)**.

**Answers 2a)** # of asympt. = 3

Char. Eq: s^3 + 7s^2+ 6s + K = 0
Im=0 gives: $om^2 = 6$
Re=0 gives: $K = 7*om^2 = 42$
Im. axis crossings at +/- j*sqrt(6)

As. stable range of K: 0 < K < 42

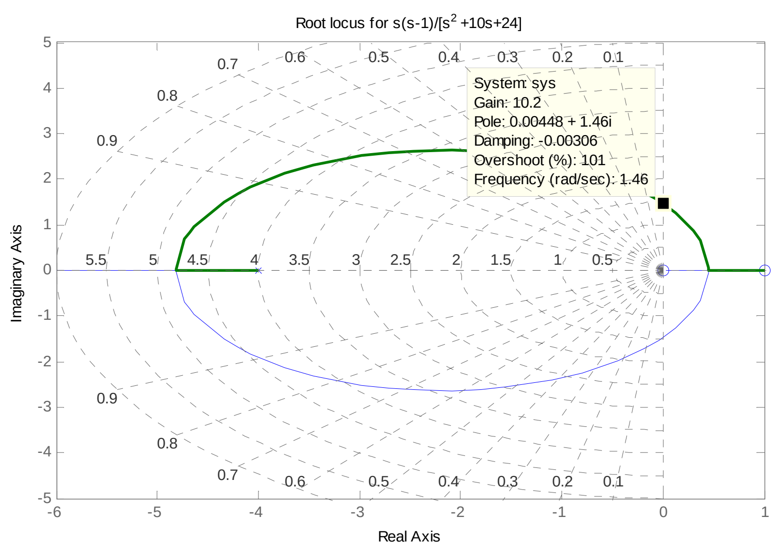


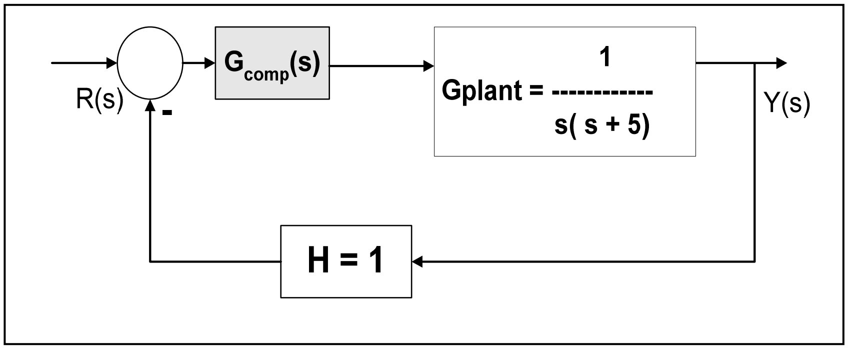

**Answers 2b** # of asympt. =0

Char eq: $s^2(1+K) + s(10-K) + 24 = 0$

Im = 10-K = 0 → $K_{crit} = 10$
Re = 0: $om^2*(1+10) = 24$
Im. axis crossings at **+/- 1.46j**

