ME 421L
Lab 11: Single Inverted Pendulum (SIP)

Pre-Lab:

- The design performance requirements are listed in p. 4 in the SIP handout.

1- Linearize the Equations of Motions (EOM) of the system. (Eqns: B-17 & B-18 p. 34)
Use the hint in page 6 of your handout.

- The linearized EOM of the SIP system is used to represent the SIP in the state space form, i.e.

$$\frac{\partial}{\partial t}X = A\,X + B\,U$$ ….. Input equation

$$Y = C\,X + \mathrm{D}\,U$$ ……. Output equation

Where

$$X^T = \left[x_c(t),\ \alpha(t),\ \frac{d}{dt}x_c(t),\ \frac{d}{dt}\alpha(t) \right]$$

The output in this case is the cart horizontal position, $\mathbf{x_c}$, and the pendulum angular position (angle), $\boldsymbol{\alpha}$. ➔ Matrix C

MATLAB and SIMULINK:

2- Open up a new MATLAB script file and save it as *LastName1_LastNam2_SIP.m* in your section's folder.

3-In the MATLAB file, define the following matrices for the SIP open loop State Space representation:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 2.2643 & -15.8866 & -0.0073 \\ 0 & 27.8203 & -36.6044 & -0.0896 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ 0 \\ 2.2772 \\ 5.2470 \end{bmatrix}$$

C= [‡ ‡ ‡ ‡ ‡] and D = [0]

4- Find the matrix C?

5- Find the poles of the open loop system using the MATLAB command eig:

eig(A) or ***poles_OL=eig(A)***

6- From the open loop poles locations, what can you infer about the open loop system stability? (Question 4, page 8, in the SIP handout)

7- Go through *section 7.2, pp. 10-14* in SIP handout to use SIMULINK to assess the state space model operating range.

- Simulation and Design of a Linear Quadratic Regulator (LQR):

1- Open the MATLAB script called *setup_lab_ip01_2_sip.m* located in *your desktop\upright pendulum\ linear experiments\Exp05 - SIP - LQR\Lab Design Files* and run it to initialize the SIP parameters into the MATLAB workspace.

2- Make sure that the CONTROLLER_TYPE flag in the script file is set to 'MANUAL' and the variable IC_ALPHA0 is set back to 0.

3- Run the following MATLAB command to define the weighting matrices Q and R, find the full state feedback gain vector, and locate the closed loop poles.

- Q=diag([q_{11}, q_{22}, 0, 0]) R(1,1)= r , q_{11}, q_{22}, r> 0

Try the following values for q_{11}, q_{22}, r and find the optimal full-state feedback gain vector and then the closed loop matrix A_{cl} and check your closed loop system poles locations each time.

q_{11}=1, q_{22}=1, r=1
q_{11}=0.6, q_{22}=10, r=0.0001
q_{11}=0.5, q_{22}=5, r=0.0001
q_{11}=0.4, q_{22}=4.5, r=0.0002

- The optimal full state feedback gain can be calculated using the following MATLAB command**:**

K=lqr(A,B,Q,R)

Define the closed loop matrix A as:

A_{CL}= A-B*K

Find the poles of the closed loop system using the MATLAB command eig:

eig(A_{CL}) or ***poles_CL=eig(A_{CL})***

4- Open the SIMULINK model *s_sip_lqr.mdl* located in *your desktop\upright pendulum\ linear experiments\ Exp05 - SIP - LQR \Lab Design Files* and open the scopes of control effort, x_c, and α.

5- Run the SIMULINK model and notice the three scopes.

6- Repeat the steps 3-5 for each group of the weighting elements given.

7- Compare the responses in each case. Remember, our goal is to keep the pendulum upright with a minimum error, to let the cart follow the step command, and to minimize the control effort.

<u>Real Time Implementation (In Group)</u>

Follow the SIP handout, *pp. 20-26* and answer the questions in the end.

- Take screenshots for your work.
- Show your MATLAB code and its outputs.