Mechanics of Materials-ME 302
Spring Semester 2007
Homework 2

Problem 2.6

2.6 A cast-iron tube is used to support a compressive load. Knowing that $E = 69$ GPa and that the maximum allowable change in length is 0.025 %, determine (*a*) the maximum normal stress in the tube, (*b*) the minimum wall thickness for a load of 7.2 kN if the outside diameter of the tube is 50 mm.

$E = 69 \text{ GPa} = 69 \times 10^9 \text{ Pa}$

$\varepsilon = \frac{\delta}{L} = \frac{0.00025 L}{L} = 0.00025$

(a) $\sigma = E\varepsilon = (69 \times 10^9)(0.00025) = 17.25 \times 10^6 \text{ Pa}$ $\quad$ 17.25 MPa

(b) $\sigma = \frac{P}{A}$ $\quad A = \frac{P}{\sigma} = \frac{7.2 \times 10^3}{17.25 \times 10^6} = 417.39 \times 10^{-6} \text{ m}^2 = 417.39 \text{ mm}^2$

$A = \frac{\pi}{4}(d_o^2 - d_i^2)$ $\quad d_i^2 = d_o^2 - \frac{4A}{\pi} = 50^2 - \frac{(4)(417.39)}{\pi} = 1968.56 \text{ mm}^2$

$d_i = 44.368 \text{ mm}$ $\quad t = \frac{1}{2}(d_o - d_i) = \frac{1}{2}(50 - 44.368) = 2.82 \text{ mm}$

Problem 2.12

2.12 A nylon thread is to be subjected to a 2.5-lb tensile load. Knowing that $E = 0.5 \times 10^6$ psi, that the maximum allowable normal stress is 6 ksi, and that the length of the thread must not increase by more than 1%, determine the required diameter of the thread.

$\sigma = 6 \text{ ksi} = 6 \times 10^3 \text{ psi}$

Stress: $\sigma = \frac{P}{A}$ $\quad A = \frac{P}{\sigma} = \frac{2.5}{6 \times 10^3} = 416.667 \times 10^{-6} \text{ in}^2$

Deformation: $\delta = \frac{PL}{AE}$

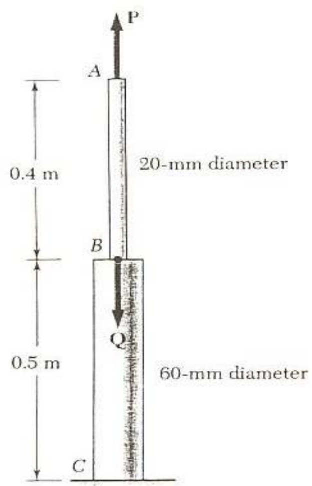
$A = \frac{P}{E}\left(\frac{L}{\delta}\right) = \left(\frac{2.5}{0.5 \times 10^6}\right)(100) = 500 \times 10^{-6} \text{ in}^2$

Larger value of A governs. $\quad A = 500 \times 10^{-6} \text{ in}^2$

$A = \frac{\pi}{4} d^2$ $\quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(500 \times 10^{-6})}{\pi}} = 25.2 \times 10^{-3} \text{ in.}$

0.0252 in.

Problem 2.17



2.17 Both portions of the rod *ABC* are made of an aluminum for which $E = 70$ GPa. Knowing that the magnitude of **P** is 4 kN, determine (*a*) the value of **Q** so that the deflection at *A* is zero, (*b*) the corresponding deflection of *B*.

(a) $A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6}\ m^2$

$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274 \times 10^{-3}\ m^2$

Force in member AB is P tension

Elongation $\delta_{AB} = \dfrac{P L_{AB}}{E A_{AB}} = \dfrac{(4\times10^3)(0.4)}{(70\times10^9)(314.16\times10^{-6})}$

$= 72.756 \times 10^{-6}$ m

Force in member BC is Q - P compression

Shortening $\delta_{BC} = \dfrac{(Q-P)L_{BC}}{E A_{BC}} = \dfrac{(Q-P)(0.5)}{(70\times10^9)(2.8274\times10^{-3})}$

$= 2.5263 \times 10^{-9}(Q-P)$

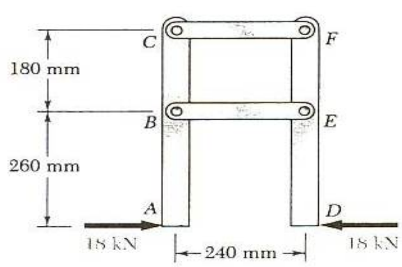
For zero deflection at A $\delta_{BC} = \delta_{AB}$