As. stable range of K **0 < K < 10**






**3. (25 points)** Consider the closed loop system at left with $G_{comp}(s) = K(s+1)/s$ (PI controller).

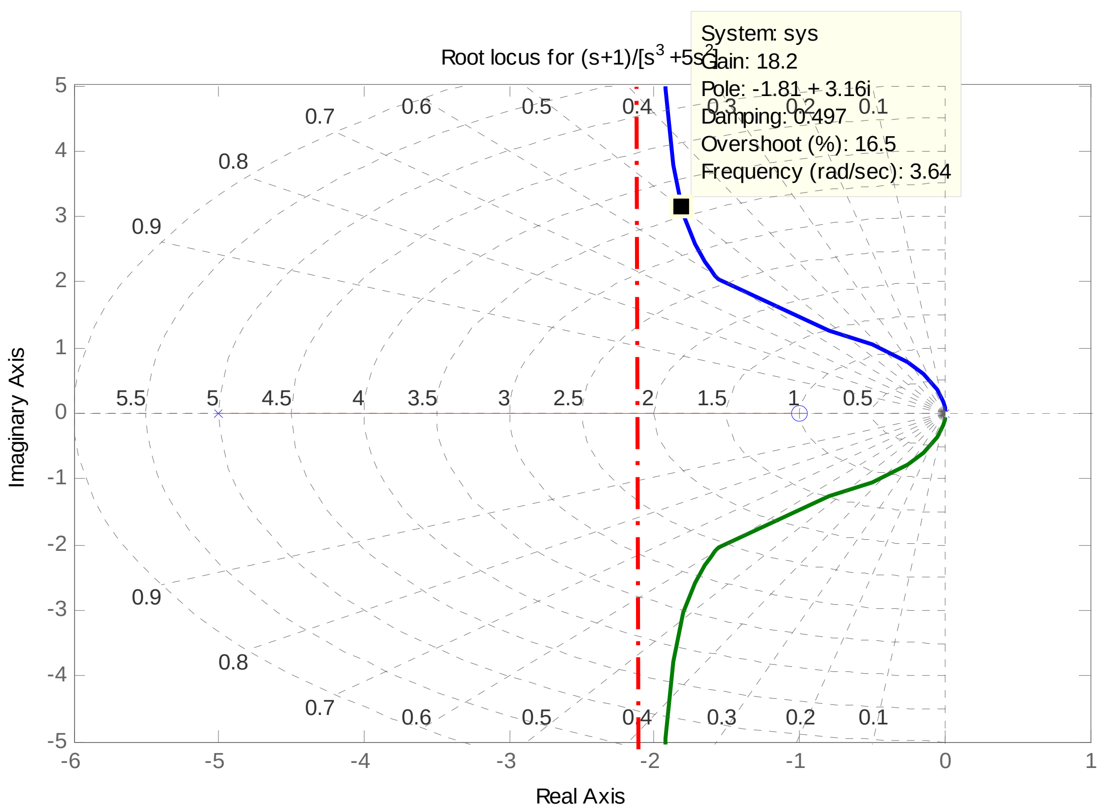
(a) determine the closed loop transfer function Y/R (5)

(b) Draw the Root locus plot. Draw all branches and asymptotes of the root locus, and indicate in each branch the directions of the R.L for increasing K. Determine all imaginary axis crossings if they exist. Determine the range of K, if any, for which the closed loop system is asymptotically stable. (10) **Label and scale all axes.** (5 )

(c) In the Root Locus graph, select a pair of closed loop poles such that the closed loop damping $\zeta = 0.5$. Determine the approximate gain K associated with $\zeta = 0.5$. (5)

Answer (b) As. stable range of K: **0 < K < infinity**

*15pts*




Answers (a) and (c)

| | |
|---|---|
| (a) Y/R = $\dfrac{K(s+1)}{s^2(s+5)+K(s+1)}$ *5pts* | (c) Approximate poles for $\zeta = 0.5$ and associated gain K: $K(\zeta = 0.5) =$ 18<br>Poles at : -1.8 +/- 3j *5pts* |

**4. (10 Points)** The characteristic equation of a closed-loop system with P-controller K was found as: $s^2 + 3.2s + 16K = 0$.

(a) For K= 4, determine the closed loop poles, undamped natural frequency $\omega_n$, and damping coefficient $\zeta$.

(b) Find a gain K at which the closed loop system's damping coefficient $\zeta$ is at least double the damping $\zeta$ found in (a).

$s^2 + 3.2s + 64 = 0$ closed loop poles at -1.6 +/- 7.84j

We have $2\zeta\omega_n = 3.2$ and $\omega_n^2 = 64$ Thus $\omega_n = 8$ and $\zeta = 3.2/16 = 0.2$

For $\zeta = 0.4$, we require $2*0.4*\omega_n = 3.2$ or $\omega_n = 3.2/0.8 = 4$ $16K = \omega_n^2 = 16$ thus K = 1

Answer

| | |
|---|---|
| **(a)** **Closed Loop Poles:** at -1.6 +/- 7.84j $\omega_n =$ 8 $\zeta = 0.2$ | *6pts* |
| **(b)** Gain for Double damping: K = 1 $\zeta =$ 0.4 | *4pts* |