

ME 421L

Lab 10: Single Pendulum Gantry (SPG)

Pre-Lab:

- 1- How many actuators and sensors in the Single Pendulum Gantry (SPG)?
- 2- Linearize the Equations of Motions (EOM) of the system. (Eqns: B-17 & B-18 p. 33)
Use the hint in page 6 of your handout.
- 3- The SPG system needs to be represented in the state space form, i.e.

$$\frac{\partial}{\partial t} X = A X + B U \quad \text{..... Input equation}$$

$$Y = C X + D U \quad \text{..... Output equation}$$

$$X^T = \left[x_c(t), \alpha(t), \frac{d}{dt} x_c(t), \frac{d}{dt} \alpha(t) \right]$$

Where

The objective is to control the displacement of the pendulum tip along the x-axis, x_t .

$$x_t = x_c + L_p \sin(\alpha) \hat{e} \text{ vector } C$$

4- Explain why it is a SISO?

MATLAB:

- 5- start a MATLAB script file and save it as LastName1_LastNam2_SPG.m in your section folder
- 6-In the MATLAB file, define the following matrices for the SPG open loop State Space representation:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1.5216 & -11.6513 & 0.0049 \\ 0 & -26.1093 & 26.8458 & -0.0841 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1.5304 \\ -3.5261 \end{bmatrix}$$

$$C = [1, L_p, 0, 0] \quad \text{and} \quad D = [0]$$

Where L_p is 25.25 inches * **25.4/1000** = meter

- 7- Define your Open Loop System from the provided State Space matrices using the MATLAB command `ss`:

$$\text{sys_OL} = \text{ss}(A, B, C, D);$$

- 8- Find the poles of the open loop system using the MATLAB command `eig`:

$$\text{eig}(A) \quad \text{or} \quad \text{poles_OL} = \text{eig}(A)$$

- 9- Find the pole-zero locations of the SISO system using the MATLAB command:

$$[z, p, k] = \text{ss2zp}(A, B, C, D) \quad \text{OR} \quad [z, p, k] = \text{ss2zp}(\text{sys_OL})$$

- 10- Plot the pole-zero locations of the SISO system in the s-plane using the MATLAB command:

$$\text{pzmap}(\text{sys_OL})$$

11- From the open loop poles locations, what can you infer about the open loop system stability?

- Pole Placement Design:

12- Go through pp. 9-10 of the SPG handout and use the design performance requirements in p. 4, to find the dominating pair of complex conjugate poles, p_1 and p_2 .

For controllability check, you can use the MATLAB command $Co=ctrb(A,B)$ and then $rank(Co)$.

13- Go through steps 3 – 8, pp. 12-13 in the SPG handout

The place command in step 4 can be used as follow:

a- First put your calculated poles in a vector in the form:

$$p=[p_1, p_2, p_3, p_4]$$

b- Use $K=place(A,B,p)$ to find the state-feedback gain vector.

c- Define the closed loop matrices as:

a. $A_{CL}=A-B*K$

b. $B_{CL}=B*K(1)$

c. $C_{CL}=C$

d. $D_{CL}=D$

d- Repeat steps from 7-10 in this handout for the closed loop system.

e- The step command can be used as follow:

$$\text{step}(\text{sys_OL}) \quad \& \quad \text{step}(\text{sys_CL})$$

f- In the resulting plots, right click to show the system characteristics (overshoot and settling time). Point at the resulting locations to get the values.

g- Re-adjust the values of p_3 , and p_4 until the requirements are satisfied.

SIMULINK

Follow the SPG handout, pp. 13-18.

Real Time Implementation

Follow the SPG handout, pp. 19-26.

- Take screenshots for your work.
- Show your MATLAB code and its outputs.