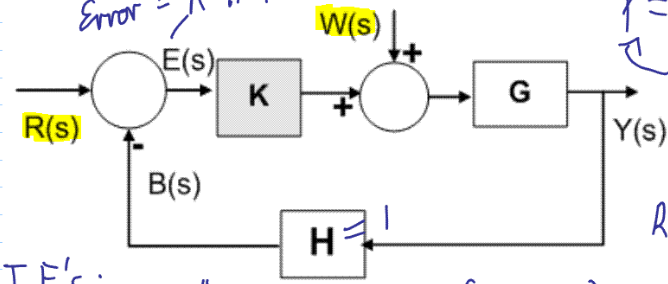
Chapter 4 Feedback

Note Title 10/3/2011

Error = R − H·Y



H = 1

$$Y = E\cdot K\cdot G + W\cdot G \quad (1)$$

$$E = R - Y \quad (2)$$

$$Y = R - E$$

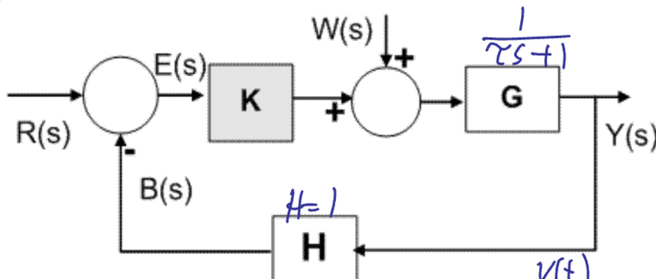
$$R - E = E\cdot K\cdot G + W\cdot G$$

$$E(1 + KG) = R - WG$$

Two T.F's:

$$\boxed{\frac{E}{R} = \frac{1}{1+KG}} \qquad \boxed{\frac{E}{W} = \frac{-G}{1+K\cdot G}}$$

Transient response



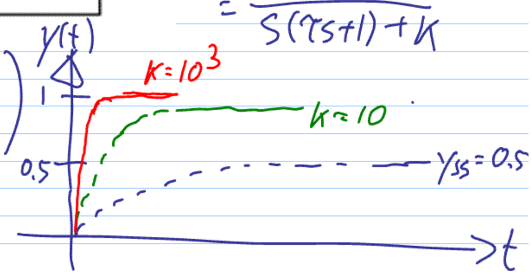
$G = \frac{1}{\tau s + 1}$, $H = 1$

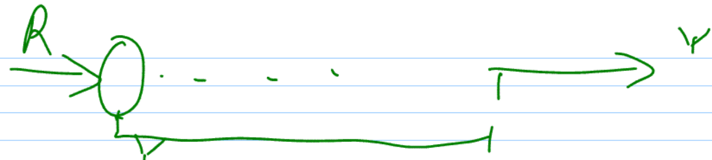
let $R(s) = \frac{1}{s}$

$$E(s) = \frac{1}{s}\,\frac{1}{1 + K\cdot\frac{1}{\tau s+1}} = \frac{\tau s + 1}{s(\tau s + 1) + K}$$

$$y(t) = \frac{K}{K+1}\left(1 - e^{-\frac{K+1}{\tau}t}\right)$$

K	y_{ss}
1	0.5
10	0.9..
10^3	≈1

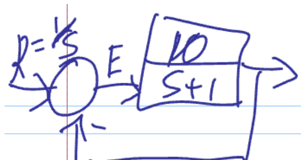




$$\frac{Y}{R} = \frac{Num(s)}{Den(s)} = \frac{Num(s)}{1+K\cdot GH(s)} = 0 \text{ char. eq.}$$

$p_1 \cdots p_n$

Solve $y(t) = \mathcal{L}^{-1}\left(\frac{Num}{Den}\cdot R(s)\right) = \mathcal{L}^{-1}\left\{\frac{A}{s-p_1}+\frac{B}{s-p_2}+\cdots\right\}$



$$G_{closed} = \frac{G}{1+G} = \frac{K}{(s+1)\left(1+\frac{K}{s+1}\right)} = \frac{10}{s+1+10}$$

$$Y_{ss} = \lim_{s\to 0} \cancel{s}\cdot\frac{1}{\cancel{s}}\,\frac{10}{s+11}$$

$$E = R - Y = 1 - \frac{10}{11} = \frac{1}{11}$$

Clicker registrations missing:

Fjare Santos

Lennon

Mun

Poblete

4.1.4 Sensitivity

F = T. F.

