Name: **KEY**

Last First

UNIVERSITY OF NEVADA, LAS VEGAS

DEPARTMENT OF MECHANICAL ENGINEERING

ME 421 Automatic Control Fall 2011

First Test, Closed Book, One page of handwritten notes and Table of Laplace Transforms allowed

1. **(20 points)** A feedback system is shown in Fig. 1.
Using block diagram reduction, determine the closed loop transfer function Y(s)/R(s) in terms of the element names in the diagram.. Express the result as a **single fraction** (5 points).

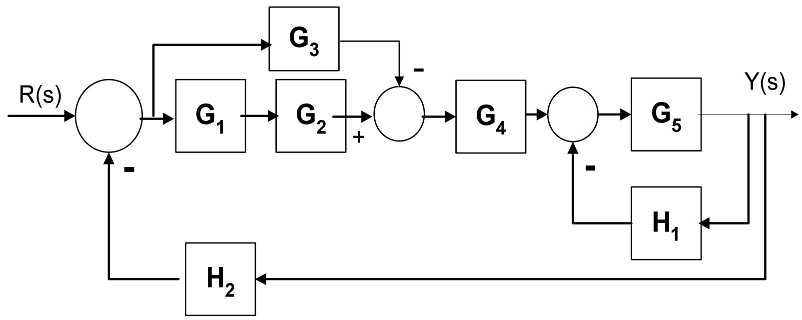


Figure 1 Feedback System

Intermediate Reduction: Forward path: Y = (G1*G2-G3)*G4*G5/(1+G5*H1)*(R-H2*Y)

$$\frac{Y(s)}{R(s)} = \frac{(\text{G1} * \text{G2 - G3}) * \text{G4} * \text{G5}}{(1+H_1G_5)(1+\frac{\text{G1} * \text{G2 - G3}) * \text{G4} * \text{G5} * \text{H2}}{(1+H_1G_5)})}$$

Answer

Answer Transfer Function	$$\frac{Y(s)}{R(s)} = \frac{(\text{G1} * \text{G2 - G3}) * \text{G4} * \text{G5}}{1+H_1G_5+(\text{G1} * \text{G2 - G3}) * \text{G4} * \text{G5} * \text{H2}}$$

2. (15 points) A dynamic system with input r(t) and output y(t) is described by the DE:

$$\dddot{y} + 7\ddot{y} + \dot{y} = 2.5\dot{r} + 7r$$

(a) Find the transfer function. (b) Determine the final value of y when r(t) is a unit step function.

(b) Final value : The system has a pole at s = 0. A step input will result in y → infinite

Answer

$\frac{Y(s)}{R(s)} = \frac{2.5s + 7}{s^3 + 7s^2 + s}$	(b) y(t → ∞) = infinity

3. (25 points) (a) Using the Laplace method, find the solution, y(t) for the system $\ddot{y} + 3\dot{y} - 10y = 2$, $y(0) = \dot{y}(0) = 0$ (15)

(b) Graph the approximate solution y(t). Scale and label all axes. Clearly show the initial value. (10)

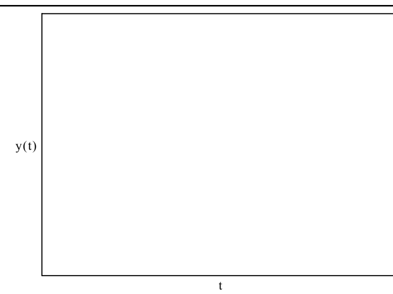
Roots or the char. Equ. are at -5 and +2.

The transform of 2 is 2/s.

$$Y(s) = \frac{2}{\left[s\cdot\left(s^2 + 3\cdot s - 10\right)\right]} = \frac{-1}{5\cdot s} + \frac{2}{35\cdot(s+5)} + \frac{1}{7\cdot(s-2)}$$

Answer (3a)

$$y(t) = \frac{-1}{5} + \frac{2}{35}\cdot e^{(-5)\cdot t} + \frac{1}{7}\cdot e^{2\cdot t}$$



Answer (3b) Plot

4. (10 points) A transfer function is given as Y(s)/R(s) = Num(s)/Den(s) Using the given information,

(a) define the characteristic equation

(b) describe the approach to determine whether the system possesses asymptotic Stability?

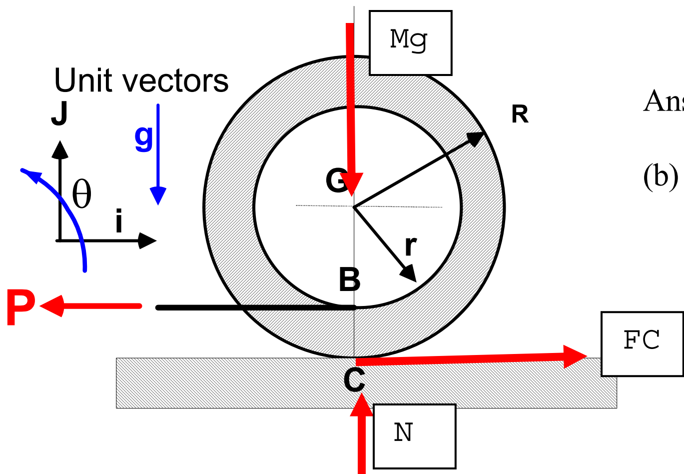
Answer

(a) Char. Equ : Den(s) = 0
(b) Asymptotic Stability: Real parts of all poles = roots of Den(s) < 0

5. (20 points) Force P(t) drives the disk with inertia J_G. The disk rolls without slipping.

(a) Draw a free-body diagram of the system.

(b) Find the time domain differential equation describing the dynamics of the output variable θ(t) as function of the input P(t). Given: Mass M, J_G, R, r.



Answer (a) Free-body diagram at left

(b) Newton's law about point C:

$$\sum Moments(C) = (R-r) * P = I_C * \ddot{\theta} = (J_G + m * R^2) * \ddot{\theta}$$

Answer (b) Differential equation *(Matrix or any other format of your choice, express in either time domain or Laplace domain)*

$$(J_G + m * R^2) * \ddot{\theta} = (R - r) * P$$

6. (10 points) A characteristic is given as: $y = 3*x^{0.5}$, where x = input variable, and y = output variable. We wish to linearize the equation around x = 0.16.

(a) Determine y at the operating point (3)

(b) Find the linearized characteristic about the operating point (7)

$m := \frac{1.5}{x\_op^{0.5}} = 3.75$ $m = 3.75$

Using y = m*x+b:

$b := y\_op - m \cdot x\_op$ $b = 0.6$

Answer

(a) $y\_op := 3 \cdot x\_op^{0.5} = 1.2$

(b) Linearized Characteristic: y_lin = 3.75*x +0.6