$2.5263\times10^{-9}(Q-P) = 72.756\times10^{-6}$ $\therefore$ $Q - P = 28.8\times10^3$ N

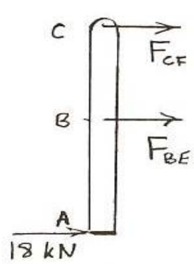
$Q = 28.3\times10^3 + 4\times10^3 = 32.8\times10^3$ N $= 32.8$ kN ◀

(b) $\delta_{AB} = \delta_{BC} = \delta_B = 72.756\times10^{-6}$ m $= 0.0728$ mm ↓ ◀

Problem 2.25



2.25 Members *ABC* and *DEF* are joined with steel links (E = 200 GPa). Each of the links is made of a pair of 25×35-mm plates. Determine the change in length of (*a*) member *BE*, (*b*) member *CF*.



Use member ABC as a free body

$\circlearrowleft \Sigma M_B = 0$

$(0.260)(18\times10^3) - (0.180)F_{CF} = 0$

$F_{CF} = \dfrac{(0.260)(18\times10^3)}{0.180} = 26\times10^3$ N

$\circlearrowleft \Sigma M_C = 0$ $(0.440)(18\times10^3) + (0.180)F_{BE} = 0$

$F_{BE} = -\dfrac{(0.440)(18\times10^3)}{0.180} = -44\times10^3$ N

Area for link made of two plates

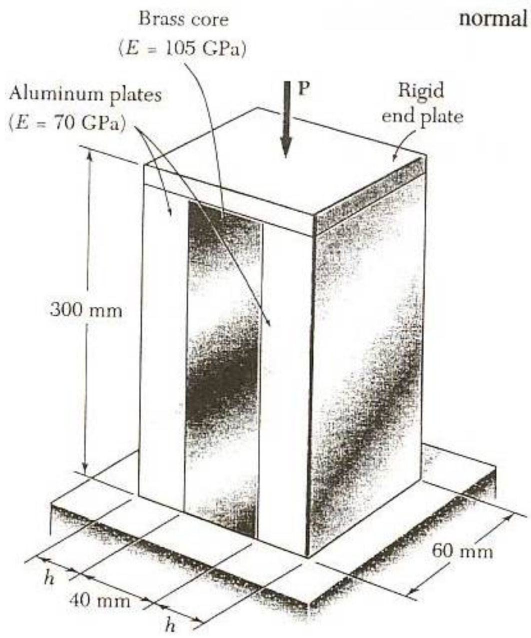
$A = (2)(0.025)(0.035) = 1.75\times10^{-3}\ m^2$

(a) $\delta_{BE} = \dfrac{F_{BE}L_{BE}}{EA} = \dfrac{(-44\times10^3)(0.240)}{(200\times10^9)(1.75\times10^{-3})} = -30.2\times10^{-6}$ m $= -0.0302$ mm ◀

(b) $\delta_{CF} = \dfrac{F_{CF}L_{CF}}{EA} = \dfrac{(26\times10^3)(0.240)}{(200\times10^9)(1.75\times10^{-3})} = 17.83\times10^{-6}$ m $= 0.01783$ mm ◀

Problem 2.35

2.35 An axial centric force of magnitude $P = 450$ kN is applied to the composite block shown by means of a rigid end plate. Knowing that $h = 10$ mm, determine the normal stress in (*a*) the brass core, (*b*) the aluminum plates.



Let P_b = portion of axial force carried by brass core

P_a = portion carried by two aluminum plates

$$\delta = \frac{P_b L}{E_b A_b} \qquad P_b = \frac{E_b A_b \delta}{L}$$

$$\delta = \frac{P_a L}{E_a A_a} \qquad P_a = \frac{E_a A_a \delta}{L}$$

$$P = P_b + P_a = (E_b A_b + E_a A_a)\frac{\delta}{L}$$

$$\varepsilon = \frac{\delta}{L} = \frac{P}{E_b A_b + E_a A_a}$$

$$A_b = (60)(40) = 2400 \text{ mm}^2 = 2400 \times 10^{-6} \text{ m}^2$$

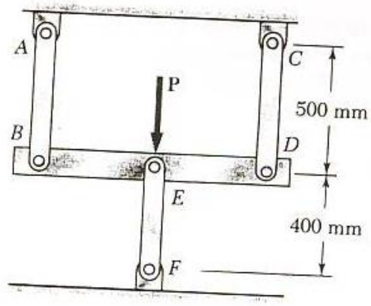
$$A_a = (2)(60)(10) = 1200 \text{ mm}^2 = 1200 \times 10^{-6} \text{ m}^2$$

$$\varepsilon = \frac{450 \times 10^3}{(105 \times 10^9)(2400 \times 10^{-6}) + (70 \times 10^9)(1200 \times 10^{-6})} = 1.3393 \times 10^{-3}$$