α = Parameter

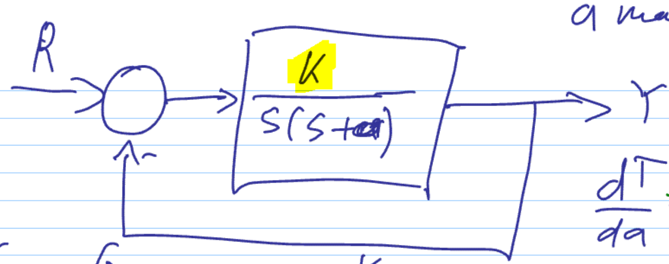
Def.: $S_{\alpha}^{F} = \frac{dF/F}{d\alpha/\alpha} = \frac{\alpha}{F} \cdot \frac{dF}{d\alpha}$

Purpose: Hardware components may change over time

Packing changes Friction coeff.

→ Instability is possible

Ex.:



a may vary

$$\frac{dT}{da} = \frac{0 - Ks}{[s(s+a)+K]^2}$$

$$T(s) = \frac{Y}{R} = \frac{G}{1+G\cdot 1} = \frac{K}{s(s+a)+K}$$

$$S_\alpha^F = \frac{dF/F}{d\alpha/\alpha} = \frac{\alpha}{F}\cdot\frac{dF}{d\alpha}$$

$$S_a^T = \frac{a}{T}\cdot\frac{dT}{da} = \frac{a\cancel{[s(s+a)+K]}}{\cancel{K}}\cdot\frac{-\cancel{K}s}{[\quad]^2}$$

$$\boxed{S_a^T = \frac{-as}{s(s+a)+K}}$$

"Closed loop is directly affected.
use quality components $\quad T \approx -a\cdot s$

Same example: Change of K

$$S_K^T = \frac{K}{T}\cdot\frac{dT}{dK} = \frac{\cancel{K}\cancel{[s(s+a)+K]}}{\cancel{K}}\cdot\frac{[\quad]\cdot 1 - K}{[\quad]^2}$$

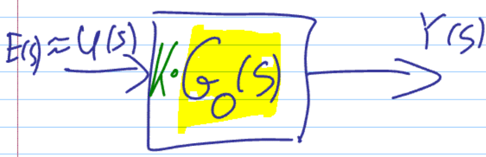
$$S_K^T = \frac{s(s+a)}{s(s+a)+K}$$ – in denom.

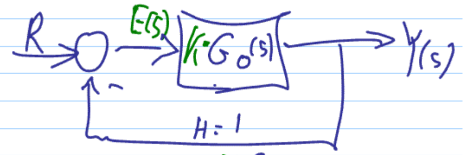
$$\approx \frac{1}{K}$$ lesser effect

4.2 System Type for Tracking

how many integrators in $G_{forward}$?

Def.: open-loop — closed Loop





$$T_{closed} = \frac{K\cdot G_0}{1+K G_0}$$

m = # of Integrators

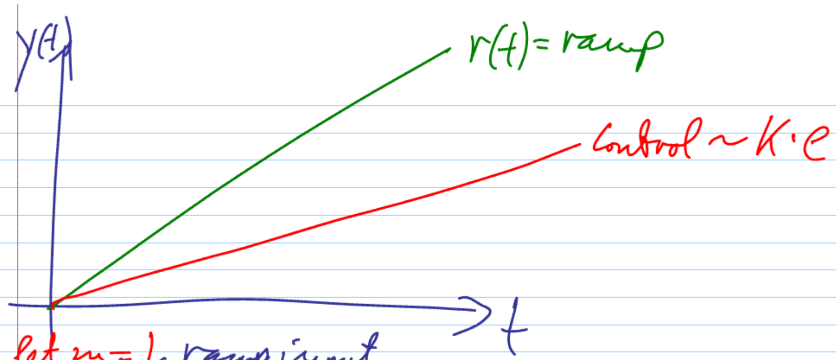
$$E(s) = \frac{1}{1+KG_0(s)}\cdot R(s)$$

Test Functions	$\mathcal{L}(s)$
$\delta(t)$	1
$u_s(t) = \int \delta(t)\,dt$	$1/s$
$u_{ramp}(t) = \int u_s\cdot dt$	$1/s^2$
$u_{accel} = \int u_{ramp}\cdot dt$	$2/s^3$

$$G_0(s) = \frac{K(1+T_a s)(1+T_b\cdot s)\cdots}{s^m(1+T_1 s)(1+T_2 s)\cdots}$$

Cases: $R(s) = \frac{1}{s}$ unit step; $m = 0$

$$e_{ss} = \lim_{s\to 0} s\cdot F(s) = \lim_{s\to 0} \cancel{s}\cdot\frac{1}{\cancel{s}}\,\frac{1}{1+\frac{K(1+T_a s)(1+T_b\cdot s)\cdots}{s^m(1+T_1 s)(1+T_2 s)\cdots}}$$



let m = 1, ramp input

$$e_{ss} = \lim_{s\to 0} s\cdot F(s) = \lim_{s\to 0} \cancel{s}\cdot\frac{1}{\cancel{s^2}}\,\frac{1}{1+\frac{K(1+T_a s)(1+T_b\cdot s)\cdots}{s\,(1+T_1 s)(1+T_2 s)\cdots}}$$

$$e_{ss} = \lim_{s\to 0}\frac{1}{s+s\frac{K(\cdots)\cdots}{s(\;)(\;)}} = \frac{1}{K}$$

