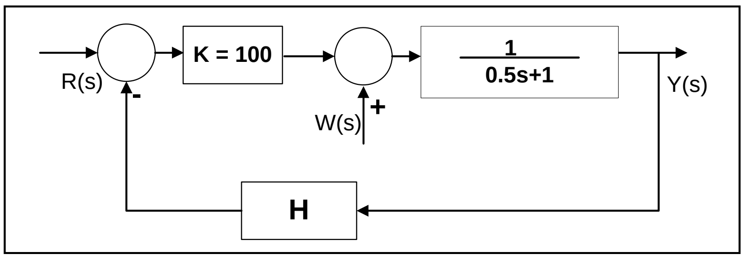
Name:______KEY_________

UNIVERSITY OF NEVADA, LAS VEGAS
DEPARTMENT OF MECHANICAL ENGINEERING

MEG 421 Automatic Control   Fall 2007
Second Test, Closed Book   One page of handwritten notes, Handout Root Locus, and Table of Laplace Transforms allowed.

1. (15 points) A control system with P-controller K =100 is shown below. The reference is a unit step signal.



(a) For H=1 and W=0, determine the closed loop system's steady state error to a unit step input R(s).
(b) For H = 0.1 and W=0, determine the closed loop system's steady state gain Y_{ss}/R_{ss}.
(c) For H= 0.1 and R = 0, determine the closed loop system's steady state error to a unit step disturbance W.

Answer (a) $e_{ss} = \lim(s \to 0)\dfrac{1}{1+GH} = \dfrac{1}{1+100}$

(b) K_{ss} = Y/R = G/(1+GH) = 100/(1+100*0.1) = 100/11

(c) $(Y/W)_{ss}$ = 1/(1+GH) = 1/(1+100*0.1) = $^1/_{11}$

Since R = 0, the error equals Y*H= 0.1*$^1/_{11}$ = 1/110

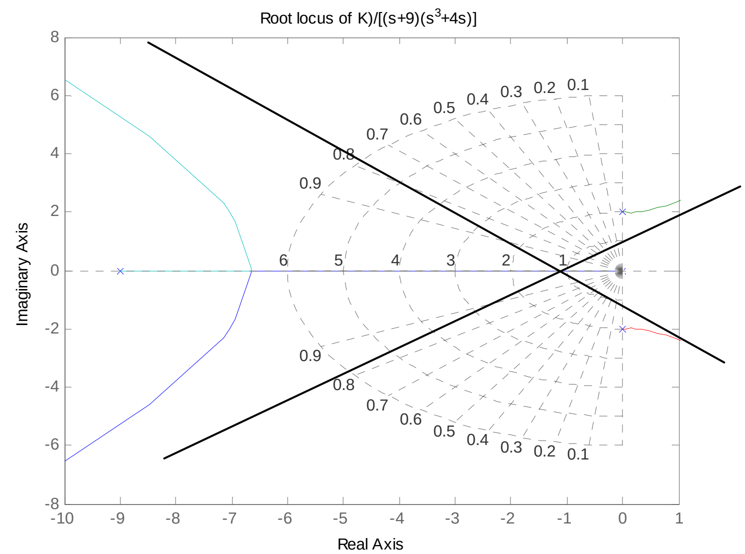
Answers

(a) Ref: $e_{ss,\ step} = {}^1/_{101}$
(b) Steady state Gain (H=0.1) = $^{100}/_{11}$
(c) Disturbance: $e_{ss,\ step}$ (H=0.1, R=0) = $^1/_{110}$

2. (25 points) The open-loop transfer function of a system is given as:

$$G(s) = \frac{K}{s(s^2+4)\,(s+5)}$$

Draw all branches and asymptotes of the root locus, and indicate in each branch the directions of the R.L for increasing K. Determine the location of all imaginary axis crossings if they exist. Determine the range of K, if any, for which the closed loop system is asymptotically stable. **Label and scale all axes.**



Answer # of asympt. = 4 σ_c = - 5/4

Im. axis crossings, if any: **no crossings**

As. stable range of K: 0 < K < 0
Closed loop is unstable for any K > 0

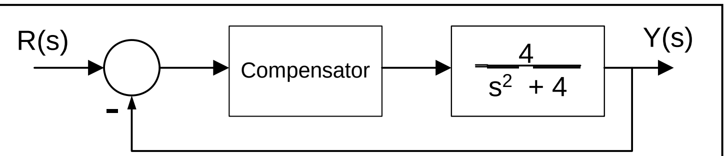
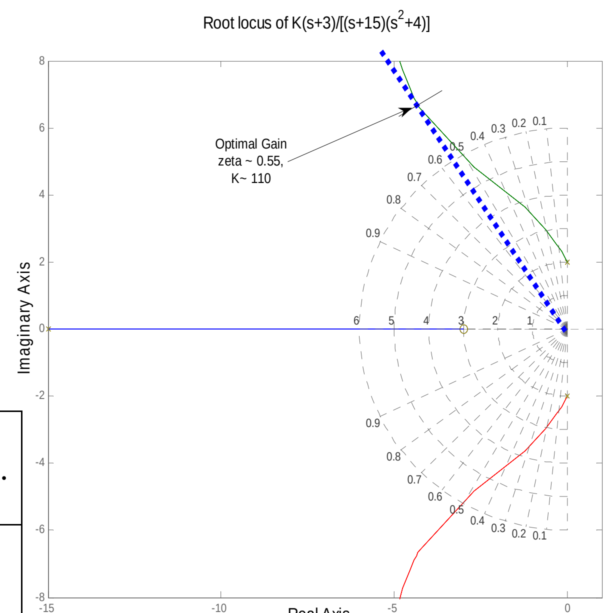


