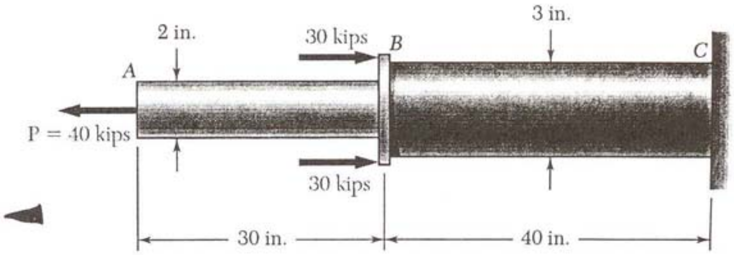
Mechanics of Materials-ME 302
Spring Semester 2007
Homework Solution 1

Problem 1.3

1.3 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Determine the average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.



(a) Rod AB.

$P = 40$ kips (tension)

$A_{AB} = \frac{\pi d_{AB}^2}{4} = \frac{\pi (2)^2}{4} = 3.1416 \text{ in}^2$

$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{40}{3.1416} = 12.73 \text{ ksi}$

(b) Rod BC.

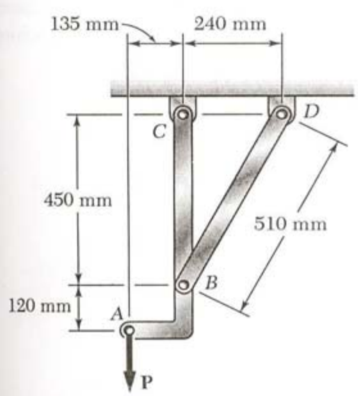
$F = 40 - (2)(30) = -20$ kips. i.e. 20 kips compression.

$A_{BC} = \frac{\pi d_{BC}^2}{4} = \frac{\pi (3)^2}{4} = 7.0686 \text{ in}^2$

$\sigma_{BC} = \frac{F}{A_{BC}} = \frac{-20}{7.0686} = -2.83 \text{ ksi}$

Problem 1.7

1.7 Knowing that the central portion of the link *BD* has a uniform cross-sectional area of 800 mm², determine the magnitude of the load **P** for which the normal stress in that portion of *BD* is 50 MPa.

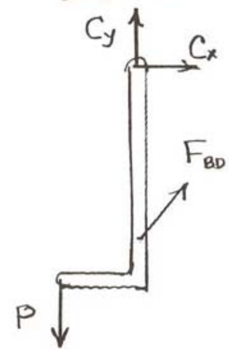


$\sigma_{BD} = 50 \text{ MPa} = 50 \times 10^6 \text{ Pa}$

$A_{BD} = 800 \text{ mm}^2 = 800 \times 10^{-6} \text{ m}^2$

$F_{BD} = \sigma_{BD} A_{BD} = (50 \times 10^6)(800 \times 10^{-6}) = 40 \times 10^3 \text{ N}$

Draw free body diagram of body ABD.



$+\circlearrowleft M_C = 0$

$(0.450)\left(\frac{240}{510} F_{BD}\right) - 0.135 P = 0$

$P = 1.5686 F_{BD}$

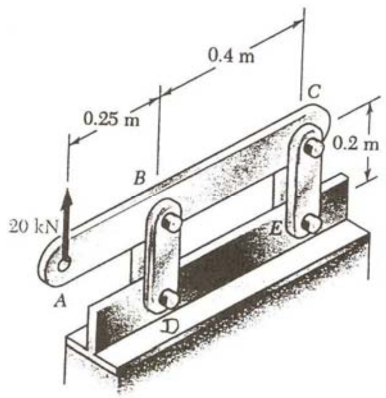
$P = (1.5686)(40 \times 10^3)$

$= 62.7 \times 10^3 \text{ N}$

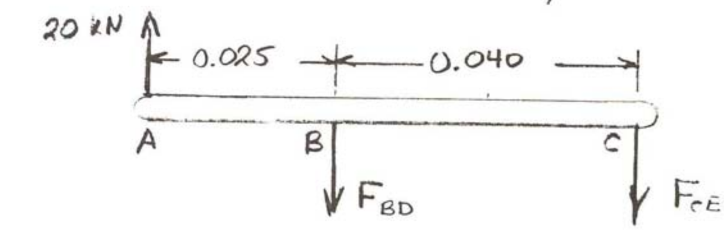
62.7 kN

Problem 1.9

1.9 Each of the four vertical links has an 8 × 36- mm uniform rectangular cross section and each of the four pins has a 16- mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.