Step Input: $$e_{ss} = \lim_{s\to 0}\frac{1}{1+\frac{K(1+T_a s)(1+T_b\cdot s)\cdots}{\cancel{s^m}_{1}(1+T_1 s)(1+T_2 s)\cdots}} = \frac{1}{1+\lim_{s\to 0} G_0} = \frac{1}{1+K}$$

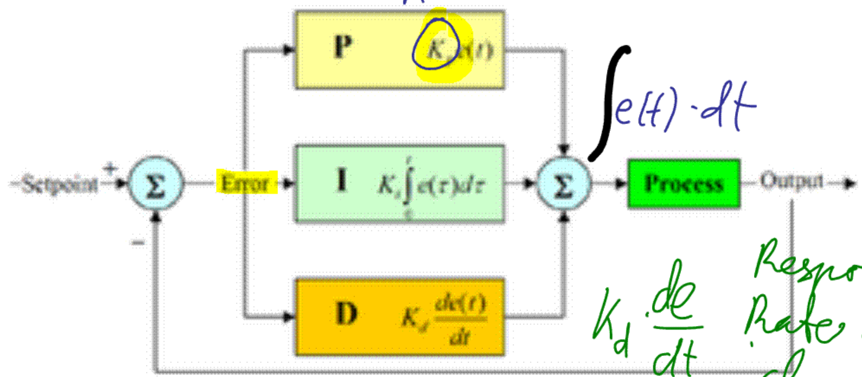
Ramp: $R(s) = \frac{1}{s^2}$

$$e_{ss} = \lim_{s\to 0}\left(\frac{1}{1+\frac{K(1+T_a s)(1+T_b\cdot s)\cdots}{\cancel{s^m}_{1}(1+T_1 s)(1+T_2 s)\cdots}}\right)\cdot\cancel{s}\cdot\frac{1}{s^{\cancel{2}}} = \frac{1}{s+s\frac{K(\cdots)}{(\;)(\;)}}$$

$$\Longrightarrow \infty$$

4.3 3-Term Controller P I D

K



$\int e(t)\cdot dt$

$K_d \cdot \frac{de}{dt}$ Response to Rate of change

P I D

$$u(t) = \text{[redacted]}\ K_p e(t) + K_i \int_0^t e(\tau)\, d\tau + K_d \frac{d}{dt} e(t)$$

Problem 4.11

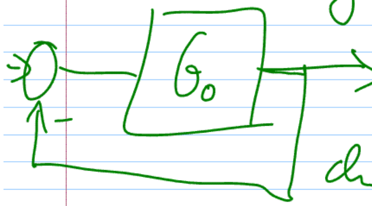
zero

Plant $G_0(s) = \frac{1}{s^2 + 2\zeta s + 1} \cdot \frac{K(s+a)}{s}$ G_{open}

$\boxed{\text{Comp}}$

Type 1 system? $b = 0$

Stability Type 1 $\quad G_o = \frac{K(s+a)}{s(s^2+2\zeta s+1)}$

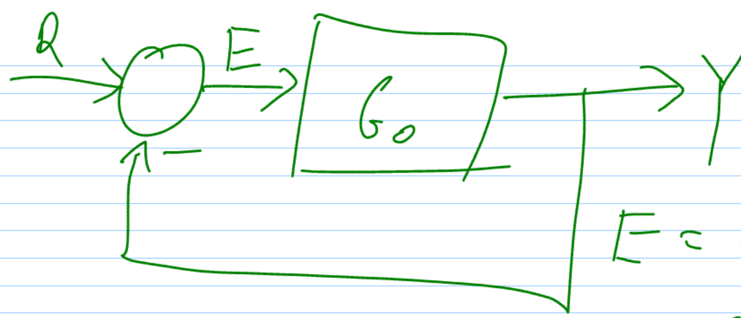


Y Stability is determined by char. eq. of G_{closed}

$$G_o + 1 = 0 = \frac{Num}{Den} + 1 = 0$$

$$Den(s) + Num(s) = 0$$

$$s^3 + 2\zeta s^2 + s(1+K) + Ka = 0$$



$$E = G_o \cdot Y \quad (1)$$

$$E = R - Y \quad (2)$$

$$\frac{Y}{R} = \frac{G_o}{1+G_o}$$

$$Y = \frac{G_o}{1+G_o} R \qquad E = R - \frac{G_o}{1+G_o} R$$