Name: ________________

UNIVERSITY OF NEVADA, LAS VEGAS
DEPARTMENT OF MECHANICAL ENGINEERING

MEG 421 Automatic Control   Fall 2004

Final examination, Closed Book,
Three pages of handwritten notes allowed

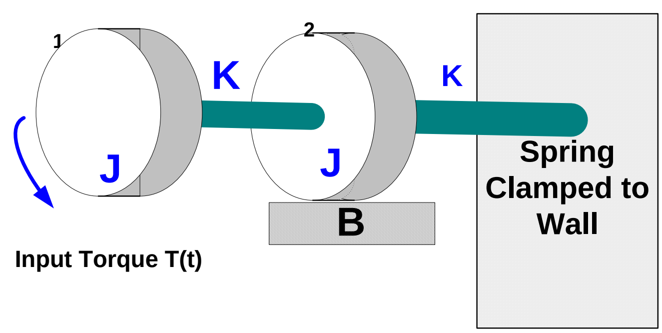


Figure 1 Coupled Elastic System

1. (20 points) Figure 1 shows schematically two coupled rotating masses each having inertia J, two springs K and viscous damping B. A torque T(t) drives inertia J at left.

(a) Sketch the free-body diagrams for each inertial mass. Using Laplace notation, find the set of differential equations in terms of the displacement $\theta_1(s)$, $\theta_2(s)$ and torque T(s) in matrix form.

(b) Find the plant's transfer function when the input is the torque T(s), and the output variable is the angle $\theta_1(s)$.

Answer

(a) Differential Equation (Matrix form, Use Laplace notation)
(b) Transfer Function = --

2. (10 points) Find the Laplace transform of the following time functions:

(a) $f(t) = t + e^{-2t} \sin 3t$

(b) $f(t) = f(t) = 3+7t + t^2 + \delta(t)$

Answer

(a)
(b)

3. (10 points) For the second-order closed-loop system with transfer function

$$\frac{Y(s)}{R(s)} = \frac{3}{s^2 + 2s - 3}$$

determine the following:

(a) the roots of the characteristic equation;

(b) DC gain;

(c) the final value to a step input (*R(s) = 1/s*).

Answer

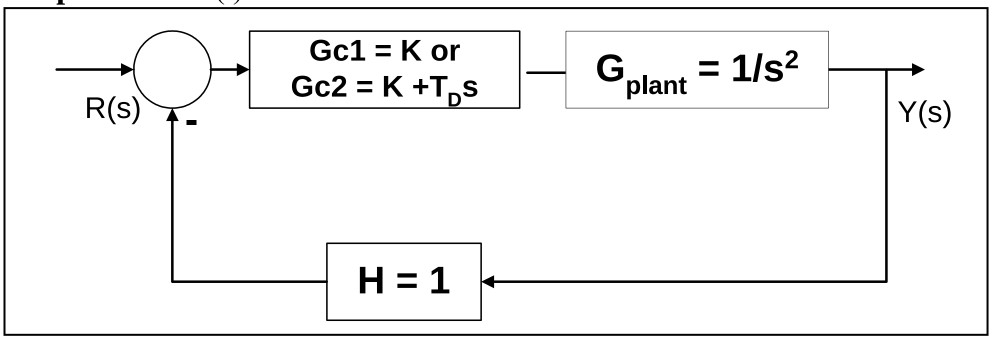
(a) $s_1 =$ $s_2 =$	(b) DC Gain =
(c) Final value =	

4. **(15 points)** The feedback system below is controlled by either a P- or a PD controller, $u(t) = K * e(t) + T_D \dot{e}$.

(a) By evaluating the control system's characteristic equation, determine the closed-loop asymptotic stability with P-control ($T_D = 0$)

(b) Using a PD-controller, compute the values of K and T_D such that the closed loop poles are located at –2 +/- 2j. Hint: compare to standard form for 2nd order underdamped systems: $s^2 + 2\zeta\omega_n s + \omega^2_n = 0$

(c) For K = 2, T_D = 1 s, use the final value theorem to determine the steady-state error for a **unit-ramp** reference r(t).