(a) $\sigma_b = E_b \varepsilon = (105 \times 10^9)(1.3393 \times 10^{-3}) = 140.6 \times 10^6 \text{ Pa} = 140.6 \text{ MPa}$ ◄

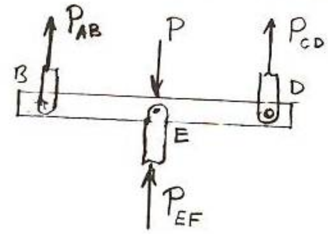
(b) $\sigma_a = E_a \varepsilon = (70 \times 10^9)(1.3393 \times 10^{-3}) = 93.75 \times 10^6 \text{ Pa} = 93.75 \text{ MPa}$ ◄

Problem 2.41



2.41 Three steel rods (E = 200 GPa) support a 36-kN load **P**. Each of the rods AB and CD has a 200-mm² cross-sectional area and rod EF has a 625- mm² cross-sectional area. Determine the (a) the change in length of rod EF, (b) the stress in each rod.

Use member BED as a free body



By symmetry, or by $\Sigma M_E = 0$

$$P_{CD} = P_{AB}$$

$$\Sigma F_y = 0$$

$$P_{AB} + P_{CD} + P_{EF} - P = 0$$

$$P = 2P_{AB} + P_{EF}$$

$$\delta_{AB} = \frac{P_{AB}L_{AB}}{EA_{AB}}, \quad \delta_{CD} = \frac{P_{CD}L_{CD}}{EA_{CD}}, \quad \delta_{EF} = \frac{P_{EF}L_{EF}}{EA_{EF}}$$

Since $L_{AB} = L_{CD}$ and $A_{AB} = A_{CD}$, $\delta_{AB} = \delta_{CD}$

Since points A, C, and E are fixed $\delta_B = \delta_{AB}$, $\delta_D = \delta_{CD}$, $\delta_E = \delta_{EF}$

Since member BED is rigid $\delta_E = \delta_B = \delta_C$

$$\frac{P_{AB}L_{AB}}{EA_{AB}} = \frac{P_{EF}L_{EF}}{EA_{EF}} \quad \therefore \quad P_{AB} = \frac{A_{AB}}{A_{EF}} \cdot \frac{L_{EF}}{L_{AB}} P_{EF} = \frac{200}{625} \cdot \frac{400}{500} P_{EF}$$

$$= 0.256\ P_{EF}$$

$$P = 2P_{AB} + P_{EF} = (2)(0.256)P_{EF} + P_{EF} = 1.512\ P_{EF}$$

$$P_{EF} = \frac{P}{1.512} = \frac{36 \times 10^3}{1.512} = 23.810 \times 10^3\ \text{N}$$

$$P_{AB} = P_{CD} = (0.256)(23.810 \times 10^3) = 6.095 \times 10^3\ \text{N}$$

(a) $$\delta = \delta_{EF} = \frac{(23.810 \times 10^3)(400 \times 10^{-3})}{(200 \times 10^9)(625 \times 10^{-6})} = 76.2 \times 10^{-6}\ \text{m}$$

$$= 0.0762\ \text{mm}$$

or

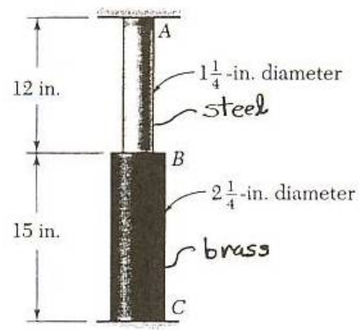
$$\delta = \delta_{AB} = \frac{(6.095 \times 10^3)(500 \times 10^{-3})}{(200 \times 10^9)(200 \times 10^{-6})} = 76.2 \times 10^{-6}\ \text{m}$$

