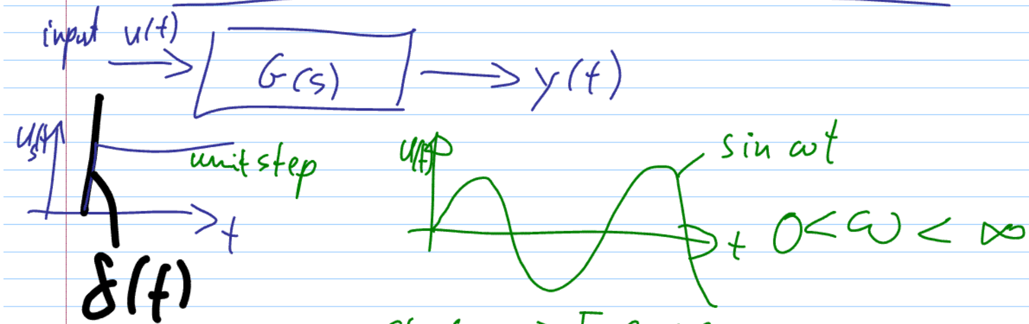
ME 421 chapter 3 Dynamic Response

Note Title 9/13/2011

3.1 Laplace

Def: $F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt.$



Ch.6 = Frequency response

Laplace Method Solve DE's

1. Time domain $\quad \tau \dot{y} + y = K \cdot u(t) \qquad y(0) = 0$ (y: outp., u(t): input)

2. Laplace domain $\quad s = \frac{d}{dt}$

$$\tau\left[sY(s) - y(0)\right] + Y(s) = K \cdot U(s)$$

3. Solve in $\mathcal{L}$-domain

$$Y(s)\left[\tau s + 1\right] - \tau y_0 = K \cdot U(s)$$

$$Y(s) = \frac{K \cdot U(s) - \tau \cdot y_0}{\tau s + 1}$$

Specify $U(s)$

Let $U(s) = \frac{1}{s}$: unit step

$$Y(s) = \frac{K \cdot 0}{s(\tau s+1)} \overset{\text{Partial Fractions}}{=} \frac{A}{s} + \frac{B}{\tau s+1}$$

$$\boxed{\begin{aligned}\tau s+1&=0\\ s&=-\frac{1}{\tau}\end{aligned}}$$

use Coefficients or Residue

$$A = \left.\frac{K \cdot \cancel{s}}{\cancel{s}(\tau s+1)}\right|_{s=0} = K$$

$$B = \left.\frac{K}{s}\right|_{s=-\frac{1}{\tau}} = -K \cdot \tau$$

$$e^{-at} \longrightarrow \frac{1}{s+a}$$

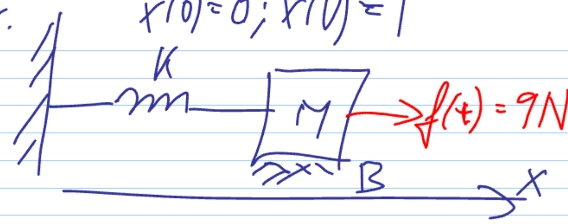
4. Inverse T. F.

Solution $Y(s) = \frac{K}{s} - \frac{K \cdot \tau}{\tau s+1} = \frac{K}{s+\frac{1}{\tau}}$ $\qquad \frac{1}{s+a}$ $\qquad \frac{1}{\tau} = a$

Solution: $\boxed{y(t) = K - K \cdot e^{-\frac{t}{\tau}}}$

$\mathcal{L}(\text{Constant}) = \frac{\text{Const.}}{s}$! $\qquad \mathcal{L}(1) = \frac{1}{s}$

Ex. $\dot{x}(0) = 0;\ x(0) = 1$



$M\ddot{x} + B\dot{x} + Kx = f(t)$

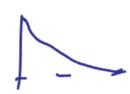
$\ddot{x} + 4\dot{x} + 3x = 9$

$$\left(s^2X(s) - s\,x(0) - \dot{x}(0)\right) + 4\left[s\,X(s) - x(0)\right] + 3X(s) = \frac{9}{s}$$

$$X(s) = \frac{s^2 + 4s + 9/s}{s^2 + 4s + 3} = \frac{s^2 + 4s + 9}{s(s+1)(s+3)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+3}$$