Answer

(a) Char. Equ. and asymptotic stability, P-control:
(b) K = $\qquad\qquad$ T_D =
(c) $e_{ss,ramp}$ =

5. (20 points) The open-loop transfer function of a system is given as:

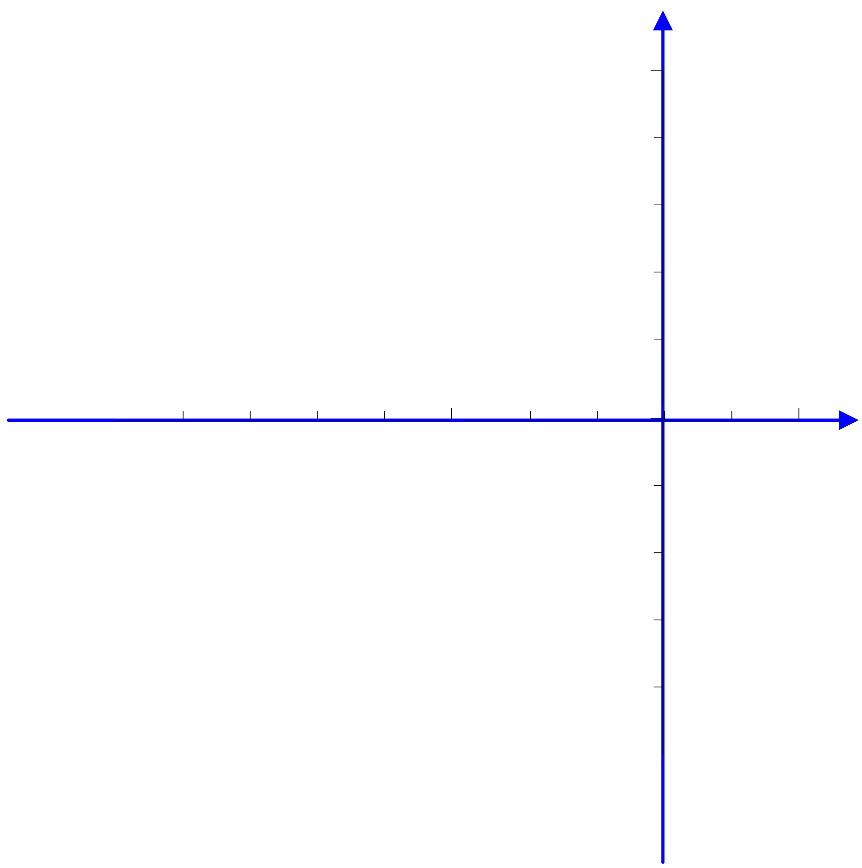
$$G(s) = \frac{K(s+1)}{s^2(s^2+5)}$$

Draw all branches and asymptotes of the root locus, and indicate in each branch the directions of the R.L for increasing K. Determine all imaginary axis crossings if they exist. Determine the range of K, if any, for which the closed loop system is asymptotically stable. Label and scale all axes.

Answer # of asympt. =

Im. axis crossings, if any:

As. stable range of K: < K <



6. (15 points) An open-loop system and a **lag compensator** are given as

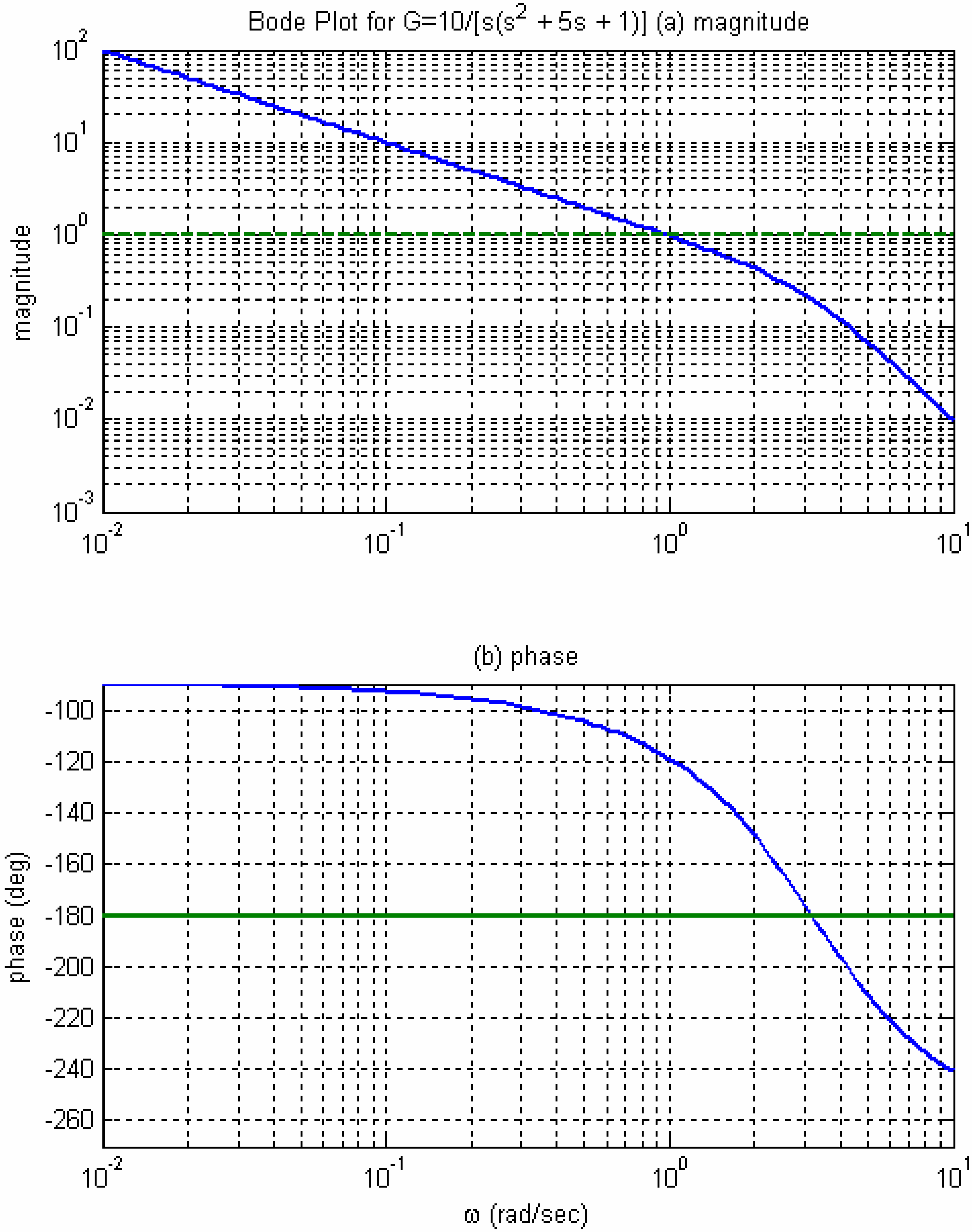
$$G_{plant}(s) = \frac{10K}{s(s^2 + 5s + 10)} \qquad \text{and} \qquad G(s) = \frac{\tau s + 1}{0.1 * \tau s + 1}$$

In the Bode plot on the following page (drawn for K = 1), construct the **lag** compensator and the complete open-loop Bode plot (Plant * Compensator). Select the compensator time constant τ such that an improved closed loop performance results. Adjust the value of gain K such that the resulting system has a phase margin of approximately 40 degrees. Use the page provided in the back for the compensator design. **Your plot must show:** -Compensator pole and zero
-Approximate Compensator Magnitude and phase plots
-Magnitude and phase plots of the compensated system
-Your determination of the controller gain K for the prescribed phase margin of 40^0.