Use bar ABC as a free body.



$\Sigma M_C = 0 \qquad (0.040) F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$

$F_{BD} = 32.5 \times 10^3 \text{ N}$ Link BD is in tension

$\Sigma M_B = 0 \qquad -(0.040) F_{CE} - (0.025)(20 \times 10^3) = 0$

$F_{CE} = -12.5 \times 10^3 \text{ N}$ Link CE is in compression

Net area of one link for tension $= (0.008)(0.036 - 0.016)$

$= 160 \times 10^{-6} \text{ m}^2$. For two parallel links $A_{net} = 320 \times 10^{-6} \text{ m}^2$

Tensile stress in link BD

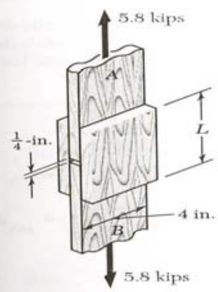
(a) $\sigma_{BD} = \dfrac{F_{BD}}{A_{net}} = \dfrac{32.5 \times 10^3}{320 \times 10^{-6}} = 101.56 \times 10^6$ or 101.6 MPa

Area for one link in compression $= (0.008)(0.036)$

$= 288 \times 10^{-6} \text{ m}^2$. For two parallel links $A = 576 \times 10^{-6} \text{ m}^2$

(b) $\sigma_{CE} = \dfrac{F_{CE}}{A} = \dfrac{-12.5 \times 10^3}{576 \times 10^{-6}} = -21.70 \times 10^6$ or -21.7 MPa

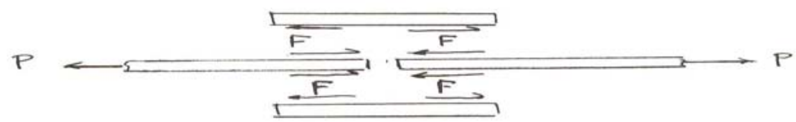
Problem 1.16



1.16 The wooden members A and B are to be joined by plywood splice plates which will be fully glued on the surfaces in contact. As part of the design of the joint, and knowing that the clearance between the ends of the members is to be $\frac{1}{4}$ in., determine the smallest allowable length L if the average shearing stress in the glue is not to exceed 120 psi.

There are four separate areas that are glued. Each of these areas transmits one half the the 5.8 kip force. Thus

$$F = \tfrac{1}{2}P = \tfrac{1}{2}(5.8) = 2.9 \text{ kips} = 2900 \text{ lb.}$$



Let ℓ = length of one glued area and $w = 4$ in. be its width.

For each glued area $\quad A = \ell w$

Average shearing stress $\quad \tau = \dfrac{F}{A} = \dfrac{F}{\ell w}$

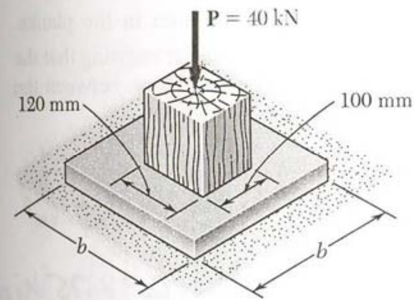
The allowable shearing stress is $\quad \tau = 120$ psi

Solving for ℓ $\quad \ell = \dfrac{F}{\tau w} = \dfrac{2900}{(120)(4)} = 6.0417$ in.

Total length L $\quad L = \ell + (\text{gap}) + \ell = 6.0417 + \frac{1}{4} + 6.0417$

$= 12.33$ in.

Problem 1.20



1.20 A 40-kN axial load is applied to a short wooden post that is supported by a concrete footing resting on undisturbed soil. Determine (*a*) the maximum bearing stress on the concrete footing, (*b*) the size of the footing for which the average bearing stress in the soil is 145 kPa.

