4.4 Solve Prob. 4.3, assuming that the wide-flange beam is bent about the y axis by a couple of moment $M_y$.

4.3 Using an allowable stress of 155 MPa, determine the largest bending moment $M$ that can be applied to the wide flange beam shown. Neglect the effect of fillets.

Moment of inertia about y-axis

$I_1 = \frac{t}{12}(12)(200)^3 = 8 \times 10^6 \text{ mm}^4$

$I_2 = \frac{t}{12}(196)(8)^3 = 8.36 \times 10^6 \text{ mm}^4$

$I_3 = I_1 = 8 \times 10^6 \text{ mm}^4$

$I = I_1 + I_2 + I_3 = 16.00836 \times 10^6 \text{ mm}^4 = 16.00836 \times 10^{-6} \text{ m}^4$

$\sigma = \frac{Mc}{I}$ with $c = \frac{1}{8}(200) = 100 \text{ mm} = 0.100 \text{ m}$

$M_y = \frac{I \sigma}{C}$ with $\sigma = 155 \times 10^6 \text{ Pa}$

$M_y = \frac{(16.00836 \times 10^6)(155 \times 10^6)}{0.100} = 24.8 \times 10^3 \text{ N} \cdot \text{m}$

$M_y = 24.8 \text{ kN} \cdot \text{m}$
Problem 4.8

4.7 through 4.9 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion BC of the beam.

Neutral axis lies 4.778 in above the base.

\[ I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (8)(1)^3 + (8)(3.222)^2 = 59.94 \text{ in}^4 \]

\[ I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12} (4)(1)^3 + (4)(4.778)^2 = 73.54 \text{ in}^4 \]

\[ I = I_1 + I_2 + I_3 = 59.94 + 21.63 + 73.54 = 155.16 \text{ in}^4 \]

\[ y_{top} = 3.222 \text{ in} \quad y_{bot} = -4.778 \text{ in} \]

\[ M = Pa = 0 \]

\[ M = Pa = (25)(20) = 500 \text{ kip-in.} \]

\[ \sigma_{top} = \frac{My_{top}}{I} = \frac{(500)(3.222)}{155.16} = -10.38 \text{ ksi} \quad (\text{compression}) \]

\[ \sigma_{bot} = \frac{My_{bot}}{I} = \frac{(500)(4.778)}{155.16} = 15.40 \text{ ksi} \quad (\text{tension}) \]
Problem 4.13

4.13 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 8 kip \cdot\text{in.}, determine the total force acting on the shaded portion of the beam.

The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

where $y$ is a coordinate with its origin on the neutral axis and $I$ is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element $dA$ the force is

$$dF = \sigma_x dA = -\frac{My}{I} dA$$

The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I} \int \bar{y} dA = -\frac{M}{I} \bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and $A^*$ is its area.

$$\begin{align*}
I &= I_1 - I_2 \\
&= \frac{1}{12} b_1 h_1^3 - \frac{1}{12} b_2 h_2^3 \\
&= \frac{1}{12} (1.8)(2.4)^3 \\
&\quad - \frac{1}{12} (1.2)(1.8)^3 \\
&= 1.4904 \text{ in}^4 \\

\bar{y}^* A^* &= \bar{y}_a A_a + \bar{y}_b A_b \\
&= (1.05)(0.9)(0.3) + (0.45)(0.3)(0.9) = 0.405 \text{ in}^2 \\
F &= \frac{M \bar{y}^* A^*}{I} = \frac{(8)(0.405)}{1.4904} = 2.17 \text{ kips}
\end{align*}$$
Problem 4.21

4.21 Knowing that \( \sigma_e = 24 \text{ ksi} \) for the steel strip \( AB \), determine (a) the largest couple \( M \) that can be applied, (b) the corresponding radius of curvature. Use \( E = 29 \times 10^6 \text{ psi} \).

\[
I = \frac{1}{12} b h^3 = \left(\frac{1}{12}\right)(1)(\frac{1}{4})^3 = 1.30208 \times 10^{-6} \text{ in}^4
\]

\[
\sigma = \frac{Mc}{I} = \frac{1}{2}\left(\frac{1}{4}\right) = 0.125 \text{ in}.
\]

(a) \( M = \frac{\sigma I}{c} = \frac{(24\times10^3)(1.30208\times10^{-5})}{0.125} \)

\[
M = 250 \text{ lb-in}
\]

(b) \( \frac{c}{\rho} = \frac{5_{max}}{E} \)

\[
\rho = \frac{E c}{5_{max}} = \frac{29\times10^6}{24\times10^3}(0.125)
\]

\[
\rho = 151.0 \text{ in}
\]

Problem 4.26

4.26 A 24 kN·m couple is applied to the W200 x 46.1 rolled-steel beam shown. (a) Assuming that the couple is applied about the \( z \) axis as shown, determine the maximum stress and the radius of curvature of the beam. (b) Solve part (a), assuming that the couple is applied about the \( y \) axis. Use \( E = 200 \text{ GPa} \).