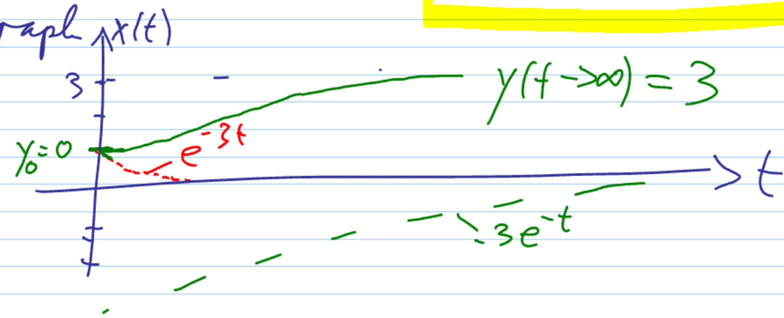
$$A = \left.\frac{s^2 + 4s + 9}{(s+1)(s+3)}\right|_{s=0} = \frac{9}{3} = 3$$

$$B = \left.\frac{s^2 + 4s + 9}{s(s+3)}\right|_{s=-1} = \frac{6}{-2} = -3\ ;\ C = 1$$



Inverse T.F. = solution = $\boxed{x(t) = 3 - 3e^{-1\cdot t} + 1\cdot e^{-3t}}$

Graph



$$\frac{s^2+4s+9}{s(s+1)(s+3)} \cdot s$$

$$x(0) = \lim_{s \to \infty} sX(s)$$

Initial value theorem:

$$x(\infty) = \lim_{s \to 0} sX(s)$$

Final value theorem:

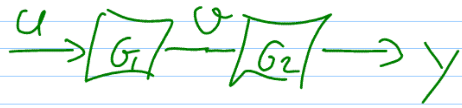
$\lim_{s \to 0} sX(s) = $ final value

Residue method

$$f(x) = \sum_i \left(\frac{a_{i1}}{x - x_i} + \frac{a_{i2}}{(x - x_i)^2} + \cdots + \frac{a_{ik_i}}{(x - x_i)^{k_i}} \right).$$

$$a_{ij} = \frac{1}{(k_i - j)!} \lim_{x \to x_i} \frac{d^{k_i - j}}{dx^{k_i - j}} \left((x - x_i)^{k_i} f(x) \right),$$

Block Diagram Reduction



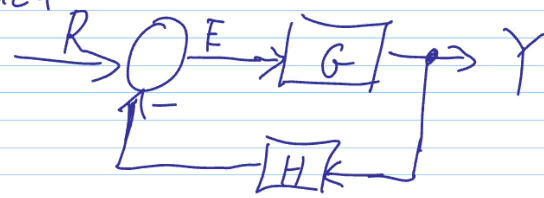
$$G_1 = \frac{v}{u} \qquad G_2 = \frac{y}{v}$$

$v = G_1 \cdot u$

$y = G_2 \cdot v = G_2 \cdot G_1 \cdot u$

G = forward
H = feedback

Feedback



$Y = G \cdot E$ (1)

$E = R - H \cdot Y$ (2) insert (2) into (1)

$Y = G(R - H \cdot Y) \Rightarrow Y(1 + GH) = G \cdot R$

$$\boxed{Y = \frac{G}{1+GH} \cdot R}$$ Memorize!