(a) Bearing stress on concrete footing.

$$P = 40 \text{ kN} = 40 \times 10^3 \text{ N}$$

$$A = (100)(120) = 12 \times 10^3 \text{ mm}^2 = 12 \times 10^{-3} \text{ m}^2$$

$$\sigma = \frac{P}{A} = \frac{40 \times 10^3}{12 \times 10^{-3}} = 3.33 \times 10^6 \text{ Pa}$$

3.33 MPa

(b) Footing area.

$$P = 40 \times 10^3 \text{ N} \qquad \sigma = 145 \text{ kPa} = 45 \times 10^3 \text{ Pa}$$

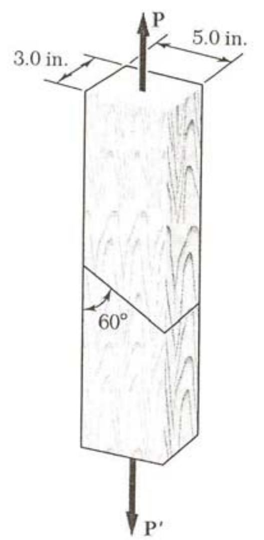
$$\sigma = \frac{P}{A} \qquad A = \frac{P}{\sigma} = \frac{40 \times 10^3}{145 \times 10^3} = 0.27586 \text{ m}^2$$

Since the area is square $A = b^2$

$$b = \sqrt{A} = \sqrt{0.27586} = 0.525 \text{ m}$$

$b = 525$ mm

Problem 1.31



1.31 The 1.4 kip load **P** is supported by two wooden members of uniform cross section that are joined by the simple glued scarf splice shown. Determine the normal and shearing stresses in the glued splice.

$P = 1400 \text{ lb} \qquad \theta = 90° - 60° = 30°$

$A_o = (5.0)(3.0) = 15 \text{ in}^2$

$$\sigma = \frac{P\cos^2\theta}{A_o} = \frac{(1400)(\cos 30°)^2}{15}$$

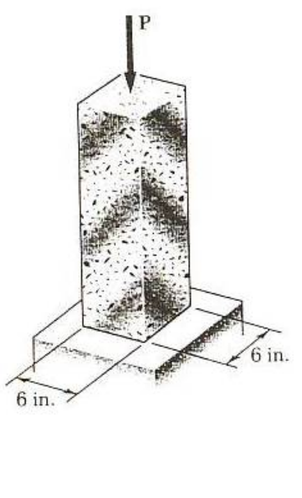
$\sigma = 70.0 \text{ psi}$

$$\tau = \frac{P\sin 2\theta}{2A_o} = \frac{(1400)\sin 60°}{(2)(15)}$$

$\tau = 40.4 \text{ psi}$

Problem 1.35

1.35 A 240-kip load **P** is applied to the granite block shown. Determine the resulting maximum value of (*a*) the normal stress, (*b*) the shearing stress. Specify the orientation of the plane on which each of these maximum values occurs.



$A_o = (6)(6) = 36 \text{ in}^2$

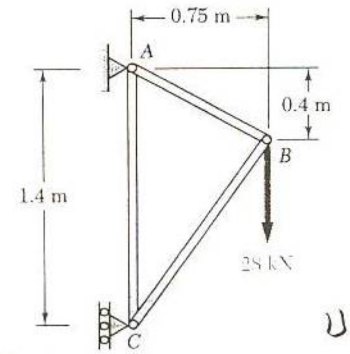
$$\sigma = \frac{P}{A_o}\cos^2\theta = \frac{-240}{36}\cos^2\theta = -6.67\cos^2\theta$$

(a) max tensile stress $= 0$ at $\theta = 90°$

max. compressive stress $= 6.67$ ksi at $\theta = 0°$

(b) $$\tau_{max} = \frac{P}{2A_o} = \frac{240}{(2)(36)} = 3.33 \text{ ksi}$$ at $\theta = 45°$

Problem 1.41

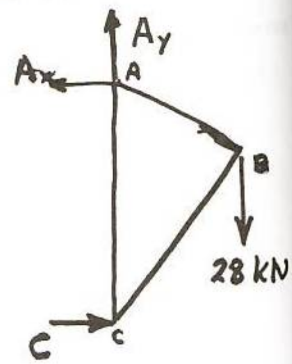


1.41 Members *AB* and *BC* of the truss shown are made of the same alloy. It is known that a 20-mm-square bar of the same alloy was tested to failure and that an ultimate load of 120 kN was recorded. If a factor of safety of 3.2 is to be achieved for both bars, determine the required cross- sectional area of (*a*) bar *AB*, (*b*) bar *AC*.