For W200 x 46.1 rolled steel section:

\[
I_x = 45.5 \times 10^6 \text{ mm}^4 = 45.5 \times 10^{-6} \text{ m}^4
\]

\[
S_x = 448 \times 10^3 \text{ mm}^3 = 448 \times 10^{-6} \text{ m}^3
\]

\[
I_y = 15.3 \times 10^4 \text{ mm}^4 = 15.3 \times 10^{-6} \text{ m}^4
\]

\[
S_y = 151 \times 10^3 \text{ mm}^3 = 151 \times 10^{-6} \text{ m}^3
\]

(a) \( M_z = 24 \text{ kn·m} = 24 \times 10^3 \text{ N·m} \)

\[
\sigma = \frac{M}{S} = \frac{24\times10^3}{448\times10^{-2}} = 53.6 \times 10^6 \text{ Pa} = 53.6 \text{ MPa}
\]

\[
\frac{1}{\rho} = \frac{M}{EI} = \frac{24\times10^3}{(200\times10^9)(45.5\times10^{-6})} = 2.637 \times 10^{-3} \text{ m}^{-1}
\]

\[
\rho = 379 \text{ m}
\]

(b) \( M_y = 24 \text{ kn·m} = 24 \times 10^3 \text{ N·m} \)

\[
\sigma = \frac{M}{S} = \frac{24\times10^3}{151\times10^{-2}} = 158.9 \times 10^6 \text{ Pa} = 158.9 \text{ MPa}
\]

\[
\frac{1}{\rho} = \frac{M}{EI} = \frac{24\times10^3}{(200\times10^9)(15.3\times10^{-6})} = 7.84 \times 10^{-3} \text{ m}^{-1}
\]

\[
\rho = 127.5 \text{ m}
\]
Problem 4.31

A W200 × 31.3 rolled-steel beam is subjected to a couple \( M \) of moment 45 kN⋅m. Knowing that \( E = 200 \) GPa and \( v = 0.29 \), determine (a) the radius of curvature \( \rho \), (b) the radius of curvature \( \rho' \) of a transverse cross section.

For \( W \) 200 × 31.3 rolled steel section

\[
I = 31.4 \times 10^6 \text{ mm}^4 = 31.4 \times 10^{-8} \text{ m}^4
\]

(a) \[
\frac{1}{\rho} = \frac{M}{EI} = \frac{45 \times 10^3}{(200 \times 10^9)(31.4 \times 10^{-8})} = 7.17 \times 10^{-5} \text{ m}^{-1}
\]

\[\rho = \frac{1}{7.17 \times 10^{-5}} = 139.6 \text{ m}\]

(b) \[
\frac{1}{\rho'} = 2 \frac{1}{\rho} = 2(0.29)(7.17 \times 10^{-5}) = 2.07 \times 10^{-5} \text{ m}^{-1}
\]

\[\rho' = 481 \text{ m}\]

Problem 4.37

Three wooden beams and two steel plates are securely bolted together to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.

<table>
<thead>
<tr>
<th>Modulus of elasticity</th>
<th>Wood</th>
<th>Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 × 10^6 psi</td>
<td>30 × 10^6 psi</td>
</tr>
<tr>
<td>Allowable stress</td>
<td>2000 psi</td>
<td>22,000 psi</td>
</tr>
</tbody>
</table>

Use wood as the reference material

For wood \( n_w = 1 \)

For steel \( n = \frac{E_s}{E_w} = \frac{30}{2} = 15 \)

Properties of the geometric section.

Steel: \( I_s = \frac{1}{12}(\frac{1}{4} + \frac{1}{4})(10)^3 = 41.6667 \text{ in}^4 \)

Wood: \( I_w = \frac{1}{12}(2 + 2 + 2)(10^2) = 500 \text{ in}^4 \)

Transformed section

\[
I_{trans} = n_s I_s + n_w I_w = (15)(41.6667) + (1)(500) = 1125 \text{ in}^4
\]

\[
I_\text{final} = \frac{n MY}{I}
\]

Wood: \( n = 1 \)

\[
M = \frac{(2000)(1125)}{(1)(5)} = 450 \times 10^3 \text{ lb⋅in}
\]

Steel: \( n = 15 \)

\[
M = \frac{(22,000)(1125)}{(15)(5)} = 330 \times 10^3 \text{ lb⋅in}
\]