Chapter Slides and HW will be posted

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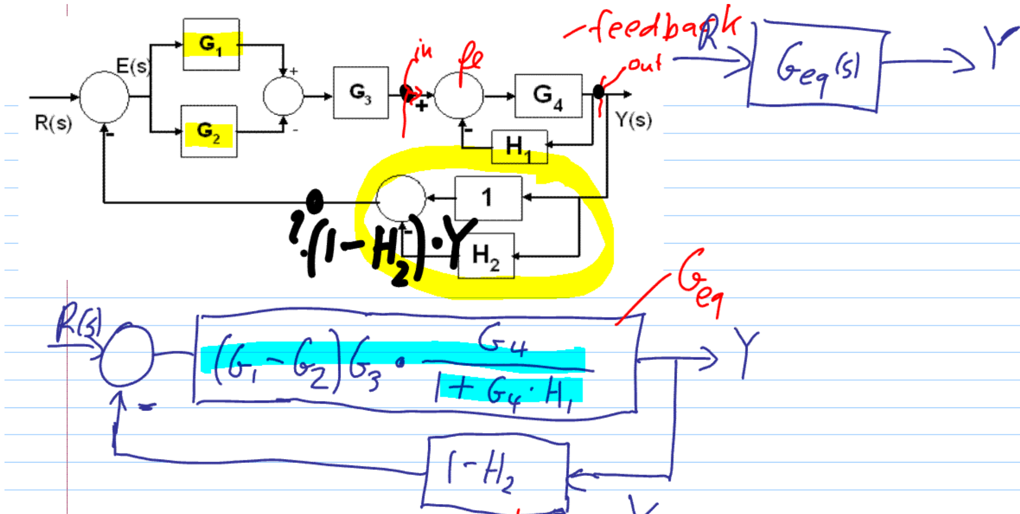
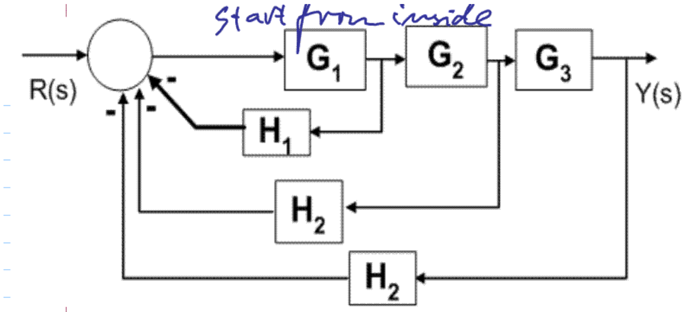


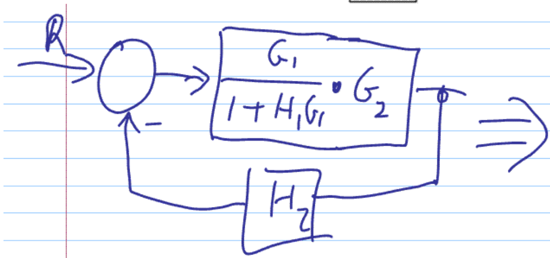
$$\frac{Y}{R}=\frac{G_{eq}}{1+G_{eq}\cdot H_{eq}}$$

$$\frac{Y}{R}=\frac{(G_1-G_2)G_3\cdot\frac{G_4}{1+G_4\cdot H_1}}{1+(1-H_2)\cdot(G_1-G_2)G_3\cdot\frac{G_4}{1+G_4\cdot H_1}}\Bigg|\cdot\frac{1+G_4\cdot H_1}{1+G_4\cdot H_1}$$

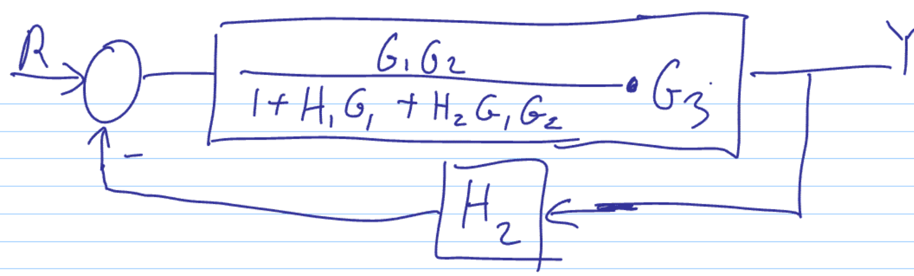
$$\frac{Y}{R}=\frac{(G_1-G_2)\cdot G_3\cdot G_4}{1+G_4\cdot H_1+(1-H_2)(G_1-G_2)\cdot G_3\cdot G_4}\Bigg|\text{clean up}$$