Figure 3 Feedback System

3. (30 points) A feedback system consisting of a plant and a lead compensator is given in Fig. 3.
(a) Define the range 0< K < ∞ for which the closed loop is asymptotically stable when using P-control only.

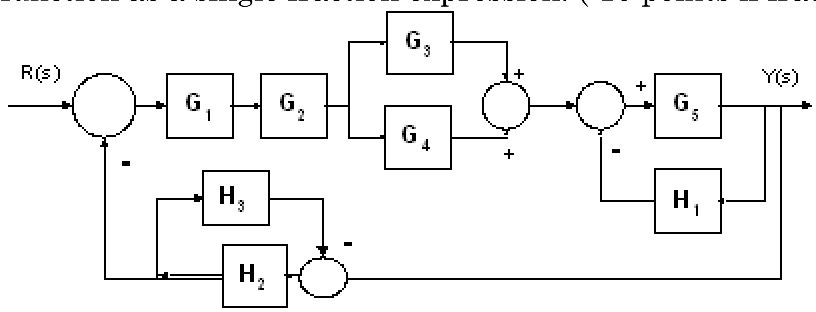
(b) Perform a root locus analysis of the system of Fig. 3 with a lead controller $Comp(s) = \frac{K(s+3)}{s+15}$. Determine all asymptotes and sketch the RL plot. **Label and scale all axes**.

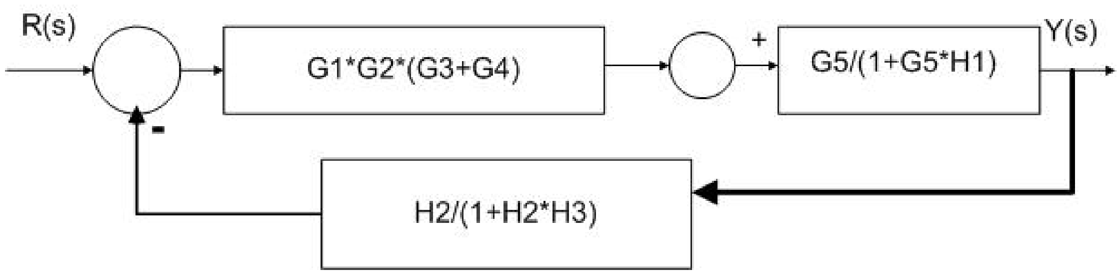
(c) We wish to obtain the largest closed loop damping possible for the system with lead compensator. In the RL plot, sketch the approximate location of the desired closed loop poles and determine the approximate ζ and ω_n of the system.



Answers (3a)
Stable range for P-Control: stable, but NOT as. stable for all K > 0
Answers (3b) # of As. 2 σ_c = -6
(3c) ζ = 0.55 approx. ω_n = 7.8 (approx.)

4. (20 points) For the block diagram shown below, determine the closed loop transfer function as a single fraction expression. (-10 points if fraction is multi-tiered)



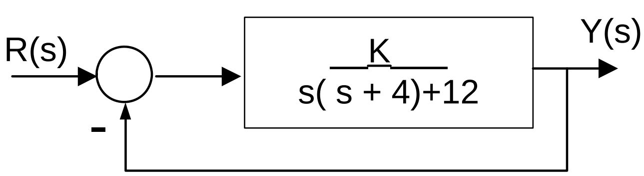


$$\frac{Y}{R}=\frac{G1*G2*G5(G3+G4)}{(1+G5*H1)*(1+\frac{G1*G2*G5(G3+G4)}{1+G5*H1}*\frac{H2}{1+H2*H3})}=\frac{G1*G2*G5(G3+G4)*(1+H2*H3)}{(1+G5*H1)*(1+H2*H3)+G1*G2*G5*H2(G3+G4)}$$

Answer

$$\frac{Y}{R}=\frac{G1*G2*G5(G3+G4)*(1+H2*H3)}{(1+G5*H1)*(1+H2*H3)+G1*G2*G5*H2(G3+G4)}$$

5. (10 Points) For the feedback system below, find K such that the closed loop damping is $\zeta = 0.5$



Closed loop transfer function:

$$\frac{Y}{R}=\frac{K}{s^2+4s+12+K}$$ We choose K such that the characteristic equation Den(s) = 0 matches the standard char. equation $s^2+2\varsigma\omega_n s+\omega_n^2 = s^2+4s+12+K$, thus $12+K=\omega_n^2$, and $2\varsigma\omega_n =$ `2*0.5*sqrt(12+K)=4` thus 12+K = 16, or K = 4

$K(\zeta = 0.5)$ = **4**