(b) $$\sigma_{AB} = \sigma_{CD} = \frac{P_{AB}}{A_{AB}} = \frac{6.095 \times 10^3}{200 \times 10^{-6}} = 30.5 \times 10^6\ \text{Pa} = 30.5\ \text{MPa}$$

$$\sigma_{EF} = -\frac{P_{EF}}{A_{EF}} = -\frac{23.810 \times 10^3}{625 \times 10^{-6}} = -38.1 \times 10^6\ \text{Pa} = 38.1\ \text{MPa}$$

Problem 2.52

2.52 A rod consisting of two cylindrical portions AB and BC is restrained at both ends. Portion AB is made of steel ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}/°F$) and portion BC is made of brass ($E_b = 17 \times 10^6$ psi, $\alpha_b = 10.4 \times 10^{-6}/°F$). Knowing that the rod is initially unstressed, determine (*a*) the normal stresses induced in portions AB and BC by a temperature rise of 65°F, (*b*) the corresponding deflection of point B.



$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(1.25)^2 = 1.2272 \text{ in}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(2.25)^2 = 3.9761 \text{ in}^2$$

Free thermal expansion

$$\delta_T = L_{AB}\alpha_s(\Delta T) + L_{BC}\alpha_b(\Delta T)$$
$$= (12)(6.5\times10^{-6})(65) + (15)(10.4\times10^{-6})(65)$$
$$= 15.21\times10^{-3} \text{ in.}$$

Shortening due to induced compressive force P

$$\delta_P = \frac{P L_{AB}}{E_s A_{AB}} + \frac{P L_{BC}}{E_b A_{BC}}$$
$$= \frac{12\ P}{(29\times10^6)(1.2272)} + \frac{15\ P}{(17\times10^6)(3.9761)} = 559.10\times10^{-9}\ P$$

For zero net deflection $\quad \delta_P = \delta_T$

$$(559.10\times10^{-9})P = 15.21\times10^{-3}$$
$$P = 27.204\times10^3 \text{ lb.}$$

(a) $\sigma_{AB} = -\dfrac{P}{A_{AB}} = -\dfrac{27.204\times10^3}{1.2272} = -22.2\times10^3 \text{ psi} = -22.2 \text{ ksi}$

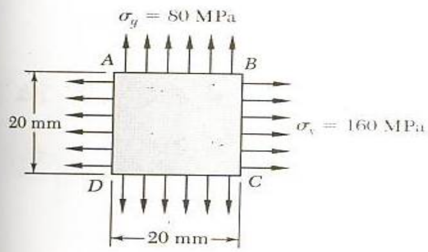
$\sigma_{BC} = -\dfrac{P}{A_{BC}} = -\dfrac{27.204\times10^3}{3.9761} = -6.84\times10^3 \text{ psi} = -6.84 \text{ ksi}$

(b) $\delta_B = +\dfrac{P L_{AB}}{E_s A_{AB}} - L_{AB}\alpha_s(\Delta T)$

$= +\dfrac{(27.204\times10^3)(12)}{(29\times10^6)(1.2272)} - (12)(6.5\times10^{-6})(65) = +4.10\times10^{-3}$ in

i.e. 4.10×10^{-3} in. ↑

Problem 2.67



2.67 A 20-mm square has been scribed on the side of a large steel pressure vessel. After pressurization, the biaxial stress condition of the square is as shown. Using the data available in Appendix B, for structural steel, determine the percent change in the slope of diagonal *DB* due to the pressurization of the vessel.

For structural steel Appendix B gives

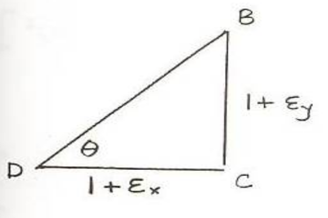
$E = 200$ GPa, $\quad G = 77.2$ GPa

$$G = \frac{E}{2(1+\nu)}$$

$$\nu = \frac{E}{2G} - 1 = \frac{200}{(2)(77.2)} - 1 = 0.2953$$

$\sigma_x = 160 \times 10^6$ Pa $\qquad \sigma_y = 80 \times 10^6$ Pa

$\sigma_z \approx 0$



$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$
$$= \frac{1}{200\times10^9}\left[160\times10^6 - (0.2953)(80\times10^6)\right]$$
$$= 0.00068187$$