Start from inside





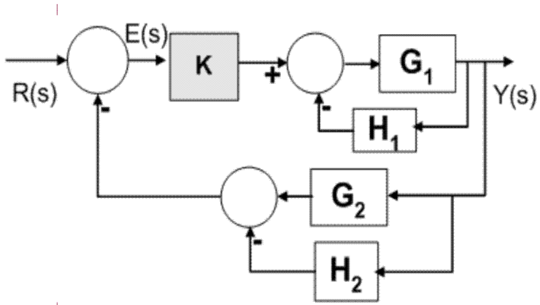
$$\Rightarrow \frac{G_1 G_2}{1+H_1 G_1} \cdot \frac{1}{1+H_2 \cdot \frac{G_1 G_2}{1+H_1 G_1}}$$

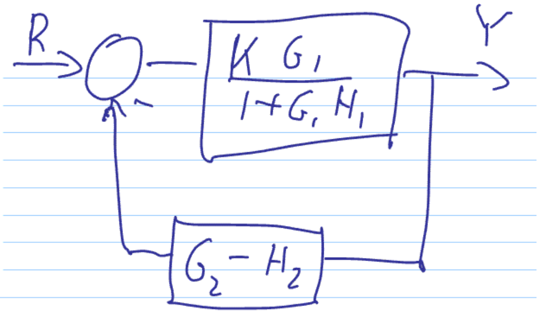


$$\frac{Y}{R} = \frac{G_1 G_2 G_3}{(1+H_1G_1+H_2G_1G_2)\cdot\left(1+ \underline{H_2 \cdot G_1 G_2 G_3}\right)} =$$

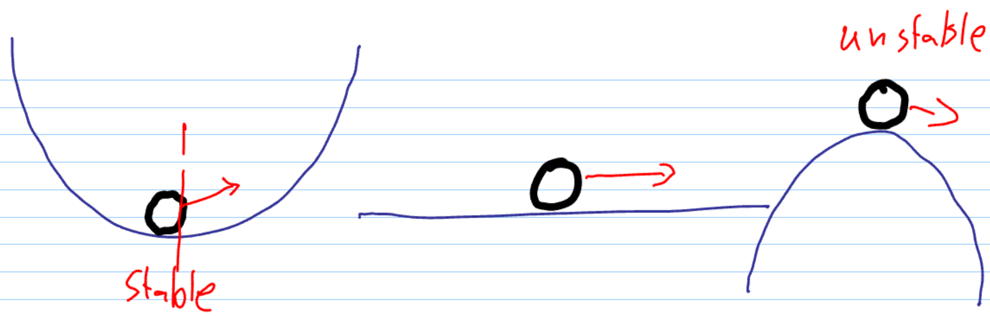
Den 1

$$= \frac{G_1 G_2 G_3}{1+H_1G_1+H_2G_1G_2+H_2G_1G_2G_3}$$





$$\frac{Y}{R} = \frac{K \cdot G_1}{1+G_1 H_1 + (G_2 - H_2) K \cdot G_1}$$



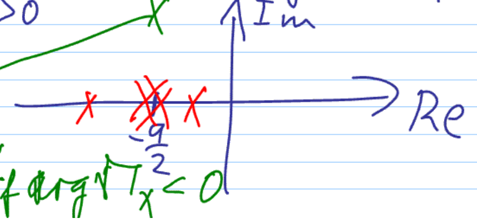
3.3 Response

Second order $\xrightarrow{R(s)} \boxed{\frac{Num(s)}{s^2+as+b}} \longrightarrow Y(s)$

char eq.: $s^2 + as + b = 0$ $\sqrt{\ } = $ Real

Poles = roots of char. eq: $s_{1,2} = -\frac{a}{2} \pm \sqrt{\left(\frac{a}{2}\right)^2 - b}$