Length of member AB

$$l_{AB} = \sqrt{0.75^2 + 0.4^2} = 0.85 \text{ m}$$

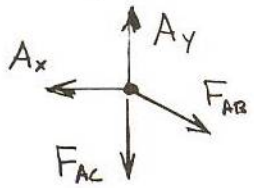
Use entire truss as a free body



$\circlearrowleft \sum M_c = 0 \qquad 1.4 A_x - (0.75)(28) = 0 \qquad A_x = 15 \text{ kN}$

$+\uparrow \sum F_y = 0 \qquad A_y - 28 = 0 \qquad A_y = 28 \text{ kN}$

Use joint A as free body



$\xrightarrow{+} \sum F_x = 0 \qquad \frac{0.75}{0.85} F_{AB} - A_x = 0$

$$F_{AB} = \frac{(0.85)(15)}{0.75} = 17 \text{ kN}$$

$+\uparrow \sum F_y = 0 \qquad A_y - F_{AC} - \frac{0.4}{0.85} F_{AB} = 0$

$$F_{AC} = 28 - \frac{(0.4)(17)}{0.85} = 20 \text{ kN}$$

For the test bar $\quad A = (0.020)^2 = 400 \times 10^{-6} \text{ m}^2 \qquad P_U = 120 \times 10^3 \text{ N}$

For the material $\quad \sigma_U = \frac{P_U}{A} = \frac{120 \times 10^3}{400 \times 10^{-6}} = 300 \times 10^6 \text{ Pa}$

(a) For member AB $\quad \text{F.S.} = \frac{P_U}{F_{AB}} = \frac{\sigma_U A_{AB}}{F_{AB}}$

$$A_{AB} = \frac{(\text{F.S.}) F_{AB}}{\sigma_U} = \frac{(3.2)(17 \times 10^3)}{300 \times 10^6} = 181.33 \times 10^{-6} \text{ m}^2$$

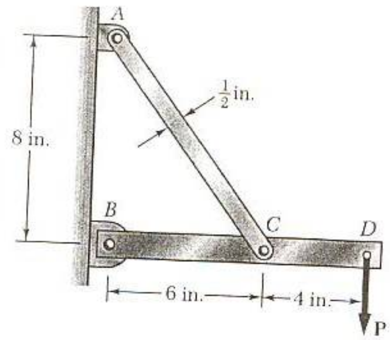
$$A_{AB} = 181.3 \text{ mm}^2 \blacktriangleleft$$

(b) For member AC $\quad \text{F.S.} = \frac{P_U}{F_{AC}} = \frac{\sigma_U A_{AC}}{F_{AC}}$

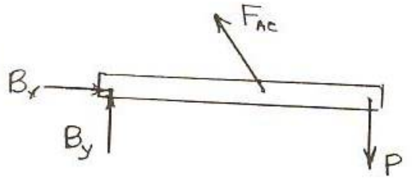
$$A_{AC} = \frac{(\text{F.S.}) F_{AC}}{\sigma_U} = \frac{(3.2)(20 \times 10^3)}{300 \times 10^6} = 213.33 \times 10^{-6} \text{ m}^2$$

$$A_{AC} = 213 \text{ mm}^2 \blacktriangleleft$$

Problem 1.51



1.51 Link AC is made of a steel with a 65-ksi ultimate normal stress and has a $\frac{1}{4} \times \frac{1}{2}$-in. uniform rectangular cross section. It is connected to a support at A and to member BCD at C by $\frac{3}{8}$-in.-diameter pins, while member BCD is connected to its support at B by a $\frac{5}{16}$-in.-diameter pin; all of the pins are made of a steel with a 25-ksi ultimate shearing stress and are in single shear. Knowing that a factor of safety of 3.25 is desired, determine the largest load **P** that can be applied at D. Note that link AC is not reinforced around the pin holes.



Use free body BCD.

$+\circlearrowleft M_B = 0: \ (6)(\frac{8}{10}F_{AC}) - 10\,P = 0$

$P = 0.48\ F_{AC}$ (1)

$\pm \Sigma F_x = 0: \ B_x - \frac{6}{10}F_{AC} = 0$

$B_x = \frac{6}{10}F_{AC} = 1.25P \rightarrow$

$+\circlearrowleft M_c = 0: \ -6B_y - 4P = 0$

$B_y = -\frac{2}{3}P$ i.e. $B_y = \frac{2}{3}P \downarrow$

$B = \sqrt{B_x^2 + B_y^2} = \sqrt{1.25^2 + (\frac{2}{3})^2}\ P = 1.41667\,P$ $\qquad P = 0.70588\,B$ (2)

Shear in pins at A and C.