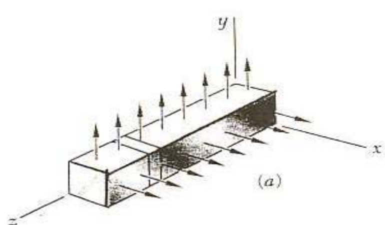
$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z)$$
$$= \frac{1}{200\times10^9}\left[-(0.2953)(160\times10^6) + 80\times10^6\right]$$
$$= 0.00016373$$

Slope of DB $\qquad \tan\theta = \dfrac{1+\varepsilon_y}{1+\varepsilon_x} = \dfrac{1 + 0.00016373}{1 + 0.00068187} = 0.9994822$

Change in slope $\qquad 0.999482 - 1 = -0.000518$

$$\frac{\Delta \tan\theta}{\tan 45^\circ} \times 100\% = -0.0518\ \%$$

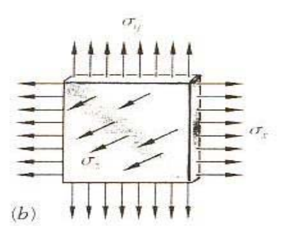
Problem 2.73



2.73 In many situations physical constraints prevent strain from occurring in a given direction, for example $\varepsilon_z = 0$ in the case shown, where longitudinal movement of the long prism is prevented at every point. Plane sections perpendicular to the longitudinal axis remain plane and the same distance apart. Show that for the this situation, which is known as *plane strain*, we can express σ_z, ε_x and ε_y as follows:

$$\sigma_z = \nu(\sigma_x + \sigma_y)$$

$$\varepsilon_x = \frac{1}{E}[(1-\nu^2)\sigma_x - \nu(1+\nu)\sigma_y] \qquad \varepsilon_y = \frac{1}{E}[(1-\nu^2)\sigma_y - \nu(1+\nu)\sigma_x]$$



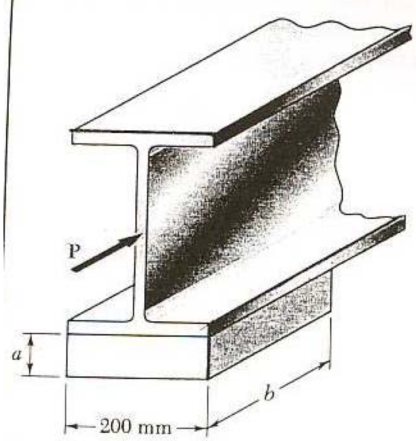
$$\varepsilon_z = 0 = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) \quad \text{or} \quad \sigma_z = \nu(\sigma_x + \sigma_y)$$

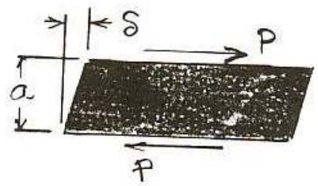
$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{1}{E}\left[\sigma_x - \nu\sigma_y - \nu^2(\sigma_x + \sigma_y)\right]$$
$$= \frac{1}{E}\left[(1-\nu^2)\sigma_x - \nu(1+\nu)\sigma_y\right]$$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \frac{1}{E}\left[-\nu\sigma_x + \sigma_y - \nu^2(\sigma_x + \sigma_y)\right]$$
$$= \frac{1}{E}\left[(1-\nu^2)\sigma_y - \nu(1+\nu)\sigma_x\right]$$

Problem 2.79

2.79 An elastomeric bearing ($G = 0.9$ MPa) is used to support a bridge girder as shown to provide flexibility during earthquakes. The beam must not displace more than 10 mm when a 22 kN lateral load is applied as shown. Knowing that the maximum allowable shearing stress is 420 kPa, determine (*a*) the smallest allowable dimension *b*, (*b*) the smallest required thickness *a*.





Shearing force $P = 22 \times 10^3$ N

Shearing stress $\tau = 420 \times 10^3$ Pa

$$\tau = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\tau} = \frac{22 \times 10^3}{420 \times 10^3} = 52.381 \times 10^{-3}\ m^2$$

$$= 52.381 \times 10^3\ mm^2$$

$$A = (200\ mm)(b)$$

$$b = \frac{A}{200} = \frac{52.381 \times 10^3}{200} = 262\ mm$$

$$\gamma = \frac{\tau}{G} = \frac{420 \times 10^3}{0.9 \times 10^6} = 466.67 \times 10^{-3}$$

But $\gamma = \frac{\delta}{a} \quad \therefore \quad a = \frac{\delta}{\gamma} = \frac{10\ mm}{466.67 \times 10^{-3}} = 21.4\ mm$