Answers
(a) Lag Compensator transfer function = ------------------------

(b) Controller gain K (dB) for 40^o phase margin =

Problem 6: Bode Plot of Plant G (s)

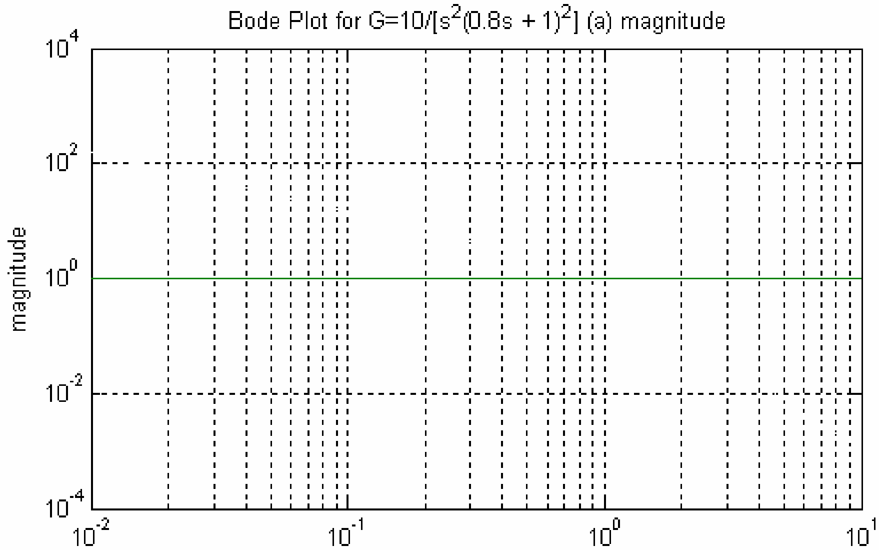


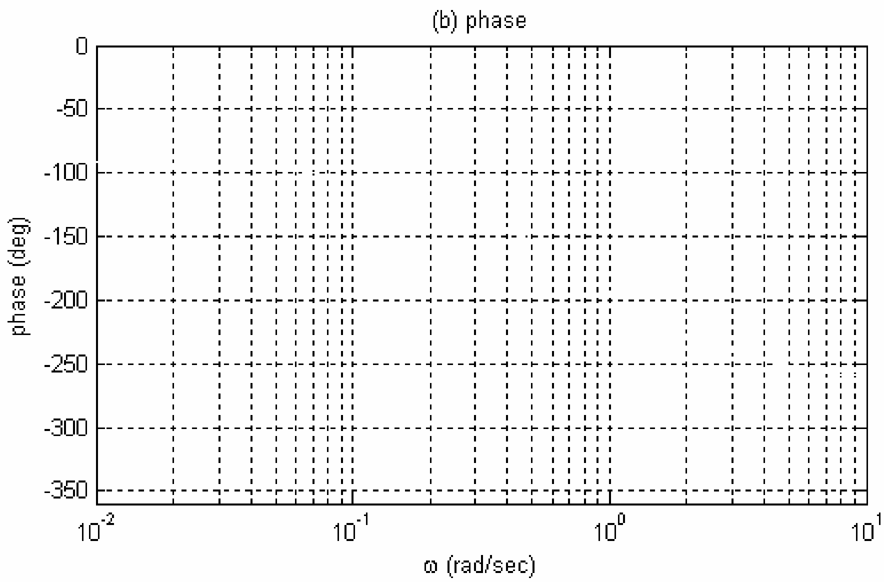
7. (10 points) Apply the Nyquist criterion to the open-loop system G(s) and determine whether the respective closed loop system is asymptotically stable.

(a) Construct the system's Bode plot below.

(d) Determine system's the gain and phase margins, and determine the closed loop stability based on your analysis of the gain and phase margins.

$$G(s) = \frac{10}{s^2(0.8s+1)^2}$$





Answers

Gain Margin =	Phase Margin =
Closed Loop Stability:	