Choose the smaller value \( M = 330 \times 10^3 \text{ lb⋅in} \)

\[M = 330 \text{ kip⋅in}\]
Problem 4.47

The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN·m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (a) the stress in the steel, (b) the maximum stress in the concrete.

\[
\begin{align*}
\eta &= \frac{E_s}{E_c} = \frac{200 \text{ GPa}}{25 \text{ GPa}} = 8.0 \\
A_s &= 4 \cdot \frac{\pi}{4} d^2 = (4)(\frac{32}{4})(32)^2 = 1.5205 \times 10^3 \text{ mm}^2 \\
nA_s &= 12.164 \times 10^3 \text{ mm}^2
\end{align*}
\]

Locate the neutral axis

\[
250 \times \frac{x}{2} - (12.164 \times 10^3)(400-x) = 0
\]

\[
125x^2 + 12.164 \times 10^3 x - 4.8657 \times 10^6 = 0
\]

Solving for \(x\)

\[
x = \frac{-12.164 \times 10^3 + \sqrt{(12.164 \times 10^3)^2 + (4)(125)(4.8657 \times 10^6)}}{2(125)}
\]

\[
x = 154.55 \text{ mm}, \quad 400 - x = 245.45 \text{ mm}
\]

\[
I = \frac{1}{3} 250 \times 3 + (12.164 \times 10^3)(400-x)^2
\]

\[
= \frac{1}{3} (250)(154.55)^3 + (12.164 \times 10^3)(45.45)^2
\]

\[
= 1.0404 \times 10^9 \text{ mm}^4 = 1.0404 \times 10^{-3} \text{ m}^4
\]

\[
\sigma = -\frac{\eta M y}{I}
\]

(a) Steel: \(y = 245.45 \text{ mm} = 0.24545 \text{ m}\)

\[
\sigma = -\frac{(8.0)(175 \times 10^3)(-0.24545)}{1.0404 \times 10^{-3}} = 330 \times 10^6 \text{ Pa} = 330 \text{ MPa}
\]

(b) Concrete: \(y = 154.55 \text{ mm} = 0.15455 \text{ m}\)

\[
\sigma = -\frac{(1.0)(175 \times 10^3)(0.15455)}{1.0404 \times 10^{-3}} = -26.0 \times 10^6 \text{ Pa} = -26.0 \text{ MPa}
\]
4.104 Solve Prob. 4.103, assuming that $t = 8$ mm.

4.103 The vertical portion of the press shown consists of a rectangular tube of wall thickness $t = 10$ mm. Knowing that the press has been tightened on wooden planks being glued together until $P = 20$ kN, determine the stress at (a) point $A$, (b) point $B$.

Rectangular cutout is $64$ mm x $44$ mm

$A = (80)(40) - (64)(44) = 1.984 \times 10^3 \text{ mm}^2$

$I = \frac{1}{12} (60)(80)^3 - \frac{1}{12} (44)(64)^3 = 1.5988 \times 10^6 \text{ mm}^4$

$c = 40$ mm = $0.004$ m

$e = 700 + 40 = 240$ mm = $0.240$ m

$M = Pe = (20 \times 10^3)(0.240) = 4.8 \times 10^3 \text{ Nm}$

(a) $\sigma_A = \frac{P}{A} - \frac{Mc}{I} = \frac{20 \times 10^3}{1.984 \times 10^3} - \frac{(4.8 \times 10^3)(0.040)}{1.5988 \times 10^6} = 130.2 \times 10^6 \text{ Pa}$

$b_A = 130.2 \text{ MPa}$

(b) $\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{20 \times 10^3}{1.984 \times 10^3} - \frac{(4.8 \times 10^3)(0.040)}{1.5988 \times 10^6} = -110.0 \times 10^6 \text{ Pa}$

$b_B = -110.0 \text{ MPa}$

Problem 4.109

4.109 Knowing that the allowable stress in section $ABD$ is 10 ksi, determine the largest force $P$ which can be applied to the bracket shown.

$A = (1.2)(0.9) = 1.08 \text{ in}^2$

$I = \frac{1}{12}(1.2)(0.9)^3 = 7.29 \times 10^{-3} \text{ in}^4$

$c = \frac{1}{2}(0.9) = 0.45 \text{ in}$

$e = 2 + 0.45 = 2.45 \text{ in}$

$\delta = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pe}{I} = PK$

$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{1.08} + \frac{(2.45)(0.45)}{7.29 \times 10^{-3}} = 16.049 \text{ in}^2$

$P = \frac{\delta}{K} = \frac{10}{16.049} = 0.623 \text{ kip}$

$P = 623 \text{ lb}$