let $a, b > 0$



$a^2 \gtrless 4 \cdot b$

$\sqrt{\ }$ is im

2 conj complex poles if arg $\sqrt{\ } < 0$

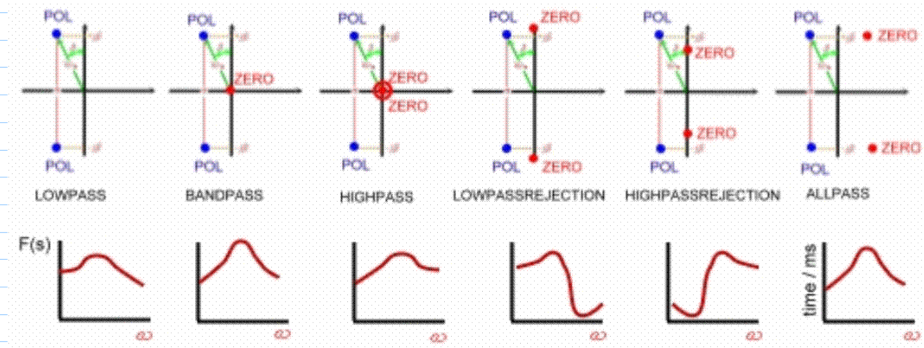
final value theorem $x(\infty) = \lim_{s \to 0} sX(s)$

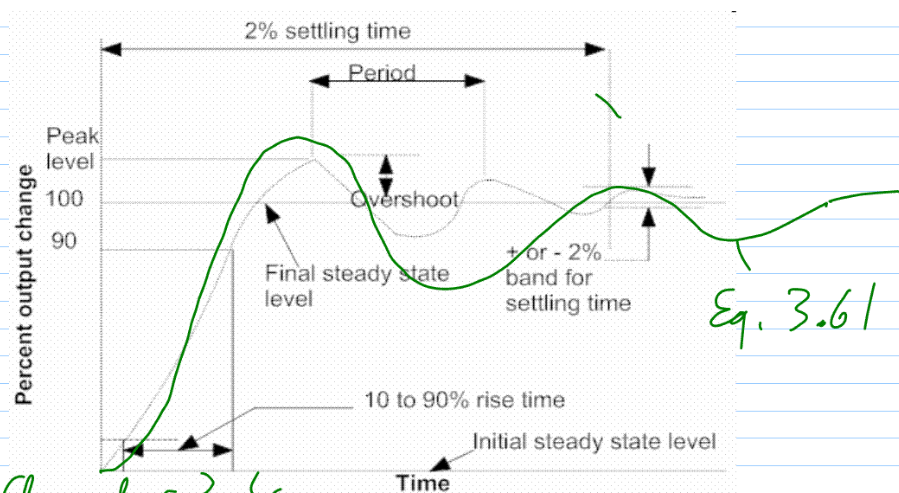
valid only if Re(poles < 0)!

Ex. $G(s) = \frac{3}{s^2 + 3^2}$ $y(t) = \sin \omega t$

apply final value theorem

3.3 Pole Locations





See Chapter 3.4

$K=1$ in

Standard form $\frac{d^2}{dt^2}x(t) + 2\cdot\zeta\cdot\omega_n\cdot\frac{d}{dt}x(t) + \omega_n^{\ 2}\cdot x(t) = K_{dc}\cdot\omega_n^{\ 2}\cdot u(t)$

out

where:

- $x(t)$ = Response of the System,
- $u(t)$ = Input to the System,
- **z** = Damping Ratio,
- w_n=Undamped Natural Frequency,

$$\frac{X(s)}{U(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

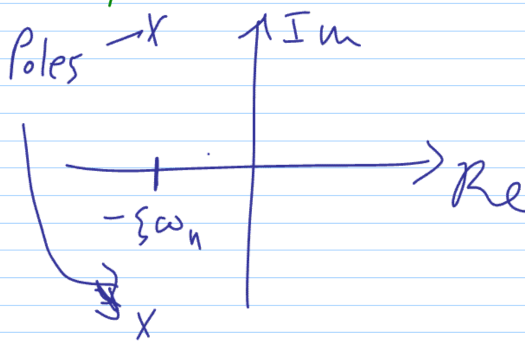
$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$s_{1,2} = -\zeta\omega_n \pm \sqrt{(\zeta\omega_n)^2 - \omega_n^2}$$

