ROOT LOCUS CONSTRUCTION RULES

Rule #1: The number of root locus branches is equal to the order of the characteristic equation. Each branch of the root locus begins at an open-loop pole (K = 0) and ends at an open-loop zero or at a zero at infinity ($K \rightarrow \infty$). The root locus is always symmetric with respect to the real axis.

Rule #2 (Real axis RL): For $K > 0$, the root locus lies on a section of the real axis if the number of finite poles and zeros to the right of the section is *odd*.

Rule #3 (Imaginary axis crossings): If branches of the root locus cross the imaginary axis, the locations of the crossings, $j\omega = j\omega_1$, and the values of the gain K at the crossing points can be found by using the Routh array. The value of K at each crossing will be the value that makes an entire row of the Routh array equal to zero. The crossing points $j\omega_1$ will be the roots of the auxiliary equation using that value of K.
An alternate method of finding the values of K and ω_1 is to form the closed-loop characteristic equation **Char-eq(s)** = Den(s) + K*Num(s) = 0. The variable s is replaced by $j\omega$, and the resulting expression is separated into its real and imaginary parts. At the imaginary axis crossing of the closed-loop pole, the real and imaginary parts of $\Delta_{CL}(j\omega)$ must **each** be zero:
Re(Char_eq($j\omega$)) = 0 **and** Im(Char_eq($j\omega$)) = 0 . The two equations can be solved for K and ω_1.

Rule #4 (Asymptotes): There will be n - m branches of the Root Locus as $K \rightarrow \infty$. For large K, they the root locus branches going to infinity will follow asymptotes that meet at a common point on the real axis, and form specified angles with respect to the positive real axis. The angles of asymptotes, φ_A, and the center of asymptotes, σ_A, are given by

$$\varphi_A = \frac{(2r+1)^* \pi}{(\# p_i) - (\# z_i)} \qquad \boldsymbol{\sigma_A} = \frac{\Sigma (p_i) - \Sigma (z_i)}{(\# p_i) - (\# z_i)}$$

where p_i and z_i are the open-loop pole and zero locations, respectively. Complex poles and zeros are included in the calculation of σ_A.

Rule #5 (Breakaway Points): For $K > 0$, the root locus breaks away from the real axis at points of relative maximum K and re-enters the real axis at points of relative minimum K. i.e., breakaway and re-entry occur at points s_B where

$$\left.\frac{dK}{ds}\right|_{s=s_B} = 0$$

See your textbook book for additional rules.