$F_{AC} = \tau\ A_{pin} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = (\frac{25}{3.25})(\frac{\pi}{4})(\frac{3}{8})^2 = 0.84959$ kips

Tension on net section at A and C.

$F_{AC} = \sigma\ A_{net} = \frac{\sigma_U}{F.S.}A_{net} = (\frac{65}{3.25})(\frac{1}{4})(\frac{1}{2} - \frac{3}{8}) = 0.625$ kips

Smaller value of F_{AC} is 0.625 kips.

From (1) $\quad P = (0.48)(0.625) = 0.300$ kips

Shear in pin at B.

$B = \tau A_{pin} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = (\frac{25}{3.25})(\frac{\pi}{4})(\frac{5}{16})^2 = 0.58999$ kips

From (2) $\quad P = (0.70588)(0.58999) = 0.416$ kips

Allowable value of P is the smaller value. $\qquad P = 0.300$ kips

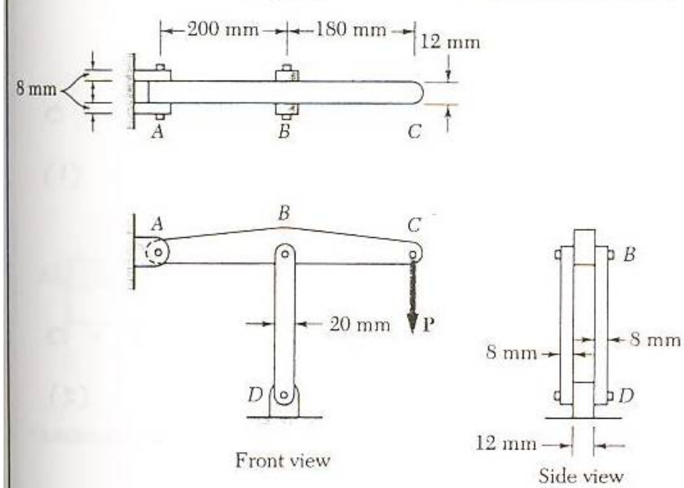
or $P = 300$ lb

Problem 1.54

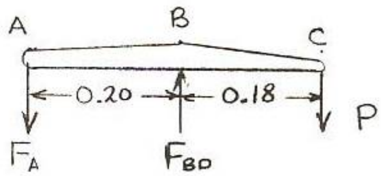
1.54 In an alternative design for the structure of Prob. 1.53, a pin of 10-mm-diameter is to be used at A. Assuming that all other specifications remain unchanged, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

1.53 In the structure shown, an 8-mm-diameter pin is used at A, and 12-mm-diameter pins are used at B and D. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining B and D, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

Top view



Statics: Use ABC as free body.



$\Sigma M_B = 0 \qquad 0.20\, F_A - 0.18\, P = 0$

$P = \frac{10}{9} F_A$

$\Sigma M_A = 0 \qquad 0.20\, F_{BD} - 0.38\, P = 0$

$P = \frac{10}{19} F_{BD}$

Based on double shear in pin A

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.010)^2 = 78.54 \times 10^{-6}\ m^2$

$F_A = \frac{2\tau_U A}{F.S.} = \frac{(2)(100 \times 10^6)(78.54 \times 10^{-6})}{3.0} = 5.236 \times 10^3\ N$

$P = \frac{10}{9} F_A = 5.82 \times 10^3\ N$

Based on double shear in pins at B and D

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.012)^2 = 113.10 \times 10^{-6}\ m^2$

$F_{BD} = \frac{2\tau_U A}{F.S.} = \frac{(2)(100 \times 10^6)(113.10 \times 10^{-6})}{3.0} = 7.54 \times 10^3\ N$

$P = \frac{10}{19} F_{BD} = 3.97 \times 10^3\ N$

Based on compression in links BD

For one link $\quad A = (0.020)(0.008) = 160 \times 10^{-6}\ m^2$

$F_{BD} = \frac{2\sigma_U A}{F.S.} = \frac{(2)(250 \times 10^6)(160 \times 10^{-6})}{3.0} = 26.7 \times 10^3\ N$

$P = \frac{10}{19} F_{BD} = 14.04 \times 10^3\ N$

Allowable value of P is smallest $\therefore \quad P = 3.97 \times 10^3\ N$

3.97 kN