$$\boxed{= -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}}$$

$$\frac{X(s)}{U(s)} = \frac{\omega_n}{s^2 + 2\zeta\omega_n s + \omega_n^2} = 0$$

Poles → X



Typo in FE book on Routh

The Routh Criterion : determine # of roots of $P(s)$ in r.h. plane

$s^3 + s^2 + 3s + 10 = 0$

s^3	1	3
s^2	$\boxed{1}$	10
s^1	$b_1 = -\frac{7}{1}$	
s^0	10	

$\Rightarrow$ 2 rhp poles

1. visual
unstable r.h.p. poles exist if terms are missing, or if there are sign changes.

2. coefficients in Routh scheme

$$b_1 = \frac{1\cdot 3 - 1\cdot 10}{1}$$

count # of sign changes in 1st column

$(s + 2)(s^2 - s + 5)$ = unstable

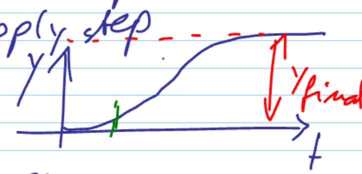
$s^4 + 10s^3 + 35s^2 + 50s + 124$

1	35	124
10	50	
$b_1 = \frac{10\cdot 35 - 50}{10} = 30$	124	
$c_1 = \frac{30\cdot 50 - 1240}{30}$		
$d_1 = 124$		

no sign changes
all $Re(p_i) < 0$

3.7 Model from Experiments

what experiment?
apply step



$$\frac{Y(s)}{U(s)} = \frac{Y_{final}(s)}{U_{step}(s)}$$

K, is variable, and that G(s) is given by:

$$G(s) = 1/[(s + 1)^4]$$

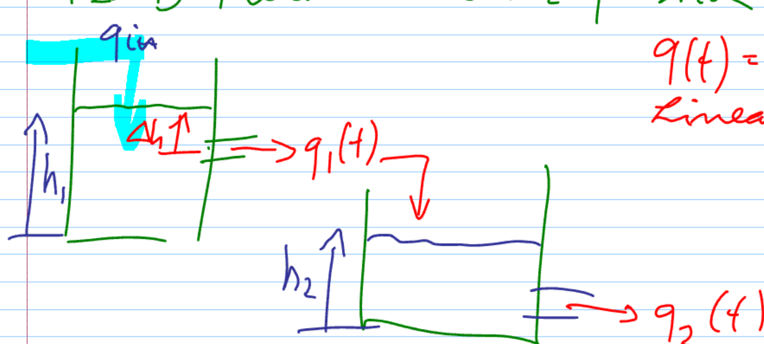
Then, the closed loop transfer function is given by:

$$G_{CL}(s) = K/[(s + 1)^4 + K]$$

The polynomial in the example is the closed loop denominator for K = 29.

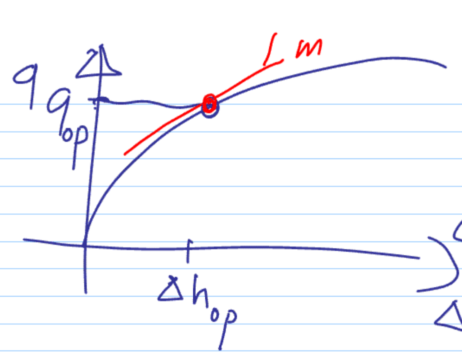
Now, think about what happens when K is variable. Then the closed loop denominator is given by:

FLUID Flow Homework question



$$q(t) = K(\Delta h)^{1/2}$$

Linearize around op. point



for Δh_{op} given
find q_{op} ✓
m_{op} ✓

Lin $\boxed{y - q_{op} = m_{op} \cdot (\Delta h - \Delta h_{op})}$

$C\dot{h} = q_{in} - Lin.out$

$$\begin{pmatrix} \cdot\cdot & \cdot\cdot \\ \cdot\cdot & \cdot\cdot \end{pmatrix} \begin{pmatrix} H_1(s) \\ H_2(s) \end{pmatrix} = \begin{pmatrix} \\ \\ \end{pmatrix}$$